Response verification book

tsm

Seeing through mathematics, Book 2
special edition

Van Engen Harding Trimble Berger Cleveland Freeman

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Response verification book

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Seeing through mathematics
special edition

Book 2

W. J. Gage Limited
Toronto

How to use this response verification book

As you work through *Seeing Through Mathematics, Book 2*, you will notice two types of exercises, developmental and practice. The developmental exercises, identified by letters, make up the greater part of each lesson and have been designed to guide your thinking. The practice exercises, identified by numerals, have been designed to help you reinforce and maintain the ideas you have studied.

Your teacher will decide how this response book should be used. He may wish to use the developmental exercises for class discussion, in which case you may not need this response book. However, he may want you to study the developmental exercises by yourself. If you proceed in this way, you should make every effort to respond to each exercise clearly and completely before using this book to verify your response.

The practice exercises are entitled "On your own," "Keeping skilful," and "Checking up," and are designed for written work. Your teacher may want you to complete an assigned block of work before verifying any responses.

If your teacher directs you to verify a response at a time, it is a good idea to cover all the responses in the column that are below the response you are concerned with at the moment. It is very difficult to avoid seeing other responses unless you use some means to conceal them. The picture shows one way to prevent unintentional "peeking."

Responses to the work entitled "Special challenge" are not included in this book. If you need help with these exercises, consult your teacher.

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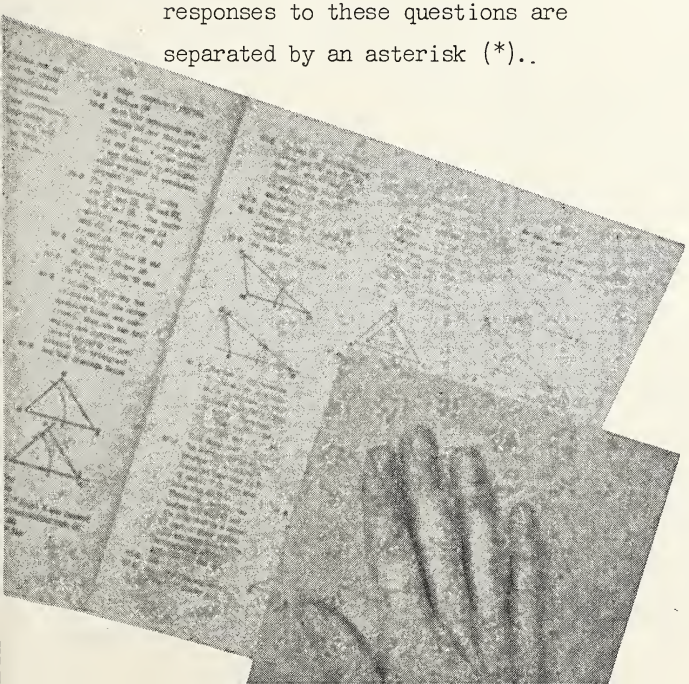
Responses are keyed to the student's book by page numeral and exercise letter or numeral. For example, the response for exercise K on page 6 is labeled *6*-**K;** the response for the "On your own" exercise 3 on page 8 is labelled *8*-**3.** At the bottom of this page, you will see the words "Responses for pages 5-6." These words indicate that this page includes the responses for pages 5 and 6 of the student's book. Each page of this key is labelled in this way. Also observe that when an exercise includes two or more questions, the responses to these questions are separated by an asterisk (*)..



Responses for Seeing Through Mathematics, Book 2

Lesson 98

5-A 0 * 1

5-B 2 * 3

5-C 0 * $\frac{5}{2}$ * $\frac{7}{2}$

5-D C and F * Yes

5-E The coördinate of C is $\frac{3}{2}$. The coördinate of F is 3.

5-F Yes

5-G $1\frac{1}{2}$ * $1\frac{1}{2}$ * $2\frac{1}{2}$ * 3 * First find the coördinates of the endpoints of the segment. Then find the difference of the greater coördinate and the lesser coördinate.

5-H $\frac{1}{2}$ * You should name three of the following: $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{FG}$.

5-I To find the measure of a segment, you first find the coördinates of the endpoints of the segment and then subtract the lesser coördinate from the greater coördinate. Therefore, $m(\overline{AC})$ is the difference of the same two numbers as $m(\overline{CA})$.

5-J Yes * Yes

6-K 1 * 1 * Yes

6-L $m(\overline{BF}) = 2$ and $m(\overline{CG}) = 2$. Since the measures of $\overline{BF}$ and $\overline{CG}$ are equal, $\overline{BF}$ is congruent to $\overline{CG}$. * $m(\overline{CG}) = 2$ and $m(\overline{AD}) = 2$. Since the measures of $\overline{CG}$ and $\overline{AD}$ are equal, $\overline{CG}$ is congruent to $\overline{AD}$.

6-M Yes * If two segments

773198

are congruent, then their measures are equal.

6-N Yes * $\overline{QR}$ is less than $\overline{QS}$.

6-O $\frac{1}{4}$ * $\frac{1}{2}$

6-P Yes

6-Q No * $\overline{QT}$ is greater than $\overline{QS}$.

6-R $\frac{3}{4}$ * $\frac{1}{2}$ * Yes

6-S $\frac{1}{2}$ * $\frac{3}{4}$ * Yes * Yes

6-T $\frac{3}{4}$ * $\frac{1}{2}$ * Yes * Yes

6-U Yes * $m(\overline{QS}) = \frac{1}{2}$ and $m(\overline{UX}) = \frac{3}{4}$. Since $m(\overline{QS}) < m(\overline{UX})$, $\overline{QS} < \overline{UX}$.

6-A 0 * 1 * 1

6-B 1 * Yes * If the measures of two segments are equal, then the segments are congruent.

6-C The measure of $\overline{IL}$ is 1. * Segments JM and LO

7-D 0 * 1

7-E The coördinate of point D is 2, of point E is 3, of point F is 4, and of point G is 5.

7-F Since $\overline{AC}$ is congruent to $\overline{MN}$ and $\overline{MN}$ is a unit segment, $\overline{AC}$ is also a unit segment because congruent segments have equal measures. Since each of segments CD, DE, EF, and FG is congruent to $\overline{AC}$, each of these segments is a unit segment.

7-G 4 * 5

7-H No * No * Each number is assigned to only one point. Since 4 is assigned to F and 5 to G, neither 4 nor 5 can be assigned to B.

7-I Yes. Since F is between A and B, $\overline{AB} > \overline{AF}$. This means that $m(\overline{AB}) > m(\overline{AF})$. * Yes. Since $\overline{AB} < \overline{AG}$, $m(\overline{AB}) < m(\overline{AG})$.

7-J $\overline{AB}$ * $m(\overline{AB})$

7-K Yes * 4 + .3, or 4.3

7-L 4.3

7-M $4\frac{2}{8}$ * $4\frac{2}{8}$

7-N Keep on subdividing the segment that contains B into congruent segments until B is an endpoint of a segment.

8-O 3

8-P 1.8

8-Q The unit segment used for exercise O is not congruent to the unit segment used for exercise P.

On your own

8-1 $\frac{2}{3}$

8-2 $8\frac{1}{6}$

8-3 8.2

8-4 $\frac{1}{16}$

8-5 $5\frac{3}{8}$

8-6 $\frac{1}{5}$

8-7 4.73

8-8 10

8-9 $6\frac{3}{10}$

8-10 $\frac{1}{2}$ and $\frac{5}{3}$ * 2 and $\frac{5}{2}$

8-11 3

8-12 $\frac{1}{4}$

8-13 $\frac{2}{3}$ * $\frac{1}{2}$ * $1\frac{1}{4}$

8-14 You should name two of the following: $\overline{MO}$, $\overline{NP}$, $\overline{OQ}$, $\overline{UV}$.

8-15 $\overline{MN}$, $\overline{NO}$, and $\overline{PQ}$

8-16 $1\frac{1}{4}$

8-17 $\frac{5}{6}$

8-18 $1\frac{1}{6}$

8-19 $1\frac{1}{4}$

8-20 Yes

8-21 $m(\overline{OQ})$

8-22 No

8-23 $\overline{MV}$

8-24 2 * 4 * 6

8-25 2 * 1 * $1\frac{1}{2}$

8-26 $\frac{1}{2}$ * $1\frac{1}{2}$ * $\frac{1}{4}$

8-27 4 * 12 * 6

8-28 $\frac{1}{3}$ * $\frac{2}{3}$ * $\frac{1}{6}$

8-29 4 * 12

8-30 1

Lesson 99

9-A All the students would not get the same measure. * One student may find that the board has a measure of 20 and another may find the board has a measure of 18. The reason is that different pencils are used.

9-B No * Again, the reason for the difference is that shoes of different sizes are used.

9-C Point O

9-D 1 inch

10-E 16

10-F Infinitely many * No

10-G $2\frac{3}{16}$

10-H $1\frac{3}{16}$ * The ends of the wire are directly above the marks 1 and $2\frac{3}{16}$. The difference of $2\frac{3}{16}$ and 1 is $1\frac{3}{16}$.

10-I $\frac{1}{16}$ of an inch * Yes

10-A ---

10-B 10 * .1 of a foot

11-C The left end of rod A is directly above the left end of the rule. So the left end of the rod suggests a point whose coördinate is 0. In the same way, the right end of rod A suggests point 2.6. So the measure of rod A is 2.6 − 0, or 2.6. * 2.6 feet

11-D The left end of rod B suggests a point whose coördinate is 0 and the

right end suggests a point whose coördinate is 2.9. The measure of rod B is 2.9 − 0, or 2.9. * 2.9 feet

11-E Yes * 5.5 * 5.5 feet

11-F 5.5 * Yes

11-G $6\frac{1}{6}$ * $6\frac{1}{6}$ yd.

11-H $11\frac{1}{2}$ ft.

11-I 9.6 in.

11-J $12\frac{1}{4}$ mi.

11-K 11.05 ft.

11-L $13\frac{23}{4}$ yd.

11-M 13.75 in.

11-N $3\frac{1}{6}$ yd.

11-O $8\frac{11}{16}$ in.

11-P $2\frac{7}{12}$ ft.

11-Q 81.50 mi.

11-R $5\frac{3}{8}$ yd.

11-S 1.5 in.

On your own

11-I $\frac{1}{8}$ of a yard

12-2 1 * $3\frac{1}{2}$

12-3 $2\frac{1}{2}$ * $2\frac{1}{2}$ yd.

12-4 $3\frac{3}{8}$ yd.

12-5 $1\frac{1}{8}$ yd.

12-6 $3\frac{3}{4}$ yd.

12-7 $10\frac{3}{4}$ * $10\frac{3}{4}$ yd.

12-8 $1\frac{1}{4}$ yd.

12-9 $\frac{3}{8}$ yd.

12-10 $5\frac{7}{8}$ yd.

12-11 $4\frac{7}{8}$ yd.

12-12 1 yd.

12-13 $8\frac{5}{8}$ in.

12-14 $19\frac{7}{12}$ yd.

12-15 21.8 mi.

12-16 $17\frac{1}{6}$ ft.

12-17 $11\frac{5}{8}$ mi.

12-18 $\frac{5}{6}$ yd.

12-19 4.6 ft.

12-20 $\frac{7}{16}$ in.

12-21 1.7 mi.

12-22 1 ft.

12-23 $1\frac{11}{12}$ yd.

12-24 $\frac{3}{4}$ rd.

12-25 $8\frac{3}{10}$ mi.

12-26 11.5 in.

12-27 $13\frac{1}{3}$ ft.

Keeping Skilful

12-1 10.565

12-2 $19\frac{1}{4}$

12-3 $84\frac{2}{7}$

12-4 6.588

12-5 11.502

12-6 39

12-7 1.814

12-8 $103\frac{9}{10}$

12-9 $16\frac{7}{24}$

12-10 $1\frac{5}{27}$

12-11 8

12-12 1.2225

12-13 $67\frac{1}{10}$

12-14 $19\frac{10}{21}$

12-15 6.664

12-16 77.806

12-17 $6\frac{3}{11}$

12-18 76.0032

12-19 2

12-20 $782\frac{5}{18}$

12-21 $\frac{63}{226}$

12-22 $92\frac{14}{45}$

12-23 7.766

12-24 $348\frac{5}{24}$

12-25 .78

12-26 1.367

12-27 306

12-28 4257.4

Lesson 100

13-A 36 inches. 3 feet * $16\frac{1}{2}$ feet. $5\frac{1}{2}$ yards * 5280 feet. 1760 yards

13-B 36/1

13-C 3/1 * $16\frac{1}{2}/1$ * 5280/1

13-D $5\frac{1}{2}/1$ * 1760/1

13-E The rate pair 3/1 represents the comparison of the number of feet in a yard with 1 yard.

13-F 2150 refers to the number of yards that is the length of the bridge. Since the second component of 3/1 refers to a number of yards, 516 is the second component of the rate pair whose first component you are to find. * The number of feet that is the length of the bridge

13-G Yes

13-H {1548} * The length of the Lions Gate Bridge is 1548 ft.

14-A (Column 1) .394 in. * 1.094 yd. * .621 mi.

14-B (Column 2, top) The rate pair 1.094/1 represents the comparison of the number of yards in a metre with 1 metre.

14-C 100 refers to the number of metres that the man has run. Since the second component of 1.094/1 refers to a number of metres, 100 is the second component of a rate pair whose first component you are to find. * The number of yards a man has run

14-D Yes

14-E {109.4} * When a man has completed the 100-metre run, he has run approximately 109.4 yd.

14-F $.394/1 \sim 1/x$. * One inch is the same length as approximately 2.54 centimetres.

14-A (Column 2) The rate pair $5\frac{1}{2}/1$ represents the comparison of the number of yards in a rod with 1 rod. * 1650 refers to the number of rods that is the

length of the east boundary. Since the second component of $5\frac{1}{2}/1$ refers to a number of rods, 1650 is the second component of a rate pair whose first component you are to find. * The number of yards that is the same length as 1650 rd.

14-B (Column 2, bottom) Yes

15-C Yes * You want to find the number of miles that is the same length as x yards.

15-D The number of yards that is the same length as 1650 rd. * The number of miles that is the same length as x yards

15-E Yes

15-F Yes

15-G 9075

15-H Yes

15-I 5.2

15-J 5.2 mi.

On your own

15-1 $\frac{1}{12}$ ft.

15-2 $\frac{1}{3}$ yd.

15-3 110 yd.

15-4 20.488 in.

15-5 3.5 mi.

15-6 164.1 yd.

15-7 10,560 yd.

15-8 217.35 mi.

15-9 .91 metre (nearer hundredth)

15-10 1.61 kilometres (nearer hundredth)

15-11 114.21 centimetres (nearer hundredth)

15-12 379.34 metres (nearer hundredth)

15-13 x, number of feet that is the average height of the sequoia.
$2\frac{3}{4}/1 \sim x/100$.
275 ft.

15-14 x, number of yards of cloth Mrs. Mills bought.
$1.094/1 \sim x/9$.
9.8 yd.

15-15 x, number of inches the diameter is.
$.394/1 \sim x/40$.
15.76 in.

15-16 x, number of miles the surface of the Dead Sea is below sea level.
$5280/1 \sim 1298/x$.
.24 mi.

15-17 x, number of feet that is the height of Angel Falls.
$5280/1 \sim x/.625$.
3300 ft.

16-18 x, number of feet that was the length of the shark.
$12/1 \sim 163/x$.
13.6 ft.

16-19 x, number of kilometres a man had run.
$.621/1 \sim 5/x$.
8 kilometres

16-20 x, number of miles the summit of Mount Everest is above sea level.
$5280/1 \sim 29028/x$.
5.5 mi.

16-21 x, number of feet that is the average depth of the Pacific Ocean.
$5280/1 \sim x/2\frac{2}{3}$.
14,080 ft.

16-22 x, number of feet that was the total length of the two boards.
$x = 1\frac{3}{4} + 2\frac{5}{12}$.
$4\frac{1}{6}$ ft.

16-23 x, number of inches that was the total length of the two boards.
$12/1 \sim x/4\frac{1}{6}$.
50 in.

16-24 x, total number of yards of material Jean bought.
y, total number of inches of material she bought.
$x = 2\frac{3}{4} + 1\frac{1}{8} \wedge 36/1 \sim y/x$.
139.5 in.

16-25 x, number of yards that is the length of the road.
y, number of rods that is the length of the road.
$1760/1 \sim x/\frac{4}{5} \wedge 5\frac{1}{2}/1 \sim x/y$.
256 rd.

16-26 x, number of yards that was the length of the tract of land.
y, number of miles that was the length of the tract of land.
$5\frac{1}{2}/1 \sim x/750 \wedge$
$1760/1 \sim x/y$.
2.34 mi.

16-27 x, number of feet Susan walks to school.
y, number of inches she walks to school.
$5280/1 \sim x/\frac{7}{8} \wedge 12/1 \sim y/x$.
55,440 in.

Applying mathematics

16-1 x, number of ounces Rajah weighs.
$16/1 \sim x/6\frac{1}{2}$.
104 oz.

16-2 x, number of bushels of apples Ed sold.
$4/1 \sim 35/x$.
$8\frac{3}{4}$ bu.

16-3 x, number of quarts of white paint Mr. Andrews used.
$4/1 \sim x/4\frac{3}{4}$.
19 qt.

16-4 x, number of kilograms

Nick weighs.
$2.2/1 \sim 110/x$.
50 kilograms

17-5 x, number of fluid ounces of milk Kay used.
$8/1 \sim x/2\frac{3}{8}$.
19 fl. oz.

17-6 x, number of pounds of peanuts Tom bought.
$16/1 \sim 24/x$.
$1\frac{1}{2}$ lb.

17-7 x, number of pecks of strawberries Susan picked.
$8/1 \sim 10/x$.
1.25 pk.

17-8 x, number of pounds of bananas Mrs. Kane bought.
$2.2/1 \sim x/1.8$.
3.96 lb.

17-9 x, number of minutes Jim spent on his homework.
$60/1 \sim x/2\frac{1}{4}$.
135 min.

17-10 x, number of pounds of coal Mr. Thompson had.
$2000/1 \sim x/1.5$.
3000 lb.

17-11 x, number of grams a 34-ounce bat weighs.
$.035/1 \sim 34/x$.
971.4 grams (nearer tenth)

17-12 x, number of pounds of steel produced.
$2000/1 \sim x/5921728$.
11,843,456,000 lb.

17-13 x, number of pints of coffee the machine holds.
y, number of quarts of coffee the machine holds.
$2\frac{1}{2}/1 \sim 50/x \wedge 2/1 \sim x/y$.
10 qt.

17-14 x, total number of minutes in six class periods.
y, number of hours in six class periods.
$45/1 \sim x/6 \wedge 60/1 \sim x/y$.
$4\frac{1}{2}$ hr.

17-15 x, number of quarts the tank will hold.
y, number of gallons the tank will hold.
$.88/1 \sim x/56 \wedge$
$4/1 \sim x/y$.
12.32 gal.

17-16 28.57 grams (nearer hundredth)

17-17 .45 kilogram (nearer hundredth)

17-18 1.14 litres (nearer hundredth)

17-19 120 hr.

17-20 2.6 tons

17-21 20 kilograms

17-22 12 qt.

17-23 48 wk.

17-24 11 lb.

17-25 9000 sec.

17-26 1168 da.

17-27 240 pt.

Lesson 101

17-A ℓ_1 and ℓ_2

17-B Rays DC, DF, DE, and DH * Yes

17-C $\angle CDF$, $\angle FDE$, $\angle EDH$, $\angle CDH$ * The sides of $\angle CDF$ are $\overrightarrow{DC}$ and $\overrightarrow{DF}$. The sides of $\angle FDE$ are $\overrightarrow{DF}$ and $\overrightarrow{DE}$. The sides of $\angle EDH$ are $\overrightarrow{DE}$ and $\overrightarrow{DH}$. The sides of $\angle CDH$ are $\overrightarrow{DC}$ and $\overrightarrow{DH}$.

17-D D * Ray DF

17-E Collinear points are contained in the same line. Since the common endpoint of rays DE and DC and a point in each of the rays are collinear, the union of the two rays is a line.

17-F Yes * Yes * Yes

17-G Yes * Yes

17-H Yes * Yes * Yes * You know that $\angle HDC$ is a right angle and that $\angle EDH$ is the supplement of $\angle HDC$. By definition, a right angle is congruent to its supplement. Therefore, $\angle EDH$ is a right angle.

18-A 3

18-B Ray MS * Ray MW

18-C $\angle VMT$ * 2

18-D 0 * 4 * 4 - 0, or 4

18-E Yes

18-F 1 * 1

18-G $\angle WMV$, $\angle VMU$, $\angle TMS$, $\angle SMR$

18-H 1 * 1 * 2 * Yes

18-I 2 * 1 * 3 * Yes

18-J Rays TA, TX, TB, TC, TD, TE, TF, TG, TH, TI, TJ, TK, TL, and TM * 0 is associated with $\overrightarrow{TA}$, 1 with $\overrightarrow{TX}$, 10 with $\overrightarrow{TB}$, 20 with $\overrightarrow{TC}$, 30 with $\overrightarrow{TD}$, 40 with $\overrightarrow{TE}$, 50 with $\overrightarrow{TF}$, 60 with $\overrightarrow{TG}$, 70 with $\overrightarrow{TH}$, 80 with $\overrightarrow{TI}$, 90 with $\overrightarrow{TJ}$, 100 with $\overrightarrow{TK}$, 110 with $\overrightarrow{TL}$, and 120 with $\overrightarrow{TM}$

18-K $\angle ATX$

19-L 10 - 0, or 10 * 120 - 100, or 20 * 30 * The measure of each angle is the difference of the numbers associated with the sides of the angle.

19-M 30

19-N 20 * 20 * $\angle DTF$ is congruent to $\angle FTH$.

19-O You should name four of the following: $\angle MTL$, $\angle LTK$, $\angle KTJ$, $\angle JTI$, $\angle ITH$, $\angle HTG$, $\angle GTF$, $\angle FTE$, $\angle ETD$, $\angle CTB$, $\angle BTA$. * You should name three of the following: $\angle MTI$, $\angle LTH$, $\angle KTG$, $\angle JTF$, $\angle HTD$, $\angle GTC$, $\angle FTB$, $\angle ETA$.

19-P Yes

19-Q 40 * Yes

19-R 60 * You should name three of the following: ∠MTG, ∠LTF, ∠KTE, ∠JTD, ∠ITC, ∠HTB.

19-S ∠WMQ * 6

19-T The unit angles used in D2 and D8 are not congruent. * You know this because of the measures of the right angles. The measure of the right angle, represented in D2, is 6. The measure of the right angle, represented in D8, is 60.

20-A 90

20-B ∠ACE

20-C 7 − 0, or 70 * 110 * 50

20-D 90 * Yes

20-E 90 * Yes * 90 * Yes

20-F You should name three of the following: ∠FCN, ∠HCP, ∠ICQ, ∠JCR, ∠KCS, ∠MCU.

20-G 90 + 90, or 180

20-H No. It is line AD.

20-I 179 * 170 * Yes

20-J When the unit is the degree, if two rays are associated with numbers whose difference is 180, then the union of these two rays is a line, not an angle. * If the difference of the numbers associated with two rays is 0, then the two rays are the same and an angle is not formed.

21-A ∠JDE and ∠TSR

21-B 40 * Less than 90

21-C ∠LMN * $74\frac{1}{2}$

21-D 132 * Greater than 90

21-E ∠OPQ * $100\frac{3}{4}$

21-F Less than * Greater than

21-G Less than

21-H ∠JDE° = 90.
∠FGH° = 132.
∠LMN° = $74\frac{1}{2}$.
∠OPQ° = $100\frac{3}{4}$.
∠RST° = 90.

22-A Rays VA and VB * An angle is the union of two rays that have a common endpoint and that do not form a line. Rays VA and VB have a common endpoint V and do not form a line.

22-B Yes

22-C 45 * It is on the counterclockwise scale; the picture of $\overrightarrow{VA}$ is under the mark that represents 0. On the clockwise scale, the picture of $\overrightarrow{VA}$ would be under the mark that represents 180. * 45

22-D 45

22-E 45 * An acute angle

22-F 140 * 0 * 140 * 140 * An obtuse angle * An obtuse angle is an angle whose measure in degrees is greater than 90.

22-G 125 * 34 * 91

22-H 91 * An obtuse angle * An obtuse angle is an angle whose measure in degrees is greater than 90.

22-I Point T

22-J Point T

22-K Above the dot that represents point T * Yes

On your own

23-1 The measure of each of the other angles is 90.

23-2 No * Yes

23-3 ∠QPL * ∠HKN

23-4 $44\frac{1}{6}$

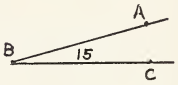
23-5 Yes * $11\frac{1}{2}$ *
∠GHI° = 90.
∠JKL° = 90.
∠RST° = $126\frac{2}{3}$.
∠MNO° = 172.

23-6 ∠ABC and ∠EDF

23-7 ∠RST and ∠MNO

23-8 ∠GHI and ∠JKL

23-9



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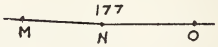
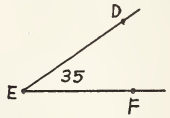
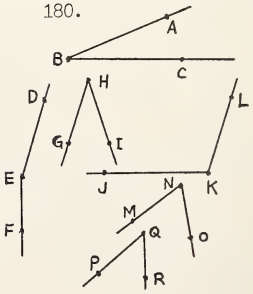


23-11



23-12

23-13 For the six angles shown below: ∠ABC° = 25; ∠DEF° = 160; ∠GHI° = 45; ∠JKL° = 100; ∠MNO° = 70; and ∠PQR° = 56. The angles whose pictures you make may have any measures between 0 and 180.



Checking up

Test 108

24-1 $\frac{3}{5}$

24-2 $\frac{6}{5}$, or $1\frac{1}{5}$

24-3 $\frac{5}{5}$, or 1

24-4 Segments AI and BJ

24-5 Segments AI, BJ, and AJ

Test 109

24-6 $3\frac{3}{4}$

24-7 $11\frac{1}{6}$

24-8 $8\frac{9}{16}$

24-9 A right angle

24-10 An acute angle

24-11 $3/1 \sim x/87$.
{111}. 111 ft.

24-12 $4/1 \sim 50/x$.
{12.5}. 12.5 gal.

24-13 $1760/1 \sim 1320/x$.
{.75}. .75 mi.

24-14 $16/1 \sim x/4.6$.
{73.6}. 73.6 oz.

Lesson 102

24-A ℓ_1 and ℓ_2 * ℓ_1 * ℓ_2

24-B Yes * They are not contained in the same line.

24-C Three non-collinear points determine a plane. If a line contains two points of a plane, the line is included in the plane. ℓ_1 and ℓ_2 each contain two points of the plane determined by points A, B, and C. * You can use the names of any three non-collinear points in the plane to name the plane. Examples of names of the plane that includes ℓ_1 and ℓ_2 are plane ABC, plane BAE and plane DAC.

24-D Yes * {A}

24-E Two lines that intersect determine a plane, and each of the lines is included in this plane.

24-F Rays AE and AC

25-G ℓ_1 * ℓ_2

25-H Yes * Side AB * Side AC

25-I ∠BAE * ∠DAE * ∠DAC

25-J Four

25-K ℓ_3 and ℓ_4 * No * Points H, I, and J are not contained in the same line.

25-L Yes * {H}

25-M ℓ_3 and ℓ_4 are included in the same plane.

25-N Point H * {H}

25-O Yes * Yes

25-P Rays HL and HJ * {H}

25-Q ℓ_4 * Yes * Rays HJ and HL have the same endpoint and are subsets of the same line.

25-R Yes * Yes * Yes * Yes

25-S Yes * The two sides of ∠IHJ are side HI and side HJ. The two sides of ∠KHL are side HK and side HL. Side HI of ∠IHJ and side HK of ∠KHL are opposite rays. Side HJ of ∠IHJ and side HL of ∠KHL are opposite rays.

25-T Rays HL and HI * Rays HJ and HK

25-U Yes * Yes * Side HL of ∠LHI and side HJ of ∠JHK are opposite rays. Side HI of ∠LHI and side HK of ∠JHK are opposite rays.

25-V The sides of ∠LHI and ∠JHK form two pairs of opposite rays.

26-A {M} * ∠NMO, ∠OMQ, ∠QMP, and ∠PMN

26-B Yes * ∠OMN and ∠QMO have a common side and a common vertex and their sides that are not common form a line.

26-C 60 * 120 * 180

26-D Yes

26-E ∠QMO and ∠PMQ have a common side and a common vertex and their sides that are not common form a line.

26-F 120 * 60 * 180

26-G Yes * Since the sum of the measures in degrees of ∠QMO and ∠PMQ is 180, the angles are supplementary angles.

26-H ∠PMQ and ∠NMP have a common side and a common vertex and their sides that are not common form a line.

26-I Yes * ∠NMP * ∠PMQ * 180

26-J Yes * 60

26-K Yes * 120

26-L Rays MO and MP * Rays MQ and MN * ∠OMN and ∠PMQ. ∠PMN and ∠QMO

26-M Yes * If the measures of two angles are equal, then the two angles are congruent.

26-N Yes. The measure of each of the angles QMO and NMP is 60. * Yes. If the measures of two angles are equal, then the two angles are congruent.

27-O ∠a and ∠b have a common vertex and a common side and their sides that are not common form a line.

27-P Yes * Yes * If two angles form a linear pair, then they are supplementary angles. An angle that is congruent to its supplement is a right angle.

27-Q 90 * 90

27-R ∠a and ∠c. ∠b and ∠d.

* Opposite angles are congruent. Since ∠a is a right angle and ∠a is congruent to ∠c, ∠c is a right angle. Since ∠b is a right angle and ∠b is congruent to ∠d, ∠d is a right angle.

27-S Opposite angles are congruent, and congruent angles have the same measure.

27-T Yes * Yes * Yes * No

27-U 360 * Two intersecting lines form two linear pairs of angles. The sum of the measures of a linear pair of angles is 180. Since two linear pairs of angles are formed, the sum of the measures of all 4 angles is 360.

27-V { }

27-W Two lines that are included in the same plane and that do not have any points in common are parallel. * None

27-A 180

27-B No * x is a variable for the measure of an angle. The measure of an angle is a rational number between 0 and 180.

27-C Yes * 82

27-D In the condition $98 + x = 180$, x is a variable for $\angle b^\circ$. So the number that satisfies $98 + x = 180$ is the measure of ∠b in degrees.

27-E Yes. Since each of the angles d and b is a supplement of a, ∠d and ∠b are opposite angles. Opposite angles are congruent. * Yes. Congruent angles have the same measure.

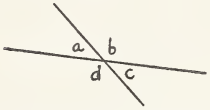
27-F 82

27-G Yes. Since ∠d and ∠b are opposite angles, so are ∠c and ∠a. Opposite angles are congruent. * Yes. Congruent angles have the same measure.

27-H 98

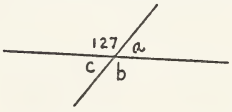
On your own

27-I Your picture should be similar to the one below.
$\angle a^\circ = 55$.
$\angle b^\circ = 125$.
$\angle c^\circ = 55$.
$\angle d^\circ = 125$.



28-2 The sum of the measures that you found for exercise 1 is 360.

28-3 Your picture should be similar to the one below.



28-4 $\angle a^\circ = 53$.
$\angle b^\circ = 127$.
$\angle c^\circ = 53$.

28-5 ∠IFG, ∠GFE, ∠EFH, and ∠HFI

28-6 ∠VWX, ∠XWZ, ∠ZWY, and ∠YWV

28-7 ∠IFG and ∠GFE. ∠GFE and ∠EFH. ∠EFH and ∠HFI. ∠HFI and ∠IFG

28-8 ∠VWX and ∠XWZ. ∠XWZ and ∠ZWY. ∠ZWY and ∠YWV. ∠YWV and ∠VWX

28-9 ∠EFH and ∠IFG. ∠HFI and ∠EFG. ∠VWX and ∠YWZ. ∠VWY and ∠XWZ

28-10 65

28-11 ∠EFH, ∠VWX, and ∠YWZ

28-12 115

28-13 ∠VWY, ∠EFG, and ∠HFI

28-14 ∠VWY and ∠XWZ

28-15 ∠EFG and ∠HFI

Lesson 103

28-A Yes * Lines that are included in the same plane are coplanar lines.

28-B Point R

28-C Rays RA, RB, RC, RD, RE, and RF

28-D Yes * ∠BRC

28-E Point R * Side RB

28-F No * { }

28-G The interior of ∠ARB and the interior of ∠BRC have no common members. Two sets that have no common members are disjoint sets.

29-H Point R * Yes * No

29-I No * Since ∠CRD and ∠ERF do not have a common side, they are not adjacent angles.

29-J The interior of ∠BRC and the interior of ∠BRD are not disjoint sets.

29-K You should name three of the following pairs: ∠BRC and ∠CRD; ∠CRD and ∠DRE; ∠DRE and ∠ERF; ∠ERF and ∠FRA; ∠FRA and ∠ARB.

29-L Yes

29-M Yes

29-N Yes

29-O 12 linear pairs of angles represented in D1. An example of another pair is ∠ERC and ∠BRC. * Yes

29-P Yes. The two angles in a linear pair of angles have a common vertex, a

common side, and their interiors are disjoint. * No. The sides of two adjacent angles that are not common do not necessarily form a line.

29-Q Yes. If two angles form a linear pair, then they are supplementary. * No. The only requirement for supplementary angles is that the sum of their measures in degrees is 180. The angles need not have a common vertex, a common side, or disjoint interiors.

29-R No. The sum of the measures of a pair of adjacent angles in degrees need not be 180. * No. Again, supplementary angles need not have a common vertex, a common side, or disjoint interiors.

29-S No. They do not have a common side. * Yes. The sides of ∠ARB and ∠ERD form two pairs of opposite rays.

29-T A pair of adjacent angles has only three sides; one of these sides is common to the two angles of the pair. A pair of opposite angles must have four sides because the sides must form two pairs of opposite rays.

29-A ℓ_4, ℓ_5, and ℓ_6 * No

29-B {S} * {T}

29-C Yes * Since the transversal intersects each of the two coplanar lines, the transversal contains two points in the plane that includes the two lines. Therefore, the transversal is included in this plane.

29-D ℓ_5 and ℓ_6 * ℓ_5

30-E ℓ_7, ℓ_8, and ℓ_9 * Yes * ℓ_7 intersects ℓ_8 in a different point than the point in which it intersects ℓ_9.

30-F No * The intersection of two parallel lines is the empty set.

30-G ∠a and ∠b. ∠b and ∠d. ∠d and ∠c. ∠c and ∠a. ∠e and ∠f. ∠f and ∠h. ∠h and ∠g. ∠g and ∠e

30-H ∠a and ∠d. ∠b and ∠c. ∠e and ∠h. ∠f and ∠g

30-I Yes

30-J Yes * Yes

30-K Yes

30-L Yes * No * No * No

30-M Their interiors are not subsets of the same half-plane bounded by the transversal.

30-N A side of one angle is not a proper subset of one of the sides of the other angle.

30-O Yes * The interiors of both ∠c and ∠g are subsets of the same half-plane bounded by ℓ_7. A side of ∠g is a proper subset of a side of ∠c.

30-P You should name one of the following pairs: ∠a and ∠e; ∠d and ∠h.

30-A Yes * { }

30-B Yes

31-C Yes * Yes * Opposite angles are congruent. * ∠t and ∠w. ∠v and ∠u

31-D Yes * The interiors of both ∠q and ∠u are subsets of the same half-plane bounded by ℓ_{12}. A side of ∠q is a proper subset of a side of ∠u.

31-E ∠r ≅ ∠q and ∠v ≅ ∠u because opposite angles are congruent. Therefore, each of the angles r, q, u, and v is congruent to the others. * Yes

31-F Yes. If two angles form a linear pair, then they are supplementary. * Yes * The angles p and t are supplementary to the congruent angles q and u. Since ∠q ≅ ∠u, ∠p and ∠t are congruent. * Yes

31-G Yes * Yes * The angles s and w are supplementary to congruent angles. * Yes

31-H ∠s, ∠t, and ∠w

31-I 75 * ∠r, ∠u, and ∠w

31-J Yes

31-K ∠g and ∠m. ∠h and ∠n. ∠k and ∠q. ∠j and ∠p.

31-L ∠p and ∠j

31-M Using opposite angles, you know that ∠j ≅ ∠p. Using supplementary angles, you know that ∠g ≅ ∠m and ∠k ≅ ∠q.

31-N Two angles are a linear pair of angles. * The angles are adjacent.

31-O No

32-P No * No * Two simple examples are the following: The statement: If an animal is a dog, then it has four legs. The converse: If an animal has four legs, then it is a dog. The statement

is true but the converse is false. The statement: If three sides of a triangle are congruent, then the triangle is equilateral. The converse: If a triangle is equilateral, then the three sides are congruent. The statement and the converse are true.

On your own

32-1 ∠VTU and ∠XTW

32-2 Their interiors are not disjoint.

32-3 ∠VTW and ∠ZTY

32-4 Examples are ∠ZTV and ∠VTW; ∠YTX and ∠XTV.

32-5 Examples are ∠UTV and ∠XTY; ∠ZTV and ∠WTY.

32-6 Yes * Opposite angles are congruent.

32-7 $\angle YTX^\circ = 25$. *
$\angle WTX^\circ = 90$. *
$\angle UTZ^\circ = 90$.

32-8 ∠ZTU, ∠UTW, ∠XTW, ∠XTZ

32-9 Examples are ∠ZTU and ∠XTW; ∠UTW and ∠ZTX; ∠VTW and ∠ZTY.

32-10 ℓ_{18}

32-11 Examples are ∠a and ∠b; ∠b and ∠d; ∠e and ∠g; ∠f and ∠h.

32-12 Examples are ∠d and ∠c; ∠c and ∠a; ∠e and ∠f; ∠g and ∠h.

32-13 ∠a and ∠d. ∠b and ∠c. ∠e and ∠h. ∠f and ∠g

32-14 ∠b and ∠f. ∠d and ∠h. ∠c and ∠g. ∠a and ∠e

32-15 $\angle b^\circ = 70$.
$\angle c^\circ = 70$.
$\angle d^\circ = 110$.
$\angle e^\circ = 110$.
$\angle f^\circ = 70$.
$\angle g^\circ = 70$.
$\angle h^\circ = 110$.

32-16 "If a line is a transversal of two coplanar lines, the line intersects one of the two coplanar lines in a point and also intersects the other coplanar line in a different point."

32-17 Yes * Yes

32-18 "If two angles are congruent, then the angles are right angles."

32-19 Yes * No

Keeping Skilful

32-1 {1.75}

32-2 $\{\frac{2}{3}\}$

32-3 $\{2\frac{11}{12}\}$

32-4 $\{\frac{1}{8}\}$

32-5 {2}

32-6 $\{10\frac{14}{17}\}$

32-7 {.565}, or $\{\frac{113}{200}\}$

32-8 $\{22\frac{1}{22}\}$

32-9 $\{7\frac{7}{8}\}$

32-10 {1.8}

32-11 $\{13\frac{17}{21}\}$

32-12 $\{\frac{1}{22}\}$

Lesson 104

33-A (Column 1) ΔDEF, ΔGHI, and ΔJKL

33-B Segment DF is greater than segment FE. * Yes * If a first segment is greater than a second segment and the second segment is greater than a third segment, then the first segment is greater than the third segment. This also means that the measure of the first segment is greater than the measure of the third segment.

33-C Yes * Since no two sides of a scalene triangle are congruent, no two sides have the same measure.

33-D ΔDEF * Yes

33-E Congruent segments have the same measure.

33-F Yes

33-G Yes

33-H Yes * If two sides have the same measure, then those sides are congruent. A triangle that has two of its sides congruent is an isosceles triangle.

33-I ΔJKL * Yes

33-J Yes * Yes * By the transitive property of equality

33-K Yes

33-L ΔJKL * Yes

33-A (Column 2) ΔMNO, ΔPQR, and ΔSTU

34-B 95 * 58 * 27

34-C Yes * 180

34-D $\angle P^\circ = 60$.
$\angle Q^\circ = 60$.
$\angle R^\circ = 60$. * 180

34-E $\angle S^\circ = 40$. $\angle T^\circ = 100$.
$\angle U^\circ = 40$. * 180.

34-F Yes * $\overline{RS}$, $\overline{ST}$, and $\overline{TR}$ * Yes

34-G Yes * The interiors of ∠d and ∠c are both subsets of the same half-plane bounded by ℓ_6, and one side of ∠c is a proper subset of a side of ∠d.

34-H Yes * If two lines are parallel, then the angles of each pair of corresponding angles formed by the two lines and a transversal are congruent.

34-I Yes. ℓ_5 intersects ℓ_4 in a point and intersects ℓ_7 in a different point.

* Yes. The interiors of $\angle e$ and $\angle a$ are subsets of the same half-plane bounded by ℓ_5 and one side of $\angle a$ is a proper subset of a side of $\angle e$. * Yes.

34-J The sides of $\angle f$ and $\angle b$ form two pairs of opposite rays.

34-K Yes * Opposite angles are congruent, and congruent angles have the same measure.

34-L Yes * From your answers to exercises H, I, and K, you know that $\angle d^\circ = \angle c^\circ$, $\angle e^\circ = \angle a^\circ$, and $\angle f^\circ = \angle b^\circ$. Therefore, $\angle d^\circ + \angle e^\circ + \angle f^\circ$ is the same sum as $\angle c^\circ + \angle a^\circ + \angle b^\circ$.

34-M They have a common vertex, a common side, and their interiors are disjoint. * Yes

34-N Yes * $\angle XTV$ and $\angle c$ have a common side and a common vertex, and their sides that are not common form a line.

34-O If two angles form a linear pair, then they are supplementary angles. The sum of the measures of a pair of supplementary angles is 180. * Yes

34-P Yes * $\angle d^\circ + \angle e^\circ + \angle f^\circ = \angle c^\circ + \angle a^\circ + \angle b^\circ$ and $\angle c^\circ + \angle a^\circ + \angle b^\circ = 180$. Therefore, using the transitive property, $\angle d^\circ + \angle e^\circ + \angle f^\circ = 180$.

35-Q Yes * $\angle Z$

35-R Yes * For any triangle, the sum of the measures of the angles in degrees is 180. * 90

35-S Yes * Yes * The measure of an angle is always greater than 0. Therefore, if the sum of the measures of two angles is 90, each of the angles has a measure less than 90. * Yes

35-T The sum of the measures of all three angles of a triangle is 180. Therefore, the sum of the measures of any two angles of a triangle must be less than 180.

35-U The sum of the measures of two obtuse angles is greater than 180.

35-V Yes

35-W No * If a triangle had only one acute angle, each of the other angles would have to be a right angle or an obtuse angle. A triangle cannot have two angles such that the sum of their measures is greater than, or equal to, 180.

On your own

35-1 { }

35-2 {C}

35-3 {C}

35-4 {B}

35-5 {A}

35-6 Sides AB, BC, and AC

35-7 $\angle BAC$, $\angle CBA$, and $\angle ACB$

35-8 $\angle ECF$

35-9 40 * Opposite angles are congruent and congruent angles have the same measure.

35-10 50 * $\angle FCG$ and $\angle CBA$ are corresponding angles formed by the parallel lines ℓ_1 and ℓ_4 and the transversal ℓ_2. These corresponding angles are congruent and congruent angles have the same measure.

35-11 90 * 90

35-12 90

35-13 ΔABC is a right triangle.

36-14 60

36-15 Yes * It is an acute triangle.

36-16 90

36-17 Yes * It is a scalene triangle. * It is a right triangle.

36-18 85

36-19 It is an isosceles triangle. * It is an obtuse triangle.

36-20 23

36-21 Yes * It is a scalene triangle. * It is an obtuse triangle.

36-22 All of them

36-23 ΔABC

36-24 ΔGHI and ΔJKL

36-25 ΔABC * Yes * Yes

36-26 ΔGHI * Yes

36-27 Yes * Yes * Yes

36-28 Yes

36-29 ΔGHI

36-30 Yes

36-31 ΔJKL

36-32 Yes * In D6, ΔDEF is both a scalene and a right triangle.

Lesson 105

37-A Yes * {D, E, F}

37-B Vertex C and vertex F will be mapped onto each other.

37-C A is the first letter in the symbol ABC, and

D is the first letter in the symbol DEF. * B is the second letter in the symbol ABC, and E is the second letter in the symbol DEF. * C is the third letter in the symbol ABC, and F is the third letter in the symbol DEF.

37-D Vertex C will be mapped onto vertex E. * Vertex E will be mapped onto vertex C. *
$ABC \longleftrightarrow DFE$

38-E For ΔMNP, the angles are: angle M, angle N, and angle P. The sides are: side MN, side NP, and side MP. For ΔRST, the angles are: angle R, angle S, and angle T. The sides are: side RS, side ST, and side RT.

38-F Vertices M and R are mapped onto each other. Vertices N and S are mapped onto each other. Vertices P and T are mapped onto each other.

38-G Side RS * Yes * Yes

38-H The endpoints of $\overline{NP}$ and $\overline{ST}$ are mapped onto each other. * The endpoints of $\overline{MP}$ and $\overline{RT}$ are mapped onto each other.

38-I Yes. The vertices of $\angle N$ and $\angle S$ are mapped onto each other. * Yes. The vertices of $\angle P$ and $\angle T$ are mapped onto each other.

38-J The corresponding sides of ΔPMN and ΔRST are: sides PM and RS, sides MN and ST, and sides PN and RT. The corresponding angles are: angles P and R, angles M and S, and angles N and T.

38-K 6 * 6

39-A The corresponding sides are: sides AB and JK, sides BC and KL, and sides AC and JL. The corresponding angles are: angles A and J, angles B and K, and angles C and L.

39-B Yes * Yes

39-C Yes * Since there is a congruence correspondence between the vertices of ΔABC and ΔJKL, ΔABC is congruent to ΔJKL.

39-D The corresponding sides are: sides PQ and ST, sides QR and TU, and sides PR and SU * Yes * If two triangles are congruent, then there is a congruence correspondence between the vertices of the triangles and corresponding sides are congruent.

39-E The corresponding angles are: angles P and S, angles Q and T, and angles R and U. * There is a congruence correspondence between the vertices of ΔPQR and ΔSTU.

39-F No

39-G No * If two triangles are congruent, each side of one triangle must be congruent to at least one side of the other triangle. Each angle of one triangle must be congruent to at least one angle of the other triangle. If every correspondence between the vertices of congruent triangles were a congruence correspondence, then each side of one triangle would have to be congruent to each of the three sides of the other triangle. And, each angle of one triangle would have to be congruent to each of the three angles of the other triangle.

39-H The corresponding sides are: sides FG and IJ, sides GH and JK, and sides FH and IK. The corresponding angles are: angles F and I, angles G and J, and angles H and K.

40-I $\overline{FG}$ and $\overline{IJ}$ are the corresponding sides determined by a congruence correspondence.

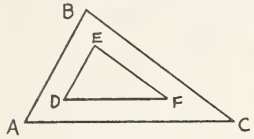
40-J Sides GH and JK, and sides FH and IK * Yes

40-K "If corresponding sides of two triangles are congruent, then the triangles are congruent." * This converse is true.

40-L $\angle F$ and $\angle I$ are corresponding angles determined by a congruence correspondence. * $\angle G$ and $\angle J$ are corresponding angles determined by a congruence correspondence. * $\angle H$ and $\angle K$ are corresponding angles determined by a congruence correspondence.

40-M "If corresponding angles of two triangles are congruent, then the two

triangles are congruent."
* This converse is false.
* See the sketch below.



$\angle A \cong \angle D$.
$\angle B \cong \angle E$.
$\angle C \cong \angle F$.

40-A Two sides of the triangle are congruent.

40-B Yes * B

40-C $\angle A$ * The intersection of $\angle A$ and $\overline{BC}$ contains exactly two points. * Yes

40-D Two sides of the triangle are congruent. * $\angle D$ is opposite side EF and $\angle F$ is opposite side DE.

41-E Yes * Yes * $\overline{EG}$

41-F Yes. A segment is congruent to itself. * Yes. $\overline{DE}$ of $\triangle DEG$ is congruent to $\overline{FE}$ of $\triangle FEG$; $\overline{DG}$ of $\triangle DEG$ is congruent to $\overline{FG}$ of $\triangle FEG$; and $\overline{EG}$ of $\triangle DEG$ is congruent to $\overline{EG}$ of $\triangle FEG$. * Yes. If corresponding sides of two triangles are congruent, then the two triangles are congruent.

41-G Yes * Yes

41-H Yes * For any triangle, if two sides are congruent, then the angles opposite these sides are congruent. If $\triangle ABC$ is equilateral, then $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{AC}$. Since $\overline{AB} \cong \overline{BC}$, $\angle C \cong \angle A$. Since $\overline{BC} \cong \overline{AC}$, $\angle A \cong \angle B$. If a first angle is congruent to a second angle and the second angle is congruent to a third angle, then all three angles are congruent to each other.

On your own

41-1 Yes * If the corresponding sides of two triangles are congruent, then the two triangles are congruent. *
FGH $\longleftrightarrow$ ZXY

41-2 7 * 8 * 4

41-3 Yes

41-4 The diagram that shows this correspondence is: FHG $\longleftrightarrow$ XZY. The corresponding sides are: sides FH and XZ, sides HG and ZY, and sides FG and XY. The corresponding angles are: angles F and X, angles H and Z, and angles G and Y.

41-5 No * The correspondence set up in exercise 4 is not a congruence correspondence.

41-6 60 * Each of the angles of $\triangle DEF$ has a measure of 60.

41-7 Yes * Yes

41-8 Yes

41-9 Yes

41-10 ABC $\longleftrightarrow$ DEF
ABC $\longleftrightarrow$ DFE
ABC $\longleftrightarrow$ EDF
ABC $\longleftrightarrow$ EFD
ABC $\longleftrightarrow$ FDE
ABC $\longleftrightarrow$ FED

41-11 All of them

41-12 Each angle of one of the equilateral triangles is congruent to each angle of the other triangle, and each side of one triangle is congruent to each side of the other triangle. Therefore, each of the six correspondences between the vertices of the two triangles is a congruence correspondence.

Lesson 106

42-A Each of these lines contains two points of the same plane and is included in this plane.

42-B Segments AB, BC, CD, and DA * Segment AB is included in ℓ_1; segment BC is included in ℓ_3; segment CD is included in ℓ_2; and segment DA is included in ℓ_4.

42-C Yes * Yes

42-D Sides AB, BC, CD, and DA. Vertices A, B, C, and D * 4

42-E No * The empty set

42-F { } * $\overline{BC}$ and $\overline{DA}$ do not intersect.

42-G Angles BAD, ABC, BCD, and CDA * The sides of angle BAD are rays AB and AD. The sides of angle ABC are rays BA and BC. The sides of angle BCD are rays CB and CD. The sides of angle CDA are rays DC and DA.

42-H Rays AB and AD * Rays CB and CD

42-I {B} * {D}

42-J 2 * B and D

42-K Point A * Point C

42-L {A, C} * $\angle B$ and $\angle D$ intersect in exactly two points.

43-A Sides EF and HG. Sides EH and FG

43-B $\ell_5 * \ell_6 * \ell_7 * \ell_8$

43-C ℓ_5 and ℓ_6 are coplanar lines whose intersection is the empty set.

43-D Sides FG and HE are parallel segments because they are subsets of parallel lines.

43-E Sides KL and JI. Sides LI and KJ * Angles KLI and KJI. Angles LIJ and LKJ

43-F Vertices I and K. Vertices L and J

43-G Segment IK

43-H Segments IJ, JK, and IK * Segments KL, LI and KI

43-I $\overline{JL}$ is a segment determined by two opposite vertices of parallelogram IJKL

43-J ΔIJL and ΔKJL

43-K The corresponding sides are: sides IJ and KL, sides JK and LI, and sides IK and KI. The corresponding angles are: angles JIK and IKL, angles J and L, and angles IKJ and LIK.

43-L Yes * Yes * Yes * If two triangles are congruent, then the corresponding sides are congruent and the corresponding angles are congruent.

43-M Yes * A parallelogram and either of its diagonals form two congruent triangles. If two triangles are congruent, then the corresponding angles are congruent.

44-A Yes

44-B Yes * A linear pair of angles are supplementary angles. The supplement of a right angle is a right angle.

44-C Yes * Corresponding angles formed by two parallel lines and a transversal are congruent.

44-D The opposite angles of a parallelogram are congruent. * The opposite angles of a parallelogram are congruent.

44-E All four angles of parallelogram NOPQ are right angles.

44-F Each of the other angles is a right angle. * A rectangle is a parallelogram in which each angle is a right angle.

44-G Segments that have the same measure are congruent segments.

45-H Yes

45-I A rectangle is a parallelogram. Opposite sides of a parallelogram are congruent. Therefore, if two adjacent sides of a rectangle are congruent, all four sides are congruent to each other. If each side of a rectangle is congruent to each of the other sides, the rectangle is a square.

45-J Yes * No

45-K Yes * Yes * No

On your own

45-1 Segment WX * Segment YZ * Segment XY * Segment WZ

45-2 No * Yes

45-3 Sides WX and YZ. Sides WZ and XY

45-4 Parallel lines * Parallel lines

45-5 $\overline{ZY} * \overline{WZ}$

45-6 A quadrilateral in which the opposite sides are parallel is a parallelogram.

45-7 $\overline{ZY} * \overline{WZ}$

45-8 Angles WXY and WZY. Angles XWZ and XYZ

45-9 $\angle Y * \angle Z$

45-10 ΔXWZ and ΔXYZ

45-11 A parallelogram and either of its diagonals form two congruent triangles.

45-12 90

Keeping Skilful

45-1 9.10

45-2 $1\frac{1}{4}$

45-3 21.74

45-4 $849\frac{7}{12}$

45-5 5.436

45-6 $18\frac{7}{20}$

45-7 $2\frac{7}{24}$

45-8 $13\frac{3}{4}$

45-9 $93\frac{43}{56}$

45-10 12.0924

45-11 1.5164

45-12 $118\frac{9}{14}$

45-13 $\frac{1}{8}$

45-14 .07200

45-15 116.026

45-16 $\frac{215}{256}$

45-17 $1800\frac{26}{45}$

45-18 $27\frac{29}{104}$

45-19 $1\frac{2}{33}$

45-20 76.23

45-21 9.22

45-22 75

45-23 .72

45-24 18.6

45-25 .052

Applying mathematics

46-1 x, number of feet of

fencing Pat needs.
$8\frac{1}{2} + 4 + 8\frac{1}{2} + 4 = x$.
{25}. 25 ft.

46–2 x, perimeter of the patio in yards.
$x = 6 + 12 + 15$.
{33}. 33 yd.

46–3 x, perimeter of the baseball diamond in feet.
$x = 90 + 90 + 90 + 90$.
{360}. 360 ft.

46–4 x, perimeter of the baseball diamond in yards.
$3/1 \sim 360/x$.
{120}. 120 yd.

46–5 x, perimeter of the piece of plywood in feet.
y, perimeter of the piece of plywood in inches.
$x = 9 + 17\frac{1}{2} + 9 + 17\frac{1}{2} \wedge$
$12/1 \sim y/x$.
{(53,636)}. 636 in.

46–6 x, number of inches of lace Sue needs.
$x = 70 + 54 + 70 + 54 + 4$.
{252}. 252 in.

46–7 x, number of yards of lace that Sue needs.
y, amount of money the lace will cost.
$36/1 \sim 252/x \wedge$
$.35/1 \sim y/x$.
{(7,2.45)}. $2.45

46–8 x, perimeter of the window in inches.
$x = 35\frac{1}{2} + 35\frac{1}{2} + 35\frac{1}{2}$.
{106$\frac{1}{2}$}. 106$\frac{1}{2}$ in.

46–9 x, perimeter of the parallelogram in feet.
y, perimeter of the parallelogram in yards.
$x = 11\frac{1}{3} + 9\frac{2}{3} + 11\frac{1}{3} + 9\frac{2}{3} \wedge$
$3/1 \sim x/y$.
{(42,14)}. 14 yd.

46–10 x, length of the rectangle in inches.
y, perimeter of the rectangle in inches.
$y = x + 64 + x + 64 \wedge$
$12/1 \sim x/3\frac{1}{2}$.
{(42,212)}. 212 in.

46–11 x, measure of $\angle n$ in degrees.
$127 + x = 180$.
{53}. $\angle n^\circ = 53$.

46–12 x, measure of $\angle h$ in degrees.
$108 + x = 253$.
{145}. $\angle h^\circ = 145$.

46–13 x, measure of third angle in degrees.
$11.5 + 11.5 + x = 180$.
{157}. The measure of the third angle in degrees is 157.

46–14 x, measure of $\angle h$ in degrees.
y, measure of $\angle m$ in degrees.
$x + 82 = 180 \wedge$
$x + y = 277$.
{(98,179)}.
$\angle h^\circ = 98$. $\angle m^\circ = 179$.

46–15 x, measure of $\angle c$ in degrees.
y, measure of $\angle d$ in degrees.
$60 + x = 110 \wedge$
$x + y = 215$.
{(50,165)}.
$\angle c^\circ = 50$. $\angle d^\circ = 165$.

Lesson 107

47–A The figures represented by pictures A, C, D, and E * The figures represented by pictures C, D, and E

47–B { } * Yes * All the points that are in either the simple closed curve or its interior

47–C Pictures C, D, and E * Picture B does not represent a simple closed curve.

47–D Yes * A rectangle is a simple closed curve.

48–E 1

48–F Picture J represents the most convenient unit region.

48–G 5

48–H 4 * No * The rectangular region ABCD

48–I 4, or 4 square units

48–J The measure of side EF is 2, of side FG is 5, of side GH is 2, and of side EH is 5.

48–K 5

48–L 2

49–M Yes * Yes

49–N Yes

49–O {10} * 10

49–P 10, or 10 square units

49–Q $3\frac{1}{2}$ * $2\frac{1}{2}$

49–R Yes * Yes * Yes

49–S $2\frac{1}{2}$

49–T The number of square units in one row

49–U Yes

49–V Yes

49–W $8\frac{3}{4}$

49–X $8.3/1 \sim x/2.2$, or $2.2/1 \sim x/8.3$.

49–Y 18.26

49–A A square yard is a square unit, each of whose sides is 1 yard long. * A square mile is a square unit, each of whose sides is 1 mile long. * A square rod is a square unit, each of whose sides is 1 rod long.

50–B In a rectangular region

15 ft. long, you can use the rate pair 15/1 to represent the fact that there are 15 square feet in 1 row.

50-C The area of the rectangular floor in square feet

50-D Yes

50-E $\{187\frac{1}{2}\}$

50-F No * No

50-G There are different numbers of square units in each row and different numbers of rows when different standard square units are used to find the area of a rectangle.

50-H x is a variable for the number of square units in $1\frac{1}{3}$ rows. Since the first component of 6/1 refers to the number of square units in 1 row, x should be the first component. Likewise, the second component of each rate pair should refer to a number of rows. * The area of the hallway

50-I Yes

50-J Yes

50-K y is the variable for the number of dollars that is the total cost, and the first component of 8.77/1 refers to the cost of 1 square yard. Therefore, y should be the first component of the rate pair you are to find. Similarly, each of the second components should refer to a number of square yards. * The number of dollars that is the total cost of the carpeting

50-L Yes

50-M Yes

50-N 8

50-O $8.77/1 \sim y/8.$

50-P 70.16

50-Q Mrs. Ames will have to pay $70.16 for the carpeting.

On your own

51-1 $13/1 \sim x/8.$
104 square feet

51-2 $24/1 \sim x/5.$
120 square inches

51-3 $6\frac{1}{2}/1 \sim x/2\frac{3}{4}.$
$17\frac{7}{8}$ square inches

51-4 $7/1 \sim x/8.$
56 square feet

51-5 $1.4/1 \sim x/.9.$
1.26 square yards

51-6 $17/1 \sim x/9\frac{1}{2}.$
$161\frac{1}{2}$ square miles

51-7 $2.8/1 \sim x/3.6.$
10.08 square inches

51-8 $4\frac{1}{2}/1 \sim x/4\frac{1}{2}.$
$20\frac{1}{4}$ square yards

51-9 $90/1 \sim x/90.$
8100 square feet

51-10 $.5/1 \sim x/1.6.$
.80 square inches

51-11 $3/1 \sim x/2.$
6 square miles

51-12 $17\frac{2}{3}/1 \sim x/3.$
53 square inches

51-13 $8.5/1 \sim x/4.6.$
39.10 square yards

51-14 $10.7/1 \sim x/6.4.$
68.48 square feet

51-15 x, area of Clay Township in square miles.
$13.9/1 \sim x/11.6.$
$\{161.24\}$.
161.24 square miles

51-16 x, area of the floor in square feet.
$15\frac{1}{3}/1 \sim x/12\frac{1}{2}.$
$\{191\frac{2}{3}\}$.
$191\frac{2}{3}$ square feet

51-17 x, number of gallons of paint that Mr. Wilson will need.
$1/600 \sim x/191\frac{2}{3}.$
$\{\frac{23}{72}\}$. Mr. Wilson will need $\frac{23}{72}$ gal. of paint but he will have to buy $\frac{1}{2}$ gal. because paint is not sold by seventy-seconds of a gallon.

51-18 x, area of rectangle in square yards.
$7\frac{1}{3}/1 \sim x/6\frac{1}{2}.$
$\{47\frac{2}{3}\}$.
$47\frac{2}{3}$ square yards

51-19 x, width of rectangle in inches.
$20/1 \sim 240/x.$
$\{12\}$. 12 in.

51-20 x, length of mirror in inches.
y, area of mirror in square inches.
$12/1 \sim x/4 \wedge x/1 \sim y/18.$
$\{(48,864)\}$.
864 square inches

51-21 x, area of lawn in square feet.
y, number of pounds of grass seed Mr. Gordon will use.
$60/1 \sim x/50 \wedge$
$1/500 \sim y/x.$
$\{(3000,6)\}$. 6 lb.

51-22 x, area of kitchen floor in square feet.
y, amount of money that was the cost per square foot.
$14/1 \sim x/11\frac{1}{2} \wedge$
$125/x \sim y/1.$
$\{(161,\frac{125}{161})\}$. $.78

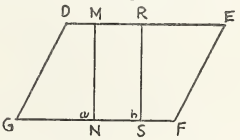
51-23 x, width in feet.
y, area in square feet.
$12/1 \sim 15/x \wedge$
$3\frac{1}{2}/1 \sim y/x.$
$\{(1\frac{1}{4},4\frac{3}{8})\}$. $4\frac{3}{8}$ square feet

Keeping Skilful

51-1 $\{19\frac{1}{4}\}$
51-2 $\{88\frac{1}{3}\}$
51-3 {.5}
51-4 $\{28\frac{8}{15}\}$
51-5 $\{\frac{1}{3}\}$
51-6 {18}
51-7 $\{10\frac{2}{11}\}$
51-8 $\{4\frac{1}{2}\}$
51-9 {.49}
51-10 {2.202}
51-11 {1.032}
51-12 $\{\frac{1}{9}\}$
51-13 $\{1\frac{7}{11}\}$
51-14 $\{2\frac{2}{3}\}$
51-15 $\{85\frac{5}{7}\}$
51-16 $\{6\frac{5}{8}\}$
51-17 $\{81\frac{9}{11}\}$
51-18 {138}
51-19 $\{29\frac{3}{4}\}$
51-20 $\{8\frac{3}{26}\}$
51-21 6
51-22 9.77
51-23 $1\frac{1}{2}$
51-24 27.44
51-25 $2\frac{9}{10}$
51-26 625,670
51-27 $6\frac{10}{21}$
51-28 8267
51-29 $6\frac{7}{8}$
51-30 1.118
51-31 $635\frac{53}{56}$
51-32 1.9577
51-33 42
51-34 64,551
51-35 $3\frac{1}{15}$
51-36 .033288

Lesson 108

52-A Sides RS and TU. Sides RU and ST
52-B {W} * {V}
52-C Yes * Yes
52-D The endpoints of $\overline{UX}$ are in the lines that include the opposite sides, $\overline{UT}$ and $\overline{RS}$. $\overline{UX}$ is perpendicular to the lines that include the opposite sides, $\overline{UT}$ and $\overline{RS}$.
52-E The endpoints of $\overline{TY}$ are in the lines that include the opposite sides, $\overline{UT}$ and $\overline{RS}$.
52-F $\overline{UR}$ is not perpendicular to lines that include opposite sides of the parallelogram. * $\overline{RS}$ is not perpendicular to lines that include opposite sides of the parallelogram.
52-G $\overline{UT}$
52-H Yes * The angles of XWVU are right angles.
52-I Yes * Opposite sides of a parallelogram are congruent.
52-J See the diagram below.



$\overline{MN}$ and $\overline{RS}$ are two altitudes. Therefore, $\overline{MN}$ and $\overline{RS}$ are perpendicular to lines DE and GF. $\angle a$ and $\angle b$ are right angles. Since the corresponding angles formed by lines MN and RS and transversal FG are congruent, $\overline{MN}$ is parallel to $\overline{RS}$. $\overline{MR}$ is parallel to $\overline{NS}$ because opposite sides of a parallelogram are parallel. This means that MRNS is a parallelogram because opposite sides are parallel. Therefore, $\overline{MN}$ is congruent to $\overline{RS}$ because opposite sides of a parallelogram are congruent.
53-K Each side of the rectangle has its endpoints in lines that include a pair of opposite sides, and each side of a rectangle is perpendicular to the lines that include a pair of opposite sides. * For each side of a rectangle, the rectangle has an altitude for which that side is a base.
53-A Yes * Yes
53-B Yes * Yes
53-C The congruent triangles AED and BFC have the same areas. This means that the area of ABCD is the sum of the same two areas as the area of EFCD. Therefore, the area of ABCD is the same as the area of EFCD.
53-D $\overline{DE}$ is perpendicular to the lines that include a pair of opposite sides of ABCD, and the endpoints of $\overline{DE}$ are contained in these lines. * $\overline{DE}$ is perpendicular to the lines that include a pair of opposite sides of EFCD, and the endpoints of $\overline{DE}$ are contained in these lines.
53-E $\overline{DC}$ * Yes
53-F 15 square units
53-G Yes * The area of parallelogram ABCD is the same as the area of rectangle EFCD.
53-H $\overline{KL}$ and $\overline{JM}$

53-I $\overline{KJ}$

54-J $5\frac{1}{4}/1 \sim x/2\frac{2}{3}$.

54-K Yes * The area of rectangle LMJK is the same as the area of parallelogram HIJK.

54-L 14

54-M Make a condition for equivalence for finding the area of a rectangle that has an altitude and a base with the same measure as the altitude and base measures given for the parallelogram. The area of this rectangle is the same as the area of the parallelogram.

54-N $4.6/1 \sim x/5.2$.

54-O 28.92 square inches

54-A Sides AB, BC, and CA. Angles CAB, ABC, and ACB

54-B Yes * {D}

54-C Yes

54-D 3 * 3

54-E $\overline{GE}$ is determined by opposite vertices of parallelogram DEFG.

54-F ΔDEG and ΔGEF

54-G The triangles determined by a parallelogram and either diagonal are congruent.

54-H Yes

55-I (Column 1) Yes * Yes

55-J (Column 1) Yes * Yes

55-K (Column 1) Yes * Yes

55-L Yes * 40 square units

55-M Multiply the number that you found for exercise L by the reciprocal of 2.

55-A $\overline{KL}$

55-B If a parallelogram and a triangle have a common base and a common altitude, then the area of the parallelogram is 2 times the area of the triangle.

55-C $4\frac{1}{2} * 2\frac{3}{4}$

55-D Use the definition of equivalent ordered pairs.

55-E Use the identity-element property of multiplication of rational numbers.

55-F Use the definition of the product of two rational numbers to obtain $\frac{99}{8}$ as the product of $4\frac{1}{2}$ and $2\frac{3}{4}$. * Multiply $\frac{99}{8}$ by the reciprocal of 2. $\frac{1}{2}$ is the reciprocal of 2. * Yes

55-G $6\frac{3}{16}$

55-H You use $2n$ to refer to the area of the parallelogram whose area is twice that of triangle EFG.

55-I (Column 2) Yes

55-J (Column 2) $\{11\frac{2}{3}\}$ * $11\frac{2}{3}$ square units

55-K (Column 2) $2.5/1 \sim 2x/3.2$. 4 square yards

On your own

56-I $12/1 \sim x/6$.
72 square inches

56-2 $3/1 \sim x/12\frac{1}{2}$.
$37\frac{1}{2}$ square feet

56-3 $2\frac{1}{2}/1 \sim x/4$.
10 square yards

56-4 $6.5/1 \sim x/11.8$.
78.45 square feet

56-5 $10\frac{3}{4}/1 \sim x/10\frac{3}{4}$.
$115\frac{9}{16}$ square inches

56-6 $3.765/1 \sim x/1.2$.
4.5180 square miles

56-7 $10/1 \sim 2x/5$.
25 square feet

56-8 $4/1 \sim 2x/1\frac{1}{2}$.
8 square yards

56-9 $8.5/1 \sim 2x/3.6$.
15.30 square inches

56-10 $7.5/1 \sim 2x/7.2$.
27.00 square feet

56-11 $9\frac{1}{2}/1 \sim 2x/12$.
57 square inches

56-12 $2.5/1 \sim 2x/.8$.
1.00 square yard

56-13 x, area of each piece of metal in square feet.
$5\frac{1}{2}/1 \sim x/8\frac{1}{2}$.
$\{46\frac{3}{4}\}$.
$46\frac{3}{4}$ square feet

56-14 x, area of the table top in square feet.
$5/1 \sim 2x/4.2$.
{10.5}.
10.5 square feet

56-15 x, length of the base in inches.
y, area of the felt in square inches.
$12/1 \sim x/2 \wedge x/1 \sim y/8$.
{(24,192)}.
192 square inches

56-16 x, area of the glass in square feet.
y, amount of money the glass will cost.
$3/1 \sim x/2\frac{2}{3} \wedge$
$1.50/1 \sim y/x$.
{(8,12.00)}. $12.00

56-17 x, length of the base in yards.
y, area of the rose garden in square yards.
$3/1 \sim 8/x \wedge x/1 \sim 2y/5$.
$\{(2\frac{2}{3}, 6\frac{2}{3})\}$.
$6\frac{2}{3}$ square yards

Keeping Skilful

56-I 485

56-2 51

56-3 13.942

56-4 $7\frac{17}{25}$

56-5 $5\frac{5}{12}$

56-6 8.37

56-7 21.0

56-8 $12\frac{7}{9}\frac{3}{6}$

56-9 $36\frac{9}{14}$

56-10 $36\frac{9}{14}$

56-11 $\frac{8}{33}$

56-12 $\frac{4}{1}$

56-13 $\frac{5}{6}$

56-14 $\frac{3}{23}$

56-15 $\frac{49}{3}$

56-16 $\frac{11}{102}$

56-17 $\frac{8}{71}$

56-18 $\frac{250}{3}$

56-19 $\frac{9}{35}$

56-20 $\frac{100}{513}$

56-21 $2\frac{1}{4}$

56-22 $21\frac{1}{3}$

56-23 $27\frac{1}{4}$

56-24 $14\frac{2}{5}$

56-25 $37\frac{6}{11}$

56-26 $14\frac{2}{7}$

56-27 $87\frac{1}{4}$

56-28 $1\frac{6}{7}$

56-29 $58\frac{14}{17}$

56-30 $203\frac{1}{3}$

56-31 {75}

56-32 {150}

56-33 $\{4\frac{8}{21}\}$

56-34 $\{\frac{1}{6}\}$

56-35 $\{22\frac{7}{9}\}$

56-36 $\{1\frac{1}{3}\}$

56-37 $\{\frac{5}{6}\frac{1}{4}\}$

56-38 $\{\frac{2}{3}\}$

56-39 $\{18\frac{3}{42}\}$

56-40 $\{\frac{1}{6}\frac{1}{4}\}$

56-41 {14.175}

56-42 $\{14\frac{2}{7}\}$

56-43 {.103}

56-44 {25}

56-45 $\{9\frac{1}{4}\}$

56-46 $\{\frac{1}{22}\}$

56-47 1967

56-48 21.7140

56-49 $11\frac{7}{24}$

56-50 9691

56-51 6.1135

56-52 $21\frac{8}{15}$

56-53 2.781

56-54 $102\frac{2}{5}$

56-55 869

56-56 14.897

56-57 $25\frac{5}{32}$

56-58 94,067

56-59 57.816

56-60 $10\frac{43}{56}$

56-61 428,770

56-62 $\frac{32}{69}$

56-63 1.29

56-64 309

56-65 105.3

56-66 65.503

Applying mathematics

57-1 x, perimeter of field in rods.
$x = 167 + 167 + 167$.
{501}. 501 rd.

57-2 x, number that was the average speed in miles per hour.
$2510/4 \sim x/1$.
$\{627\frac{1}{2}\}$.
$627\frac{1}{2}$ miles per hour

57-3 x, number of cups of milk Carol used to make the custard.
$\frac{5}{6}/1 \sim x/4$.
$\{2\frac{1}{2}\}$. $2\frac{1}{2}$ cups

57-4 x, length of the fence in feet.
$\frac{2}{3}/1 \sim 12\frac{1}{2}/x$.
$\{18\frac{3}{4}\}$. $18\frac{3}{4}$ ft.

57-5 x, area of floor in square feet.
$113\frac{1}{4}/1 \sim x/80\frac{1}{4}$.
$\{9088\frac{5}{16}\}$.
$9088\frac{5}{16}$ square feet

57-6 x, number of square inches of material Marilyn bought.
$72/1 \sim x/54$.
{3888}.
3888 square inches

57-7 x, number of square feet of material Marilyn bought.
$144/1 \sim 3888/x$.
{27}.
27 square feet

57-8 x, area of driveway in square feet.
y, area of driveway in square yards.
$50\frac{1}{2}/1 \sim x/6 \wedge 9/1 \sim x/y$.
$\{(303, 33\frac{2}{3})\}$.
$33\frac{2}{3}$ square yards

57-9 x, number of persons that is the population of Hamilton.
$\frac{1}{12}/1 \sim 20000/x$.
{240000}.
240,000 persons

57-10 x, number of square miles of land the company bought.
y, number of acres of land the company bought.
$\frac{1}{4}/1 \sim x/\frac{1}{2} \wedge 640/1 \sim y/x$.
$\{(\frac{1}{8}, 80)\}$. 80 acres

57-11 x, area of glass in square inches.
$14/1 \sim x/9\frac{1}{4}$.
$\{129\frac{1}{2}\}$.
$129\frac{1}{2}$ square inches

57-12 x, perimeter of the field in feet.
y, perimeter of the field in yards.
$x = 60 + 60 + 60 + 60 \wedge 3/1 \sim x/y$.
{(240,80)}. 80 yd.

57-13 x, number of miles Bill walked on Friday.
y, number of miles Bill walked on Saturday.
$x + y < 11 \wedge y - x = 5$.
{(1,6), (2,7)}. Bill walked 1 mi. on Friday and 6 mi. on Saturday, or Bill walked 2 mi. on Friday

and 7 mi. on Saturday.

57-14 x, area of flag in square feet.
$4/1 \sim 2x/2\frac{1}{2}$.
$\{5\}$.
5 square feet

57-15 x, number that is how many times longer Lake Superior is than Lake St. Clair.
$383/26 \sim x/1$.
$\{14\frac{19}{26}\}$. Lake Superior is 14.73 times as long as Lake St. Clair.

57-16 x, number that was the fraction of the total length of board Roger used.
$3\frac{3}{4}/6 \sim x/1$.
$\{\frac{5}{8}\}$. Roger used $\frac{5}{8}$ of the board.

58-17 x, number of gallons of coffee ice cream Mrs. Hayes ordered.
y, number of gallons of ice cream Mrs. Hayes ordered in all.
$4/1 \sim 13/x \wedge$
$y = x + 11 + 8\frac{1}{2}$.
$\{(3\frac{1}{4}, 22\frac{3}{4})\}$. $22\frac{3}{4}$ gal.

58-18 x, number of square feet that is the total area of the three lots.
y, number that is the average area per lot in square feet.
$x = 18750 + 12750 +$
$18000 \wedge x/3 \sim y/1$.
$\{(49500, 16500)\}$.
16,500 square feet

58-19 x, length of the floor in feet.
$x/1 \sim 12788/92$.
$\{139\}$. 139 ft.

58-20 x, area of the rug in square feet.
y, area of the rug in square yards.
$12/1 \sim x/8 \wedge 9/1 \sim x/y$.
$\{(96, 10\frac{2}{3})\}$.
$10\frac{2}{3}$ square yards

58-21 x, amount of money that was the total cost of the rug.
$13.50/1 \sim x/10\frac{2}{3}$.
$\{144.00\}$. $144.00

58-22 x, number that is the first component of the per cent of increase.
y, number of pecks of potatoes that was the increase.
$y/420 \sim x/100 \wedge$
$442 - 420 = y$.
$\{(5\frac{5}{21}, 22)\}$. 5.2%

58-23 x, area of the kitchen in square feet.
y, area of the kitchen in square yards.
$8/1 \sim x/8 \wedge 9/1 \sim x/y$.
$\{(64, 7\frac{1}{9})\}$.
$7\frac{1}{9}$ square yards.

58-24 x, area of the patio in square feet.
$26/1 \sim 2x/16\frac{1}{4}$.
$\{211\frac{1}{4}\}$.
$211\frac{1}{4}$ square feet

58-25 x, number that is the fraction the area of Mr. Jay's lot is of the area of Mr. Hunt's lot.
$3150/10800 \sim x/1$.
$\{\frac{7}{24}\}$. The area of Mr. Jay's lot is .29 the area of Mr. Hunt's lot.

58-26 x, area of Mr. Horton's field in square rods.
y, number of acres that are included in Mr. Horton's field.
$320.5/1 \sim x/160 \wedge$
$160/1 \sim x/y$.
$\{(51280, 320\frac{1}{2})\}$.
$320\frac{1}{2}$ acres

58-27 x, area of the piece of material in square metres.
$2.50/1 \sim 2x/1.75$.
$\{2.1875\}$.
2.1875 square metres

58-28 x, area of the field in square rods.
y, number of acres that are in the field.
$45/1 \sim x/50 \wedge$
$160/1 \sim x/y$.
$\{(2250, 14\frac{1}{16})\}$. 14.1 acres

58-29 x, area of the rectangle in square feet.
y, area of the rectangle in square inches.
$7\frac{1}{2}/1 \sim x/3\frac{2}{3} \wedge$
$144/1 \sim y/x$.
$\{(27\frac{1}{2}, 3960)\}$.
3960 square inches

58-30 x, perimeter of the triangle in feet.
y, perimeter of the triangle in yards.
$x = 4\frac{1}{3} + 4\frac{1}{3} + 4\frac{1}{3} \wedge$
$3/1 \sim x/y$.
$\{(13, 4\frac{1}{3})\}$. $4\frac{1}{3}$ yd.

58-31 x, length of the altitude in rods.
$64/1 \sim 4992/x$.
$\{78\}$. 78 rods

58-32 x, width of the rectangle in feet.
y, perimeter of the rectangle in feet.
$3/1 \sim x/16 \wedge y = x +$
$59 + x + 59$.
$\{(48, 214)\}$. 214 ft.

58-33 x, length of the base of the triangle in inches.
$x/1 \sim 2(23.12)/3.4$.
$\{13.6\}$. 13.6 in.

58-34 x, area of the parallelogram in square inches.

y, area of the parallelogram in square feet.
$36\frac{1}{2}/1 \sim x/24 \wedge$
$144/1 \sim x/y$.
$\{(876, 6\frac{1}{12})\}$.
$6\frac{1}{12}$ square feet

58-35 x, number of yards that is the same length as 45 ft.
y, perimeter of the triangle in yards.
$3/1 \sim 45/x \wedge$
$y = 17\frac{1}{2} + x + 16\frac{1}{2}$.
$\{(15, 49)\}$. 49 yd.

58-36 x, perimeter of the triangle in inches.
$36/1 \sim x/49$.
{1764}. 1764 in.

58-37 x, width of the rectangle in yards.
$3.4/1 \sim 46.24/x$.
{13.6}. 13.6 yd.

58-38 x, area of the triangle in square metres.
$3.8/1 \sim 2x/7.6$.
{14.44}.
14.44 square metres

58-39 x, length of the base in inches.
y, length of the base in feet.
$x/1 \sim 86/24 \wedge 12/1 \sim x/y$.
$\{(8\frac{7}{12}, \frac{43}{144})\}$. $\frac{43}{144}$ ft.

Applying mathematics

59-1 x, amount of money Phil earned.
$5.17 + x < 7.00$.
$\{x \mid x < 1.83\}$. Phil earned less than \$1.83.

59-2 x, number of cents Mr. Carter had to pay for 14 gal. of gasoline.
$38.5/1 \sim x/14$.
{539}. 539¢, or \$5.39

59-3 x, number of miles that Mr. Smith drove.
$.5/1 \sim x/128.5$.
{64.25}. 64.25 mi.

59-4 x, amount of money that is Richard's take-home pay.
$\frac{1}{6}/1 \sim 17.75/x$.
{142}. \$142

59-5 x, number of provincial parks in British Columbia.
$3.65/1 \sim x/40$.
{146}. 146 parks

59-6 x, number of miles per hour that was the speed of the winning boat.
$x - 102.935 = 5.151$.
{108.086}.
108.086 miles per hour

59-7 x, number of cans that together contain $1\frac{1}{4}$ lb. of nuts.
$\frac{3}{8}/1 \sim 1\frac{1}{4}/x$.
$\{3\frac{1}{3}\}$. Ellen has to buy 4 cans.

59-8 x, number of bushels of apples Mr. Keniston sold.
$x < 15\frac{1}{2} \wedge 15\frac{1}{2} - x < 7\frac{1}{8}$.
$\{x \mid x < 15\frac{1}{2} \wedge x > 8\frac{3}{8}\}$.
Mr. Keniston sold between $8\frac{3}{8}$ and $15\frac{1}{2}$ bushels of apples.

59-9 x, amount of money that was the sales tax Mr. Morris had to pay.
$1.5/100 \sim x/2400$.
{36}. \$36

59-10 x, number that is the first component of the per cent of increase in the number of hits.
y, number of hits that is the increase.
$x/100 \sim y/128 \wedge$
$205 - 128 = y$.
$\{(60\frac{5}{32}, 77)\}$. 60.16%

59-11 x, number of pounds of pork sausage Mrs. Hanson bought.
$1\frac{3}{4}/1 \sim 3\frac{1}{2}/x$.
{2}. 2 lb.

59-12 x, average number of weekly newspapers per province in the four provinces.
y, total number of weekly newspapers in the four provinces.
$x/1 \sim y/4 \wedge y = 68 + 87 + 14 + 30$.
$\{(49.8, 199)\}$. 49.8 weekly newspapers per province

59-13 x, number of times as many bushels of blueberries as peaches.
$x/1 \sim 16000000/3000000$.
$\{5\frac{1}{3}\}$. $5\frac{1}{3}$ times as many

59-14 x, number of inches of material Martha had to make the dress.
y, number of inches of material Martha used to make the dress.
$x - y = 12 \wedge 36/1 \sim x/4\frac{1}{2}$.
$\{(162, 150)\}$. 150 in.
Another way in which you can work the problem is given below.
x, number of yards of material Martha used to make the dress.
y, number of yards of material Martha had left.
$36/1 \sim 12/y \wedge 4\frac{1}{2} - x = y$.
$\{(4\frac{1}{6}, \frac{1}{3})\}$. $4\frac{1}{6}$ yd.

59-15 x, number that is the first component of the per cent of increase.
y, number of motor vehicles that was the increase.
$x/100 \sim y/48000 \wedge$
$48,000 - 16000 = y$.
$\{(66\frac{2}{3}, 32,000)\}$. $66\frac{2}{3}\%$

59–16 x, number of inches that is the average height of the 5 players.
y, number of inches that is the total of the heights of the 5 players.
$x/1 \sim y/5 \wedge y = 74 + 71 + 70 + 72 + 75$.
{(72.4, 362)}. 72.4 in.

59–17 x, number that is the first component of the per cent of increase.
y, number of persons that the population increased.
$x/100 \sim y/36000 \wedge 51000 - 36000 = y$.
{($41\frac{2}{3}$, 15000)}. $41\frac{2}{3}\%$

Checking up

Test 110

60–1 measure

60–2 1

60–3 $10\frac{5}{12}$

60–4 interior

60–5 5280

60–6 geometric figures, or simple closed curves, or polygons

60–7 acute

60–8 area

Test 111

60–9 F

60–10 F

60–11 T

60–12 T

60–13 F

60–14 T

60–15 F

60–16 T

60–17 T

Test 112

60–18 (d) coördinate

60–19 (c) 8

60–20 (b) 46

60–21 (d) 3

60–22 (d) =

60–23 (c) adjacent

60–24 (c) 180

60–25 (a) 90

60–26 (d) scalene

Test 113a

61–27 Isosceles

61–28 Side GH

61–29 $\angle FHG$

61–30 x, perimeter of ΔEFG in feet.
$x = 7.07 + 7.07 + 10$.
{24.14}. 24.14 ft.

61–31 x, perimeter of rectangle ABDC in feet.
$x = 7 + 3 + 7 + 3$.
{20}. 20 ft.

61–32 x, perimeter of parallelogram IJKL in feet.
y, perimeter of parallelogram IJKL in yards.
$x = 8\frac{1}{3} + 3\frac{1}{4} + 8\frac{1}{3} + 3\frac{1}{4} \wedge 3/1 \sim x/y$.
{($23\frac{1}{6}$, $7\frac{13}{18}$)}. $7\frac{13}{18}$ yd.

61–33 x, area of rectangle ABDC in square feet.
$3/1 \sim x/7$.
{21}.
21 square feet.

61–34 x, area of triangle EFG in square feet.
$10/1 \sim 2x/5$.
{25}.
25 square feet.

61–35 x, area of parallelogram IJKL in square feet.
y, area of parallelogram IJKL in square yards.
$8\frac{1}{3}/1 \sim x/2\frac{1}{2} \wedge 9/1 \sim x/y$.
{($20\frac{5}{6}$, $2\frac{17}{54}$)}.
$2\frac{17}{54}$ square yards

Checking up

Test 113b

61–36 2.8

61–37 a

61–38 d

61–39 NM

61–40 c

61–41 $\angle U$ (or vertex U)

61–42 square

Test 113c

61–43 congruent

61–44 c

61–45 a

61–46 parallel

62–47 b

62–48 four

62–49 $19\frac{1}{4}$

Test 113d

62–50 a

62–51 $2\frac{7}{8}$ ft.

62–52 b

62–53 c

62–54 six

62–55 diagonal

62–56 d

Lesson 109

63-A Set L * Set K * Set M

63-B Yes * Yes * No

63-C Yes * K $\cap$ L contains those members, and only those members, that belong to both K and L. Set M contains exactly the same members.

63-D Each member of K, L, and M is also a member of the set of natural numbers.

63-E 5 and 7. If an object belongs to two sets, it is a common member of the sets. The portion of the diagram that has both vertical and horizontal shading

64-F Yes * Yes

64-G No. A geometric figure cannot belong to both the set of triangles and the set of rectangles. * Yes. Sets that have no common members are disjoint sets.

64-H Yes. The intersection of two sets contains all those members, and only those members, that belong to both sets. Since A and B are disjoint, the intersection of A and B is the empty set. The empty set is the set that contains no members. * { }. The symbol "{ }" names the empty set.

64-I The portion of the diagram that represents A and the portion of the diagram that represents B do not touch each other.

64-A $x - \frac{2}{3} > \frac{1}{2}$. $\frac{1}{4} + x < 1\frac{5}{8}$. $x + 7.2 > 8.6$. $6.5 - x > 1.8$.

64-B Yes

64-C $\{\frac{7}{6}\}$, or $\{1\frac{1}{6}\}$ * You should find the sum of $\frac{2}{3}$ and $\frac{1}{2}$.

64-D Yes

64-E Yes

65-F (Column 1) Yes. There are infinitely many rational numbers that are greater than $\frac{7}{6}$. * You cannot name the least member of $\{x \mid x > \frac{7}{6}\}$.

65-G (Column 1) $\frac{1}{4} + x = 1\frac{5}{8}$.

65-H Yes

65-I $\{1\frac{3}{8}\}$ * You should find the difference of $1\frac{5}{8}$ and $\frac{1}{4}$.

65-J Yes * It is an infinite set.

65-K $\{x \mid x < 1\frac{3}{8}\}$

65-L $x + 7.2 = 8.6$.

65-M Yes

65-N {1.4}

65-O Yes * $\{x \mid x > 1.4\}$

65-P $6.5 - x = 1.8$.

65-Q Yes * Yes

65-R {4.7}

65-S Yes * $\{x \mid x < 4.7\}$

65-A $\frac{1}{8} + x < 1\frac{1}{8}$ and $\frac{4}{3} - x > \frac{2}{3}$.

65-B R_a

65-C The connective "and"

65-D $\frac{1}{8} + x < 1\frac{1}{8}$.

65-E $\{x \mid x < \frac{2}{3}\}$

65-F (Column 2) Each member of $\{x \mid x < \frac{5}{4} \wedge x < \frac{2}{3}\}$ satisfies not only $\frac{1}{8} + x < 1\frac{1}{8}$ but also $\frac{4}{3} - x > \frac{2}{3}$.

65-G (Column 2) Yes

66-H Yes * Each member of $\{x \mid x < \frac{5}{4} \wedge x < \frac{2}{3}\}$ must satisfy both $x < \frac{5}{4}$ and $x < \frac{2}{3}$. Likewise, each member in the intersection of $\{x \mid x < \frac{5}{4}\}$ and $\{x \mid x < \frac{2}{3}\}$ must satisfy both $x < \frac{5}{4}$ and $x < \frac{2}{3}$.

66-I Yes * Each solution of a compound condition must satisfy both simple conditions. Only the members of $\{x \mid x < \frac{2}{3}\}$ satisfy both $x < \frac{5}{4}$ and $x < \frac{2}{3}$.

66-J Yes

66-K By the bar below the picture of the number line * Yes. 0 is less than $\frac{2}{3}$. The vertical stroke through the dot for point 0 tells you that this dot is contained in the graph of $\{x \mid x < \frac{2}{3}\}$. * No. $\frac{2}{3}$ is not less than $\frac{2}{3}$. The curved stroke tells you that the dot for point $\frac{2}{3}$ is not contained in the graph of $\{x \mid x < \frac{2}{3}\}$.

66-L Yes * No * Yes

66-M $\{x \mid x < \frac{5}{4} \wedge x < \frac{2}{3}\}$ is the same set as $\{x \mid x < \frac{2}{3}\}$.

66-N Z

67-O $\{y \mid y > 1\frac{1}{4}\}$ * $\{y \mid y < 1\frac{7}{4}\}$

67-P Yes

67-Q Yes

67-R Yes * $y > 1\frac{1}{4}$ also means $1\frac{1}{4} < y$.

67-S Yes * Since $y > 1\frac{1}{4}$ and $1\frac{1}{4} < y$ have the same solution set, you can replace $y > 1\frac{1}{4}$ by $1\frac{1}{4} < y$.

67-T Yes

67-U There are infinitely many members of $\{y \mid 1\frac{1}{4} < y < 1\frac{7}{4}\}$. Examples are $1\frac{5}{4}$, $\frac{32}{8}$, and $\frac{7}{2}$. * Yes

67-V Yes

67-W Yes

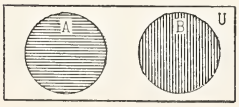
67-X Since 0 is not a member of the universe for y, 0 is not in the solution set of $\{y \mid \frac{11}{4} < y < \frac{17}{4}\}$.

67-Y Yes * By the bars above and below the picture of the number line

67-Z Yes * The density property of the set of rational numbers of arithmetic is as follows: The universe for x, y, and r is R_a. For each replacement of x and y, if $x < y$, then there is a rational number r such that $x < r < y$. (See page 389, Book 1.) This means that no matter how many rational numbers you find between $\frac{11}{4}$ and $\frac{17}{4}$, you can still find more. Therefore, there are infinitely many rational numbers between $\frac{11}{4}$ and $\frac{17}{4}$ and $\{y \mid \frac{11}{4} < y < \frac{17}{4}\}$ is an infinite set.

On your own

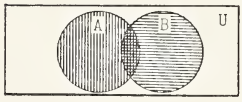
67-1 A = {0, 1, 2, 3, 4}.
B = {6, 7, 8, ...}.
A ∩ B = { }.



67-2 A = {39, 40, 41, ...}.
B = {10, 11, 12, ...}.
A ∩ B = {39, 40, 41, ...}.

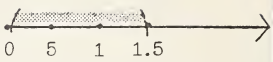
67-3 A = {0, 1, 2, ..., 10}.
B = {8}.
A ∩ B = {8}.

68-4 A = {7, 8, 9, ..., 18}.
B = {11, 12, 13, ..., 24}.
A ∩ B = {11, 12, 13, ..., 18}.

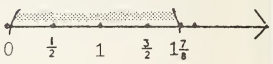


68-5 False statement
68-6 True statement
68-7 True statement
68-8 False statement
68-9 False statement
68-10 True statement
68-11 False statement
68-12 True Statement

68-13



68-14



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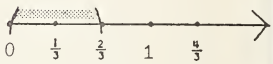

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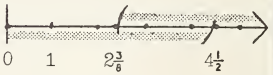
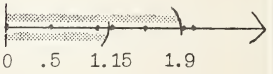


68-17



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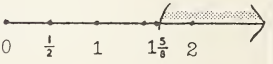


Keeping skilful

68-1 14,528
68-2 10,811
68-3 18,916
68-4 16,570
68-5 92, remainder 0
68-6 1057
68-7 2616
68-8 28,547
68-9 114,910
68-10 30,737
68-11 851, remainder 0
68-12 297,960
68-13 2,415,564
68-14 25,763

68-15 120,792
68-16 111, remainder 6
68-17 405,720
68-18 43,706
68-19 1144
68-20 85, remainder 32
68-21 205, remainder 0
68-22 11,815
68-23 250, remainder 97
68-24 7,366,392

Lesson 110

68-A The set of even numbers from 0 through 10 * The set of multiples of 5 from 5 through 15

68-B Yes * Yes * No

68-C Yes * $A \cup B = \{0, 2, 4, 6, 8, 10, 5, 15\}$. C contains exactly the same members as $A \cup B$.

68-D Yes

69-E The portion of the diagram that represents A and the portion of the diagram that represents B overlap each other.

69-F The portion of the diagram that is shaded vertically, the portion that is shaded horizontally, and the portion that is shaded both vertically and horizontally

69-G Yes * Yes

69-H No * The portion of the diagram that represents L and the portion of the diagram that represents M do not touch each other.

69-I Set $L \cup M$ is the set that contains all the seventh-grade students as well as all the eighth-grade students. $L \cup M$ contains no other members. * The portion of the diagram that is shaded either vertically or horizontally

69-A $s + 2 < 7$. $s + 4 > 6$.

69-B The set of natural numbers from 1 through 8 * $\{1, 2, 3, 4\} * \{3, 4, 5, 6, 7, 8\}$

70-C Yes

70-D Yes

70-E Yes

70-F $\{1, 2, 3, 4, 5, 6, 7, 8\}$ * Yes * No

70-G Yes * The union of the solution set of $s + 2 < 7$ and the solution set of $s + 4 > 6$ contains exactly the same members as the solution set of $s + 2 < 7 \vee s + 4 > 6$.

70-H Yes

70-I $m = 4\frac{1}{2}$. $m > 4\frac{1}{2}$.

70-J R_a * $\{m \mid m = 4\frac{1}{2}\}$. $\{m \mid m > 4\frac{1}{2}\}$

70-K R_a * Yes * Yes

70-L No

70-M Yes * $\{m \mid m = 4\frac{1}{2}\} \cup \{m \mid m > 4\frac{1}{2}\}$, as well as $\{m \mid m = 4\frac{1}{2} \vee m > 4\frac{1}{2}\}$, is made up of those numbers, and only those numbers, that satisfy either or both of the conditions $m = 4\frac{1}{2}$ and $m > 4\frac{1}{2}$.

71-N Yes * Yes

71-O $x + 2.25 < 5.5 \vee x + 6 = 9.25$. * The connective "or" * Z

71-P The solution set of $x = 3.25$ is $\{3.25\}$. The solution set of $x + 6 = 9.25$ is $\{3.25\}$.

71-Q $\{x \mid x < 3.25\}$

71-R Yes

71-S Yes. $x \leq 3.25$ is the same condition as $x < 3.25 \vee x = 3.25$. * The union. The solution set of $x \leq 3.25$ is the same set as the union of the solution sets of $x < 3.25$ and $x = 3.25$.

71-T No * 0 is not a member of the universe.

71-A They are enclosed by squares.

72-B Yes * Each dot in the graph of $\{s \mid s + 2 < 7\}$ is also in the graph of $\{s \mid s + 2 < 7 \vee s + 4 > 6\}$.

72-C Yes * Each dot in the graph of $\{s \mid s + 4 > 6\}$ is also in the graph of $\{s \mid s + 2 < 7 \vee s + 4 > 6\}$.

72-D Yes * Yes

72-E By the bar above the picture of the number line * Yes

72-F Yes

72-G Yes

72-H 0 is not a member of the universe. * By the curved stroke through the dot for point 0

73-I Yes * By the vertical stroke through the dot for point 3.25

73-J Yes * By the bar above the picture of the number line

On your own

73-1 True statement

73-2 False statement

73-3 True statement

73-4 False statement 0 is not a member of the universe.

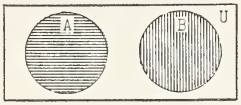
73-5 False statement

73-6 True statement

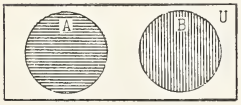
73-7 True statement

73-8 False statement

73-9 True statement
73-10 True statement
73-11 False statement
73-12 False statement
73-13 True statement
73-14 False statement
73-15 True statement
73-16 $A = \{x \mid x = 5\}$.
$B = \{x \mid x < 5\}$.
$A \cup B = \{x \mid x \leq 5\}$.

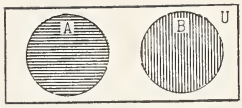


73-17 $A = \{x \mid x > 8.1\}$.
$B = \{x \mid x < 1.6\}$.
$A \cup B = \{x \mid x > 8.1 \vee x < 1.6\}$.



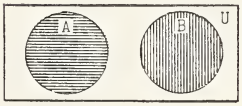
73-18 $A = \{x \mid x = 3\frac{2}{3}\}$.
$B = \{x \mid x > 2\frac{1}{2}\}$.
$A \cup B = \{x \mid x > 2\frac{1}{2}\}$.

73-19 $A = \{x \mid x < 2\}$.
$B = \{x \mid x > 8.5\}$.
$A \cup B = \{x \mid x < 2 \vee x > 8.5\}$.



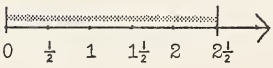
73-20 $A = \{x \mid x > 3\}$.
$B = \{x \mid x > 16\}$.
$A \cup B = \{x \mid x > 3\}$.

73-21 $A = \{x \mid x > 7\frac{1}{2}\}$.
$B = \{x \mid x < 1\}$.
$A \cup B = \{x \mid x > 7\frac{1}{2} \vee x < 1\}$.

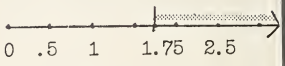


73-22 $A = \{x \mid x > 1\frac{1}{2}\}$.
$B = \{x \mid x = 3\frac{1}{3}\}$.
$A \cup B = \{x \mid x > 1\frac{1}{2}\}$.

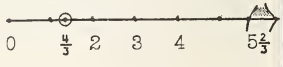
73-23 $\{x \mid x \leq 2\frac{1}{2}\}$



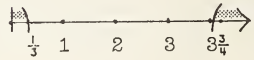
73-24 $\{x \mid x \geq 1.75\}$



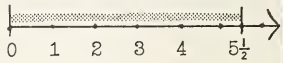
73-25 $\{x \mid x = \frac{4}{3} \vee x > 5\frac{2}{3}\}$



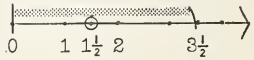
73-26 $\{x \mid x < \frac{1}{3} \vee x > 3\frac{3}{4}\}$



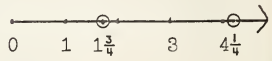
73-27 $\{x \mid x \leq 5\frac{1}{2}\}$



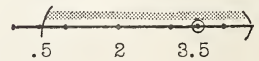
73-28 $\{x \mid x < 3\frac{1}{2} \vee x = 1\frac{1}{2}\}$, or $\{x \mid x < 3\frac{1}{2}\}$



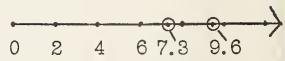
73-29 $\{1\frac{3}{4}, 4\frac{1}{4}\}$



73-30 $\{x \mid x = 3.5 \vee x > .5\}$, or $\{x \mid x > .5\}$



73-31 {7.3, 9.6}



Lesson 111

74-A $x + y = 7$. * x and y * N × N

74-B Yes * N * N * The set of natural numbers

74-C Two ordered pairs are the same only if their first components are the same and their second components are the same. The first component of (2,3) is not the same as the first component of (3,2). The second component (2,3) is not the same as the second component of (3,2). Therefore, (2,3) is not the same ordered pair as (3,2).

74-D Yes

74-E 7 * In the row opposite the letter y and directly below the numeral 0 * Yes

74-F No. There is no natural number that can be added to 8 to obtain a sum of 7. * If x in $x + y = 7$ is replaced by a number greater than 7, then the condition obtained cannot be satisfied by a member of N.

74-G

x	0	1	2	3	4	5	6	7
y	7	6	5	4	3	2	1	0

74-H Yes * 8 * Yes * No * {(0,7), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (7,0)}

74-I N × N * There are two variables in $x + y = 7 \wedge y > x + 1$, and the universe is a set of ordered pairs.

75-J Each solution of a compound condition that includes two simple conditions and the connective "and" must satisfy both of the simple conditions.

75-K (0,7), (1,6), and (2,5)

75-L {(0,7), (1,6), (2,5)} * It is a finite set

75-M Yes

75-N They indicate that the graph of N × N is an infinite set.

75-O Yes

75-P 5 * 7

75-Q The first coördinate is 0 and the second coördinate is 4.

75-R Yes * Yes

75-S Yes. Each encircled dot represents a point associated with an ordered pair of natural numbers that satisfies $y > x + 1$. * The encircled dots are not all of the dots that are in the graph of $\{(x,y) \mid y > x + 1\}$.

75-T Each of these dots is both encircled and enclosed by a square.

75-U Yes * 3

76-A $m + n < 5 \vee m = 3 - n$. * Two * N × N

76-B There are two variables in $m + n < 5 \vee m = 3 - n$ and the universe is a set of ordered pairs.

76-C The connective "or" tells you.

76-D If you replace m in $m + n < 5$ by a number greater than 4, there is no natural number that will satisfy the condition obtained.

76-E 0, 1, 2, 3, 4

76-F Yes * Yes * (0,2), (0,3), and (0,4)

76-G

m	0	1	2	3	4
n	0	0	0	0	0
	1	1	1	1	
	2	2	2		
	3	3			
	4				

76-H 15 * No

76-I It is a finite set. * {(0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (3,0), (3,1), (4,0)}

76-J If you replace n in $m = 3 - n$ by a number greater than 3, there is no natural number that will satisfy the condition obtained. *

m	3	2	1	0
n	0	1	2	3

76-K 4 * Finite * {(3,0), (2,1), (1,2), (0,3)}

76-L Finite * {(0,0), (0,1), (0,2), (0,3), (0,4), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (3,0), (3,1), (4,0)}

76-M Yes

76-N They are enclosed by circles.

76-O They are enclosed by squares.

76-P (0,0), (0,1), (0,2), (0,4), (1,0), (1,1), (1,3), (2,0), (2,2), (3,1), (4,0), * (0,3), (1,2), (2,1), (3,0) * No

76-Q All the dots for the points whose coördinates you named in exercise P

represent ordered pairs that satisfy either or both of the conditions $m + n < 5$ and $m = 3 - n$. * Complete

77-A $6 - x = y \vee x + 3 < 4$.

77-B

x	0	1	2	3	4	5	6
y	6	5	4	3	2	1	0

77-C A finite set * {(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0)}

77-D One * Yes * Yes * Yes * Yes * The condition $x + 3 < 4$ makes no requirement about the second component of each member of the solution set.

77-E Yes

77-F Yes * Since each member of the solution set of $x + 3 < 4$ is a member of the solution set of $6 - x = y \vee x + 3 < 4$ and the solution set of $x + 3 < 4$ is an infinite set, the solution set of $6 - x = y \vee x + 3 < 4$ must also be an infinite set.

77-G They are enclosed by circles. * They are enclosed by squares.

77-H They are either encircled or enclosed by squares or both encircled and enclosed by squares. * All the dots in the graph are not shown in the display.

On your own

77-1 Finite * {(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0), (0,12), (1,10), (2,8), (3,6), (4,4), (5,2)}

77-2 Infinite

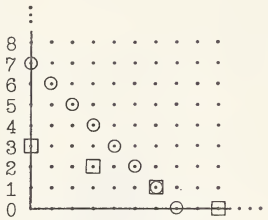
77-3 Infinite

77-4 Infinite

77-5 Finite * {(0,7), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (7,0)}

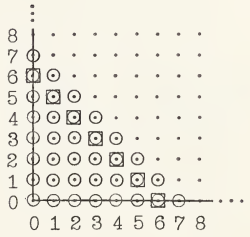
77-6 Infinite

77-7



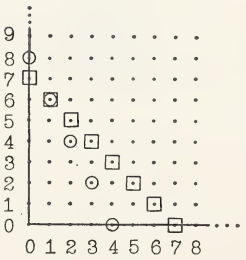
{(0,3), (0,7), (1,6), (2,5), (3,2), (3,4), (4,3), (5,2), (6,1), (7,0), (9,0)}

77-8



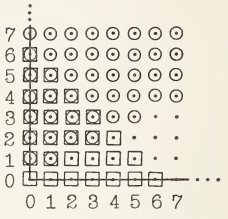
{(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0)}

77-9



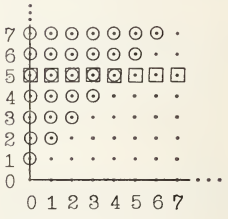
{(0,7), (0,8), (1,6), (2,4), (2,5), (3,2), (3,4), (4,0), (4,3), (5,2), (6,1), (7,0)}

77-10



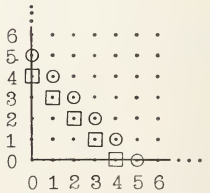
{(0,1), (0,2), (0,3), (0,4), (0,5), (0,6), (1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (3,2), (3,3)}

77-11



Incomplete graph

77-12



{(0,4), (0,5), (1,3), (1,4), (2,2), (2,3), (3,1), (3,2), (4,0), (4,1), (5,0)}

Lesson 112

78-A Yes

78-B Yes

78-C Yes * No

78-D All the members of the universe that are not in subset A are in subset B. These are the only members in subset B.

78-E The set of natural numbers from 1 through 5 * Each member of set M is also a member of the universe.

78-F 3 is a member of the universe but is not a member of M. Therefore, 3 is in the complement of M. Since 4 is a member of M, 4 is not in the complement of M.

79-G 1 and 5 are in set M. * 6 is not a member of the universe. * {2} does not contain all the members of the universe that are not in set M.

79-H {2, 3}

79-I N * A finite set * {0, 1, 2, 3, 4}

79-J Yes * No. * No

79-K Yes. 5 belongs to the universe but is not a member of set D. * Yes. Each natural number greater than 4 is a member of the universe but is not a member of set D. * 4 is a member of D.

79-L An infinite set * {5, 6, 7, ...}

79-M Yes. {5, 6, 7, ...} contains all those members of N, and only those members, that are not in $\{x \mid x < 5\}$.

79-N {0, 2, 4, 6, ...} * {1, 3, 5, 7, ...}

79-O Yes

79-P By the picture of the circle and its interior, which is shaded horizontally

79-Q No portion of the diagram has both vertical and horizontal shading.

79-R The universe * The empty set

79-S Set K

79-T Set M

79-U The universe * The empty set

80-A $x > 6$. * N * {7, 8, 9, ...}

80-B No * No * 1, 2, 4, 5, 6

80-C No * No * Yes

80-D {0, 1, 2, 3, 4, 5, 6}

80-E Yes

80-F Yes

80-G Yes

80-H {0, 1, 2, 3, 4, 5, 6}

80-I Yes * Both $\sim(x > 6)$ and the complement of the solution set of $x > 6$ contain those members of the universe, and only those members, that do not satisfy $x > 6$.

80-J They are enclosed by circles.

80-K The dot for point 6 * Yes

80-L The dots for points 0, 1, 2, 3, 4, and 5 * Yes

81-M The dots for points 0, 1, 2, 3, 4, 5, and 6

81-N Yes

81-O Yes

81-P Yes

81-A $\sim(m = 2)$. * R_a

81-B Yes

81-C 2 * Yes

81-D By the bar above the picture of the number line * No * There is no vertical stroke through the dot for point 2.

81-E Yes

81-F Yes. $\{m \mid \sim(m = 2)\}$ is the same set as $\{m \mid m \neq 2\}$. Therefore, their graphs are the same.

81-G Yes

81-H The bar above the picture of the number line and to the left of the dot for point 2 * The bar above the picture of the number line and to the right of the dot for point 2

81-I By both bars above the picture of the number line * Yes

81-J Yes * Yes

81-K The solution set of $b + 1 < 4$ is the same as the solution set of $b < 3$. Therefore, the solution set of $\sim(b + 1 < 4)$ is the same as the solution set of $\sim(b < 3)$. * The members of N that are not less than 3 are either equal to 3 or greater than 3. * The solution set of $\sim(b < 3)$, or $\sim(b + 1 < 4)$, is the same as the complement of the solution set of $b < 3$.

On your own

81-1 $\overline{A} = \{0, 1, 2, 3\}$.

81-2 $\overline{A} = \{4, 5, 6, ...\}$.

81-3 $\overline{A} = \{\frac{7}{5}\}$.

81-4 $\overline{A} = \{2, 3, 4, ...\}$.

81-5 $\overline{A} = \{0, 1, 2, ..., 10\}$.

81-6 $\overline{A} = \{x \mid x \leq 8\}$.

81-7 $\overline{A} = \{5, 10, 15, \ldots\}$.

81-8 $\overline{A} = \{x \mid x \neq 7\}$, or
$\overline{A} = \{x \mid \sim(x = 7)\}$, or
$\overline{A} = \{x \mid x < 7 \vee x > 7\}$.

81-9 False statement

81-10 True statement

81-11 False statement

81-12 True statement

81-13 True statement

82-14 Yes

82-15 By the picture of the rectangle and its interior

82-16 By the portion of the diagram that has vertical shading * By the portion of the diagram that has horizontal shading * The portion of the diagram that represents set A is completely within the portion of the diagram that represents set B. This is also indicated by the fact that the portion of the diagram that represents set A has both vertical and horizontal shading.

82-17 A * B

82-18 The portion of the diagram that is not shaded * The portion of the diagram that does not have vertical shading

82-19 Yes * Any musical instrument that is not a stringed instrument is not a violin.

82-20 The universe * The universe * The complement of A, or $\overline{A}$

82-21 A * B

82-22 The empty set * The empty set * The complement of B, or $\overline{B}$

82-23 $\{y \mid y \leq 4\}$

82-24 {5, 6, 7, ...}

82-25 $\{m \mid m \leq 6\}$

82-26 {5, 6, 7, ...}

82-27 $\{y \mid \sim(y = 7)\}$, or
$\{y \mid y < 7 \vee y > 7\}$, or
$\{y \mid y \neq 7\}$

82-28 {6}

Checking up

Test 114

82-1 {1, 3, 4, 6}

82-2 {2, 3, 4, 5, 6, 7}

82-3 {8}

82-4 {1, 2, 5, 7, 8}

82-5 {3, 4, 6}

82-6 {1, 2, 3, 4, 5, 6, 7}

82-7 {1, 2, 3, 4, 5, 6, 7, 8}

82-8 { }

82-9 {1, 3, 4, 6, 8}

82-10 True statement

82-11 True statement

82-12 False statement

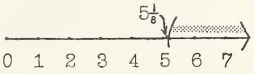
82-13 True statement

82-14 True statement

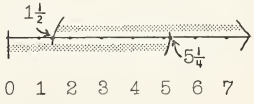
82-15 False statement

Test 115

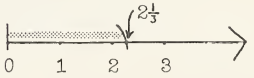
82-16 $\{x \mid x > 5\frac{1}{6}\}$



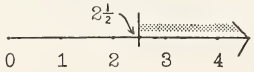
82-17 $\{x \mid 1\frac{1}{2} < x < 5\frac{1}{4}\}$



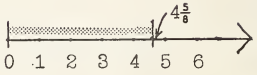
82-18 $\{a \mid a < 2\frac{1}{3}\}$



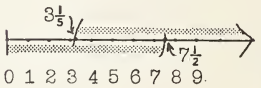
82-19 $\{y \mid y \geq 2\frac{1}{2}\}$



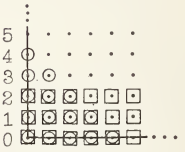
82-20 $\{x \mid \sim(x > 4\frac{5}{8})\}$, or
$\{x \mid x \leq 4\frac{5}{8}\}$



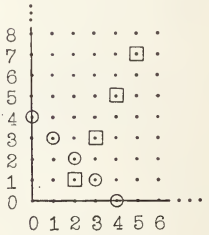
82-21 $\{b \mid 3\frac{1}{5} < b < 7\frac{1}{2}\}$



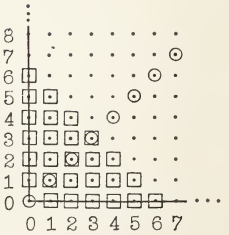
82-22



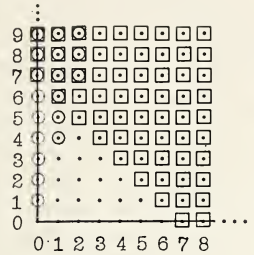
82-23



82-24



82-25



Lesson 113

83-A a was replaced by $1\frac{1}{2}$ and b by $4\frac{2}{3}$. * a was replaced by $3\frac{1}{7}$ and b by $7\frac{1}{3}$. * a was replaced by 1.2 and b by 8.6. * Yes

83-B $3\frac{1}{7} < x \wedge x < 7\frac{1}{3}$. * $1.2 < x \wedge x < 8.6$.

83-C Yes

83-D No. Since $1\frac{1}{2}$ is not less than $1\frac{1}{2}$, $1\frac{1}{2}$ does not satisfy $1\frac{1}{2} < x$. * No. Since $4\frac{2}{3}$ is not less than $4\frac{2}{3}$, $4\frac{2}{3}$ does not satisfy $x < 4\frac{2}{3}$. * Yes. Each rational number of arithmetic that is greater than $1\frac{1}{2}$ and less than $4\frac{2}{3}$ satisfies both $1\frac{1}{2} < x$ and $x < 4\frac{2}{3}$.

83-E No * Yes

83-F 1.2 is not greater than 1.2. 8.6 is not less than 8.6. * The set of rational numbers of arithmetic between 1.2 and 8.6

83-G $6\frac{1}{2} < x < 2$. * No. $6\frac{1}{2}$ is not less than $6\frac{1}{2}$. * No. 2 is not less than 2. * No. There is no member of R_a that is both greater than $6\frac{1}{2}$ and less than 2.

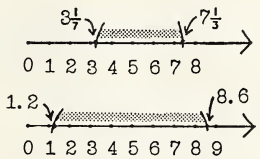
83-H No * No * Yes

83-I The limits of $\{x \mid 1\frac{1}{2} < x < 4\frac{2}{3}\}$ are $1\frac{1}{2}$ and $4\frac{2}{3}$. The limits of $\{x \mid 3\frac{1}{7} < x < 7\frac{1}{3}$ are $3\frac{1}{7}$ and $7\frac{1}{3}$. The limits of $\{x \mid 1.2 < x < 8.6\}$ are 1.2 and 8.6.

84-J Yes * By the bar above the picture of the number line

84-K By the curved stroke through the dot for point $1\frac{1}{2}$ and the dot for point $4\frac{2}{3}$

84-L



84-M R_a

84-N a was replaced by $4\frac{1}{3}$ and b by $6\frac{2}{5}$. * a was replaced by 6.5 and b by 7.2. * Yes

84-O Yes. Since $4\frac{1}{3}$ is equal to $4\frac{1}{3}$, $4\frac{1}{3}$ satisfies $4\frac{1}{3} \leq x$, and since $4\frac{1}{3}$ is less than $6\frac{2}{5}$, $4\frac{1}{3}$ satisfies $x \leq 6\frac{2}{5}$. * Yes. Since $6\frac{2}{5}$ is greater than $4\frac{1}{3}$, $6\frac{2}{5}$ satisfies $4\frac{1}{3} \leq x$, and since $6\frac{2}{5}$ is equal to $6\frac{2}{5}$, $6\frac{2}{5}$ satisfies $x \leq 6\frac{2}{5}$.

84-P Yes * Yes * Yes * Yes

84-Q Any number less than $4\frac{1}{3}$ does not satisfy $4\frac{1}{3} \leq x$. * Any number greater than $6\frac{2}{5}$ does not satisfy $x \leq 6\frac{2}{5}$.

84-R Yes

84-S The sentence $6.5 \leq x \leq 7.2$ expresses the same condition as the sentence $6.5 \leq x \wedge x \leq 7.2$. * Yes * Yes * Yes

84-T Yes

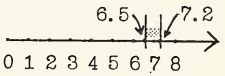
84-U Yes. a satisfies both $a \leq x$ and $x \leq b$. b satisfies both $a \leq x$ and $x \leq b$. * The set of rational numbers of arithmetic that contains a and b and all the rational numbers between a and b

85-V The limits of the interval obtained from condition D are $4\frac{1}{3}$ and $6\frac{2}{5}$. The limits of the interval obtained from condition E are 6.5 and 7.2.

85-W Yes * By the bar above the picture of the number line

85-X By the vertical stroke through the dot for point $4\frac{1}{3}$ and the dot for point $6\frac{2}{5}$

85-Y



85-Z No * $\frac{5}{6}$ and $\frac{7}{4}$ are members of the interval.

85-A R_a

85-B a was replaced by $5\frac{1}{2}$ and b by $7\frac{1}{4}$. * a was replaced by 2 and b by 8. * Yes

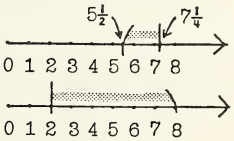
85-C Yes * No * Yes * Yes

85-D Yes

85-E Yes * Yes * No * The set of rational numbers of arithmetic that contains 2 and all the rational numbers between 2 and 8

86-F $5\frac{1}{2}$ and $7\frac{1}{4}$ * 2 and 8

86-G



86-H $\{x \mid 5 < x < 11\}$ *
$\{x \mid 5 \leq x \leq 11\}$ *
$\{x \mid 5 \leq x < 11\}$ or
$\{x \mid 5 < x \leq 11\}$

86-I $\{x \mid 7.8 < x < 9.5\}$ *
$\{x \mid 7.8 \leq x \leq 9.5\}$ *
$\{x \mid 7.8 \leq x < 9.5\}$ or
$\{x \mid 7.8 < x \leq 9.5\}$

86-J $\{x \mid \frac{4}{11} < x < \frac{7}{8}\}$ *
$\{x \mid \frac{4}{11} \leq x \leq \frac{7}{8}\}$ *
$\{x \mid \frac{4}{11} \leq x < \frac{7}{8}\}$ or
$\{x \mid \frac{4}{11} < x \leq \frac{7}{8}\}$

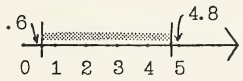
86-K $\{x \mid 6\frac{1}{2} < x < 12\frac{2}{3}\}$ *
$\{x \mid 6\frac{1}{2} \leq x \leq 12\frac{2}{3}\}$ *
$\{x \mid 6\frac{1}{2} \leq x < 12\frac{2}{3}\}$ or
$\{x \mid 6\frac{1}{2} < x \leq 12\frac{2}{3}\}$

86-L $\{x \mid .013 < x < .099\}$ *
$\{x \mid .013 \leq x \leq .099\}$ *
$\{x \mid .013 \leq x < .099\}$ or
$\{x \mid .013 < x \leq .099\}$

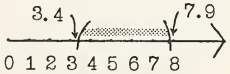
86-M $\{x \mid 162 < x < 204\}$ *
$\{x \mid 162 \leq x \leq 204\}$ *
$\{x \mid 162 \leq x < 204\}$ or
$\{x \mid 162 < x \leq 204\}$

On your own

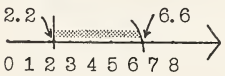
86-1 .6 and 4.8 * Closed interval *



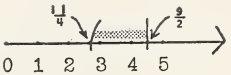
86-2 3.4 and 7.9 * Open interval *



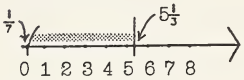
86-3 2.2 and 6.6 * Half-open interval *



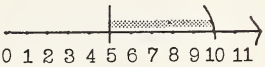
86-4 $\frac{1}{4}$ and $\frac{9}{2}$ * Half-open interval *



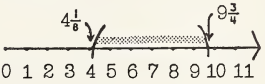
86-5 $\frac{1}{7}$ and $5\frac{1}{3}$ * Half-open interval *



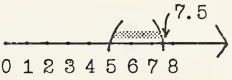
86-6 5 and 10 * Half-open interval *



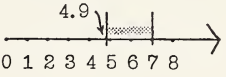
86-7 $4\frac{1}{8}$ and $9\frac{3}{4}$ * Open interval *



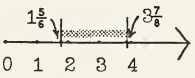
86-8 5 and 7.5 * Open interval *



86-9 4.9 and 7 * Closed interval *



86-10 $1\frac{5}{6}$ and $3\frac{7}{8}$ * Closed interval *



86-11 $\{n \mid 2\frac{1}{2} < n < 4\}$ is an open interval. $\{n \mid 3 \leq n < 6\}$ is a half-open interval. * The limits of $\{n \mid 2\frac{1}{2} < n < 4\}$ are $2\frac{1}{2}$ and 4. The limits of $\{n \mid 3 \leq n < 6\}$ are 3 and 6.

86-12 The union of two given sets is a set that contains all those members, and only those members, that are in either or both of the given sets. $\{n \mid 2\frac{1}{2} < n < 6\}$ contains all the members, and only the members, that are in either or both of the sets $\{n \mid 2\frac{1}{2} < n < 4\}$ and $\{n \mid 3 \leq n < 6\}$.

86-13 3 is the least member of $\{n \mid 3 \leq n < 6\}$ that is also contained in $\{n \mid 2\frac{1}{2} < n < 4\}$. * No

86-14 Yes

86-15 The intersection of two given sets is a set that contains all those members, and only those members, that are in both given sets. $\{n \mid 3 \leq n < 4\}$ contains all those members, and only those members, that are in both $\{n \mid 2\frac{1}{2} < n < 4\}$ and $\{n \mid 3 \leq n < 6\}$.

86-16 $\{h \mid 5.9 \leq h \leq 9.5\}$ *
$\{h \mid 8 \leq h \leq 8.6\}$

86-17 $\{y \mid 1\frac{3}{4} \leq y < 5\frac{1}{9}\}$ *
$\{y \mid 2\frac{1}{4} < y \leq 3\frac{1}{2}\}$

86-18 $\{z \mid \frac{4}{5} < z < \frac{21}{4}\}$ *
$\{z \mid \frac{10}{3} < z < \frac{13}{3}\}$

86-19 $\{m \mid 4 \leq m < 5\frac{1}{3}\}$ *
$\{m \mid 4\frac{7}{8} < m < 5\}$

86-20 $\{x \mid 98 < x \leq 105\}$ *
$\{x \mid 100 \leq x < 101\}$

86-21 $\{b \mid \frac{1}{7} \leq b < \frac{4}{5}\}$ *
$\{b \mid \frac{1}{5} < b \leq \frac{2}{3}\}$

Lesson 114

87-A $r - 8\frac{1}{2} = 11$. $y \neq 20.59$.
$4\frac{1}{3} + s < 12$. $a + 6\frac{2}{5} > 15$.

* R_a

87-B The sentence that expresses condition A includes the symbol for equality. * $\{19\frac{1}{2}\}$

87-C A condition for inequality * 20.59

87-D A condition for inequality * Yes * Yes * No * No * $7\frac{2}{3}$

87-E A condition for inequality * No * Yes * No * $8\frac{3}{5}$

87-F Three is an element of the solution set of $x > 1$. * True statement

87-G Nine is an element of the solution set of $q \geq 9$. * True statement

87-H Four tenths is an element of the solution set of $.4 < y < .6$. * False statement

87-I Eleven halves is an element of the solution set of $2\frac{1}{2} \leq z \leq 10$. * True statement

88-A z is approximately equal to ten

88-B d is approximately equal to fifty and five tenths.

88-C r is approximately equal to three and one half.

88-D y is approximately equal to thirteen.

88-E 6 * 1 * The absolute difference of 7 and 6 is 1. According to the rule, this means that 6 is a solution of $x \approx 7$.

88-F 8 * 1 * Yes

88-G .9 * Yes * Yes * .9 * Yes

88-H .5 * .5 * Yes * Yes

88-I Yes. The absolute difference of 7 and 6.99 is .01, which is less than 1. * Yes. The absolute difference of 7 and 7.01 is .01, which is less than 1.

88-J Yes * Yes

88-K Yes

88-L 1.5 * 1.7

89-M No * Neither the absolute difference of 7 and 5.5 nor the absolute difference of 7 and 8.7 is equal to 1 or less than 1. * No

89-N Yes * The solution set of $x \approx 7$ is the same as the solution set of $6 \leq x \leq 8$. This set is the set that contains 6 and 8 and all the rational numbers between 6 and 8.

89-O 6 and 8 * The limits of $\{x \mid 6 \leq x \leq 8\}$ are members of the interval.

89-P Since $\{x \mid 6 \leq x \leq 8\}$ is the same set as $\{x \mid x \approx 7\}$, their graphs are the same.

89-A Subtract .5 from 7. * 6.5 * Yes

89-B .5 * 7.5 * .5

89-C The absolute difference of 7 and 6.5 is equal to .5. The absolute difference of 7 and 7.5 is equal to .5.

89-D .45 * .459 * Yes

89-E Yes. The absolute difference of 7 and 6.9 is less than .5. * Yes. The absolute difference of 7 and 7 is less than .5. * Yes. The absolute difference of 7 and 7.35 is less than .5.

89-F Yes

89-G No * The absolute difference of 7 and 8 is greater than .5. * No

89-H Yes. $\{x \mid x \approx 7\}$ is the same set as $\{x \mid 6.5 \leq x \leq 7.5\}$. This set contains 6.5 and 7.5 and all the rational numbers between 6.5 and 7.5.

89-I A closed interval * 6.5 and 7.5

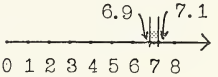
89-J Yes * Different limits were used to determine the solution sets.

90-K Subtract .1 from 7. * Add .1 to 7. * 6.9 * 7.1

90-L Yes * Yes

90-M A closed interval * The limits of $\{x \mid 6.9 \leq x \leq 7.1\}$ are members of the interval.

90-N



* Yes

90-O $x \approx_{.5} 7$. $x \approx_{.1} 7$.

90-P Z

90-Q To find the least number that is a solution of $x \approx_t k$, the tolerance t is subtracted from the given number k. * $k + t$

90-R Yes

90-S $x \approx_{.2} 17$.

91-T $\{x \mid 16.8 \leq x \leq 17.2\}$

91-U Replace k by 11 and t by 4. * $\{x \mid 7 \leq x \leq 15\}$

91-V Yes, except when t is replaced by 0

On your own

91-1 False statement

91-2 True statement

91-3 False statement

91-4 False statement

91-5 False statement

91-6 True statement

91-7 False statement

91-8 True statement

91-9 False statement

91-10 False statement

91-11 $.3 * 3.7$ and $4.3 *$
$\{x \mid 3.7 \leq x \leq 4.3\}$

91-12 $7 * 43$ and $57 *$
$\{z \mid 43 \leq z \leq 57\}$

91-13 $\frac{1}{4} * 16\frac{3}{4}$ and $17\frac{1}{4} *$
$\{x \mid 16\frac{3}{4} \leq x \leq 17\frac{1}{4}\}$

91-14 $4 * 21$ and $29 *$
$\{a \mid 21 \leq a \leq 29\}$

91-15 $.5 * 2.5$ and $3.5 *$
$\{n \mid 2.5 \leq n \leq 3.5\}$

91-16 $1 * 20$ and $22 *$
$\{m \mid 20 \leq m \leq 22\}$

91-17 $.6 * 8.4$ and $9.6 *$
$\{x \mid 8.4 \leq x \leq 9.6\}$

91-18 $.09 * 1.41$ and $1.59 *$
$\{a \mid 1.41 \leq a \leq 1.59\}$

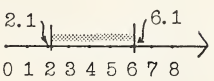
91-19 $\frac{1}{8} * 70\frac{7}{8}$ and $71\frac{1}{8} *$
$\{b \mid 70\frac{7}{8} \leq b \leq 71\frac{1}{8}\}$

91-20 $10 * 90$ and $110 *$
$\{q \mid 90 \leq q \leq 110\}$

91-21 $.25 * 3.75$ and $4.25 *$
$\{x \mid 3.75 \leq x \leq 4.25\}$

91-22 $.05 * .55$ and $.65 *$
$\{m \mid .55 \leq m \leq .65\}$

91-23



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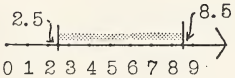
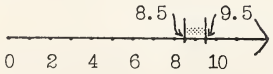
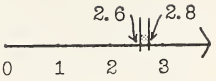


91-25



91-26



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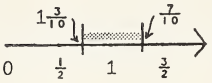
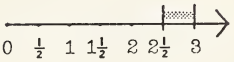


91-28



Keeping skilful

91-1 $\frac{5}{6}$

91-2 $\frac{5}{1}$

91-3 $\frac{37}{3}$

91-4 $\frac{14}{15}$

91-5 $\frac{11}{12}$

91-6 $\frac{19}{21}$

91-7 $\frac{7}{11}$

91-8 $\frac{1}{1}$

91-9 $\frac{25}{2}$

91-10 $\frac{11}{1}$

91-11 $\frac{3}{2}$

91-12 $\frac{1}{16}$

91-13 $4\frac{1}{12}$

91-14 $10\frac{3}{5}$

91-15 $1\frac{1}{2}$

91-16 $101\frac{1}{2}$

91-17 $12\frac{1}{9}$

91-18 $18\frac{1}{3}$

91-19 $17\frac{1}{20}$

91-20 $3\frac{1}{3}$

91-21 $1\frac{12}{19}$

91-22 $1\frac{1}{5}$

91-23 $166\frac{2}{3}$

91-24 $12\frac{3}{8}$

91-25 $26\frac{1}{6}$

91-26 $9\frac{1}{6}$

91-27 $2\frac{11}{12}$

91-28 $29\frac{1}{20}$

91-29 $\frac{1}{40}$

91-30 $1\frac{11}{15}$

91-31 $2\frac{2}{3}$

91-32 $32\frac{34}{35}$

91-33 $2\frac{22}{25}$

91-34 $5\frac{13}{15}$

91-35 $1\frac{14}{17}$

91-36 $40\frac{13}{20}$

91-37 34

91-38 $1\frac{9}{100}$

91-39 10

91-40 $78\frac{2}{5}$

91-41 130

91-42 $14\frac{10}{19}$

91-43 $5\frac{15}{16}$

91-44 $34\frac{4}{5}$

91-45 2649

91-46 11, remainder 27

91-47 5846

91-48 55,432

91-49 54,907

91-50 3, remainder 119

91-51 3,057,054

91-52 7006

91-53 851,088

91-54 2019

91-55 78,636

91-56 7600

91-57 119, remainder 51

91-58 191,563

91-59 205

91-60 71,478

Lesson 115

92-A No * Sentence A contains placeholders. A sentence that contains placeholders expresses neither a true nor a false idea.

92-B Three

92-C Yes * Yes

92-D All three of them

92-E x, y, and z

92-F Two * Three * Two

92-G Replace x by 4, y by 2, and z by 6.

93-H {(4,2,3), (4,2,6), (4,3,3), (4,3,6),

(4,5,3), (4,5,6),
(7,2,3), (7,2,6),
(7,3,3), (7,3,6),
(7,5,3), (7,5,6)}

93-I 2 * 3 * 2 * 12

93-J Find the product of the number of members in Q, the number of members in V, and the number of members in W. The product you obtain is the number of ordered triples whose first components are members of Q, whose second components are members of V, and whose third components are members of W.

93-K Statements C and E * {(4,2,3), (4,3,3)}

94-A Yes * Yes * No

94-B Three * $\{(x,y,z) \mid x \in N \wedge y \in N \wedge z \in N\}$

94-C An infinite set * N is an infinite set. Therefore, the set of ordered triples formed from N is an infinite set.

94-D $x + y + z = 12$. $x + y < 7$. $y = 3$. * The connective "and"

94-E There are three variables in condition A and the universe is a set of ordered triples.

94-F The connective "and" means that each solution of the compound condition must satisfy each of the simple conditions that make up the compound condition.

94-G $y = 3$.

94-H 3 * An infinite set

94-I $x + y < 7$.

94-J Yes * 0, 1, 2, 3

94-K 0, 1, 2, 3 * 3 is the only solution of $y = 3$ and y is the second component of (x,y,z).

94-L 9 * 8 * 7 * 6 * Yes

94-M 9

94-N 8

94-O (2,3,7) and (3,3,6)

94-P {(0,3,9), (1,3,8), (2,3,7), (3,3,6)} * Yes

95-A Condition F is made up of three simple conditions and the connective "and." * 3

95-B $\{(x,y,z) \mid x \in N \wedge y \in N \wedge z \in N\}$

95-C $y < 4$.

95-D The second component is less than 4. * An infinite set

95-E 5 * Yes

95-F 4 * Yes

95-G When y is replaced by 2 and x is replaced by 3, both $y < 4$ and $x + y = 5$ become true statements. * When y is replaced by 3 and x is replaced by 2 both $y < 4$ and $x + y = 5$ become true statements.

95-H No * No * No

95-I 0 * 0 * Yes

95-J {(2,3,0)}

On your own

95-1 (10:15 P.M., 9:15 P.M., 8:15 P.M.)

95-2 ($4\frac{2}{3}$ ft., 7 ft., $5\frac{1}{2}$ ft.)

96-3 (4,5,1)

96-4 No * Two ordered triples are the same only if their first components are the same, their second components are the same, and their third components are the same.

96-5 (2,4,6)

96-6 (5,10,15)

96-7 ($1\frac{1}{2}$, 3, $4\frac{1}{2}$)

96-8 ($\frac{2}{3}, \frac{4}{3}, \frac{6}{3}$), or ($\frac{2}{3}, \frac{4}{3}$,2)

96-9 (.25,.50,.75)

96-10 (15,30,45)

96-11 {(2.5,.3,4), (2.5,.3,.25), (2.5),.1,4), (2.5,.1,.25), (3.4,.3,4), (3.4,.3,.25), (3.4,.1,4), (3.4,.1,.25)}

96-12 2(2.5) + 3(.3) − 4 < 5. T
2(2.5) + 3(.3) − .25 < 5.
2(2.5) + 3(.1) − 4 < 5. T
2(2.5) + 3(.1) − .25 < 5.
2(3.4) + 3(.3) − 4 < 5. T
2(3.4) + 3(.3) − .25 < 5.
2(3.4) + 3(.1) − 4 < 5. T
2(3.4) + 3(.1) − .25 < 5.

96-13 2.5 + .3 + 4 = 6.8. T
2.5 + .3 + .25 = 6.8.
2.5 + .1 + 4 = 6.8.
2.5 + .1 + .25 = 6.8.
3.4 + .3 + 4 = 6.8.
3.4 + .3 + .25 = 6.8.
3.4 + .1 + 4 = 6.8.
3.4 + .1 + .25 = 6.8.

96-14 125

96-15 {(3,9,1)}

96-16 {(5,0,2), (5,1,2), (5,2,2)}

96-17 {(0,0,4), (0,1,3), (0,2,2), (0,3,1), (0,4,0), (1,0,3), (1,1,2), (1,2,1), (1,3,0), (2,0,2), (2,1,1), (2,2,0), (3,0,1), (3,1,0), (4,0,0)}

96-18 {(2,0,0), (1,0,0), (1,0,1), (1,1,0), (0,0,0), (0,0,1), (0,1,0), (0,1,1)}

96-19 {(4,1,0), (4,1,1), (5,0,0), (5,0,1)}

96-20 {(0,0,5), (0,1,4), (0,2,3), (0,3,2), (1,0,5), (1,1,4),

(1,2,3), (2,0,5), (2,1,4)}

Checking up

Test 116

96-1 $r < s \wedge s < t$.

96-2 The idea of "approximately equal to"

96-3 Closed interval * $\frac{1}{2}$ and $\frac{3}{4}$

96-4 Half-open interval * 2.5 and 7.6

96-5 Half-open interval * 50 and 51

96-6 Closed interval * $2\frac{1}{6}$ and $8\frac{1}{3}$

96-7 Open interval * 4.9 and 5.7

96-8 Closed interval * $3\frac{1}{2}$ and $3\frac{3}{4}$

96-9 Closed interval * 8.5 and 9.5

96-10 Open interval * $\frac{1}{980}$ and $\frac{1}{713}$

96-11 8

96-12 $\frac{2}{3}$ and $2\frac{1}{3}$

96-13 Neither

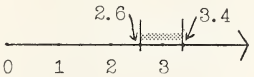
96-14 4.5 and 4.7

96-15 $2\frac{2}{3}$ and $3\frac{2}{3}$

96-16 $\frac{1}{6}$

96-17 $\{x \mid 2.6 \leq x \leq 3.4\}$

96-18



96-19 $\{x \mid 1\frac{7}{8} < y \leq 2\}$

96-20 Half-open interval * $1\frac{7}{8}$ and 2

Test 117

96-21 120

96-22 {(0,0,3), (0,1,2), (0,2,1), (0,3,0), (1,0,2), (1,1,1), (1,2,0), (2,0,1), (2,1,0), (3,0,0)}

96-23 {(0,3,0), (0,3,1)}

96-24 {(0,6,0), (1,6,6), (2,6,12), (3,6,18)}

96-25 {(8,0,0), (7,1,0), (6,2,0), (5,3,0), (4,4,0)}

Lesson 116

97-A (Column 1) No

97-B (Column 1) 27 * The number of feet of copper tubing Mr. Ames had after he bought 27 ft. more

97-C (Column 1) $x + 27$ refers to the number of feet of copper tubing Mr. Ames had after he bought the 27 feet more. The problem states that this amount of tubing is less than $40\frac{3}{8}$ feet.

97-D (Column 1) Each non-zero rational number of arithmetic that satisfies $x + 27 < 40\frac{3}{8}$ must be less than $13\frac{3}{8}$. The symbols $\{x \mid x + 27 < 40\frac{3}{8}\}$ and $\{x \mid x < 13\frac{3}{8}\}$ are both standard descriptions of the solution set of the condition $x + 27 < 40\frac{3}{8}$. The sets represented by $\{x \mid x + 27 < 40\frac{3}{8}\}$ and $\{x \mid x < 13\frac{3}{8}\}$ contain the same members.

97-E (Column 1) No. Since 0 is not a member of the universe, it cannot be a member of the solution set. * You cannot name the least member of the solution set of $x + 27 < 40\frac{3}{8}$.

97-F (Column 2, top) Yes

97-A (Column 2) $n - 5.25$ refers to the amount of money Sam had left after he spent \$5.25. The problem tells you that this amount of money is greater than \$20.00 * The amount of money Sam had before he spent \$5.25

97-B (Column 2) The problem states that the amount of money Sam had before he spent \$5.25 was less than \$30.00.

97-C (Column 2) Yes

97-D (Column 2) You can use $n - 5.25 = 20.00$ to help you find the members of R_a that satisfy $n - 5.25 > 20.00$. Since 25.25 satisfies $n - 5.25 = 20.00$, each number that satisfies $n - 5.25 > 20.00$ must be greater than 25.25. Therefore, $\{n \mid n > 25.25\}$ contains those numbers, and only those numbers, that satisfy $n - 5.25 > 20.00$.

97-E (Column 2) No * Only those members of $\{n \mid n > 25.25\}$ that are less than 30.00 satisfy $n < 30.00$.

97-F (Column 2, bottom) Yes * $\{n \mid 25.25 < n < 30.00\}$ contains those members, and only those members, of Z that satisfy both $n - 5.25 > 20.00$ and $n < 30.00$.

98-G (Column 1) It is an open interval in R_a. The limits are not members of the interval. * 25.25 and 30.00. An open interval is the set of all members of R_a between

two given numbers. The two given numbers are the limits of the interval. In this case, all the members of R_a between 25.25 and 30.00 satisfy $n - 5.25 > 20.00 \wedge n < 30.00$.

98-H (Column 1) Yes * No. Only those solutions given in hundredths would be sensible answers because of the monetary system.

98-I (Column 1) Yes. 25.26 is the least number in hundredths that is greater than 25.25. * 29.99

98-A No

98-B No

98-C The number of hits the two boys made together

98-D $x + y$ refers to the number of hits the boys made together. The problem states that the total number of hits is less than 4. * Two

98-E When a condition contains two variables, a replacement must be made for each variable in order to obtain a statement. Since the variables occur in a special order, the replacement must also occur in a special order. You know that a pair of objects that occur in a special order is an ordered pair. * The solution set of the condition consists of all members of the universe that satisfy the condition. Since the solution set is a subset of the universe and is a set of ordered pairs, the universe must be a set of ordered pairs.

98-F 3 * 0, 1, 2, 3

98-G (Column 2)

x	0	1	2	3
y	0	0	0	0
	1	1	1	
	2	2		
	3			

* {(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (2,0), (2,1), (3,0)}

98-H (Column 2) The number of hits Nick made * The number of hits Scott made

98-I (Column 2) If Nick made 1 hit, Scott made 0 hits, 1 hit, or 2 hits. If Nick made 2 hits, Scott made 0 hits or 1 hit. If Nick made 3 hits, Scott made 0 hits.

99-J No

99-K No

99-L The number of boys and girls who are on the staff of the school newspaper

99-M $a + b$ refers to the number of boys and girls who are on the staff. The problem states that there are fewer than 9 boys and girls on the staff.

99-N The problem states that the number of girls on the staff is less than the number of boys. Since a is a variable for the number of girls on the staff and b is a variable for the number of boys, $a < b$, and not $b < a$, is another simple condition for the problem.

99-O Yes

99-P Yes * The set whose members are all ordered pairs (a,b) such that a is an element of C and b is an element of C

99-Q The numbers from 1 through 7 are the only members of C that, when used to replace a in $a + b < 9$, give conditions that can be satisfied by members of C. *

a	1	2	3	4	5	6	7
b	1	1	1	1	1	1	1
	2	2	2	2	2	2	
	3	3	3	3	3		
	4	4	4	4			
	5	5	5				
	6	6					
	7						

99-R Twelve

99-S {(1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5)}

99-T If there is 1 girl on the staff, there are 2, 3, 4, 5, 6, or 7 boys. If there are 2 girls on the staff, there are 3, 4, 5, or 6 boys. If there are 3 girls on the staff, there are 4 or 5 boys.

99-A No

99-B No

99-C The problem states that Jane has 35 more stamps

than Beverly. This means that when you subtract the number of stamps Beverly has (m) from the number of stamps Jane has (n), the difference is equal to 35.

99-D Yes * Yes * The problem states that Beverly has more than 145 stamps but fewer than 150 stamps. This means that 145 is less than the number of stamps Beverly has (m), and the number of stamps Beverly has (m) is less than 150. Remember that the sentence $145 < m < 150$ expresses the same condition as the sentence $145 < m \wedge m < 150$.

99-E Yes

99-F $\{(m,n) \mid m \in N \wedge n \in N\}$

100-G 146, 147, 148, and 149 * 146, 147, 148, and 149 *

m	146	147	148	149
n	181	182	183	184

100-H Each ordered pair that satisfies $n - m = 35 \wedge 145 < m < 150$ must have a first component that is between 145 and 150, and a second component that is 35 greater than its first component. The ordered pairs named in the chart meet both of these requirements.

100-I {(146,181), (147,182), (148,183), (149,184)}

100-J If Beverly has 146 stamps, Jane has 181. If Beverly has 147 stamps, Jane has 182. If Beverly has 148 stamps, Jane has 183. If Beverly has 149 stamps, Jane has 184.

100-K No

100-L No

100-M The number of stoves and refrigerators that were installed

100-N Yes * The number of stoves and refrigerators installed last year ($x + y$) is equal to 7.

100-O $x \geq 3$. * x, not y, is the variable for the number of stoves that were installed.

100-P Yes

100-Q The universe for (x,y) must be a set of ordered pairs. 5 is not an ordered pair. * Yes * $\{(x,y) \mid x \in C \wedge y \in C\}$

100-R The first component must be equal to 3 or greater than 3.

100-S Neither 1 nor 2 satisfies $x \geq 3$. * If a number greater than 6 is used as a replacement for x in $x + y = 7$, the condition obtained cannot be satisfied by a member of C. *

x	3	4	5	6
y	4	3	2	1

100-T {(3,4), (4,3), (5,2), (6,1)}

100-U If 3 new stoves were installed, 4 new refrigerators were installed. If 4 new stoves were installed, 3 new refrigerators were installed. If 5 new stoves were installed, 2 new refrigerators were installed. If 6 new stoves were installed, 1 new refrigerator was installed.

100-A Yes

100-B $\sim(x = 3)$.

101-C Yes

101-D {(0,5), (1,4), (2,3), (3,2), (4,1), (5,0)}

101-E The first component of each ordered pair that satisfies $x + y = 5 \wedge \sim(x = 3)$ can be any member of N except 3. * Five

101-F {(0,5), (1,4), (2,3), (4,1), (5,0)} * Yes

On your own

101-1 x, number of tulip bulbs Mr. Butler had in his garden before he planted the 48.
$x + 48 < 200$.
{1, 2, 3, ..., 151}.
From 1 through 151 tulip bulbs

101-2 x, area of Puerto Rico in square miles.
$4411 - x = 976$.
{3435}. 3435 sq. mi.

101-3 x, amount of money Sally has now.
$x < 25.00 \wedge x + 10 > 32.50$.
$\{x \mid 22.50 < x < 25.00\}$.
Between \$22.50 and \$25.00

101-4 x, amount of wire Paul used.
$6\frac{3}{4} - x < 1\frac{1}{2} \wedge x < 6\frac{3}{4}$.
$\{x \mid 5\frac{1}{4} < x < 6\frac{3}{4}\}$.
Between $5\frac{1}{4}$ ft. and $6\frac{3}{4}$ ft. of wire

101-5 x, number of place mats Sandy made.
y, number of place mats

Marcia made.
$x + y < 6$.
$U = \{(x,y) \mid x \in C \wedge y \in C\}$.
{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}.
Each member of the solution set can be used to obtain an answer. The answer obtained from (2,3) is: Sandy made 2 place mats and Marcia made 3 place mats.

101-6 x, number of tickets Kay sold for the first performance.
y, number of tickets Kay sold for the second performance.
$x + y = 15 \wedge x \geq 7$.
$U = \{(x,y) \mid x \in C \wedge y \in C\}$.
{(7,8), (8,7), (9,6), (10,5), (11,4), (12,3), (13,2), (14,1)}.
From 1 through 8 tickets

101-7 x, number of typewriters the company bought.
y, number of typewriters the company had after they bought more.
$35 + x = y \wedge x \leq 5$.
$U = \{(x,y) \mid x \in C \wedge y \in C\}$.
{(1,36), (2,37), (3,38), (4,39), (5,40)}.
Each member of the solution set can be used to obtain an answer. The answer obtained from (3,38) is: The Peerless Letter Service bought 3 typewriters and then had 38 typewriters in all.

101-8 x, number of boys' gowns ordered.
y, number of girls' gowns ordered.
$x + y = 427 \wedge 200 < y < 215$.
$U = \{(x,y) \mid x \in N \wedge y \in N\}$.
{(226,201), (225,202), (224,203), (223,204), (222,205), (221,206), (220,207), (219,208), (218,209), (217,210), (216,211), (215,212), (214,213), (213,214)}.
Each member of the solution set can be used to obtain an answer. The answer obtained from (226,201) is 226 boys' gowns and 201 girls' gowns.

101-9 $x + y = 6886 \wedge x - 2829 = 1238$.
$U = \{(x,y) \mid x \in N \wedge y \in N\}$.
{(4067,2819)}

101-10 $y - x = 8 \wedge \sim(x + 2 > 6)$.
$U = \{(x,y) \mid x \in N \wedge y \in N\}$.
{(0,8), (1,9), (2,10), (3,11), (4,12)}

Lesson 117

102-A The number of miles that is the distance between the two airports * The number of minutes it takes the helicopter to fly the 16 miles

102-B Since the first component of 16/9 refers to a number of miles, the first component of the rate pair you are to find must also refer to a number of miles. * Since the second component of 16/9 refers to a number of minutes, the second component of the rate pair you are to find must also refer to a number of minutes.

102-C The sentence that expresses the condition $16/9 \sim x/60$ includes the symbol for equivalence ($\sim$). The condition $16/9 \sim x/60$ means that the rate pair $x/60$ is equivalent to the rate pair 16/9.

102-D Yes

103-E $9x$ has been multiplied by $\frac{1}{9}$. You know that $\frac{1}{9}$ is the reciprocal of 9 because the product of 9 and $\frac{1}{9}$ is 1.

103-F The associative property of multiplication

103-G Yes

103-H $106\frac{2}{5}$ * Yes

103-I $106\frac{2}{5}$ * $106\frac{2}{5}$ * Yes

103-J $\{106\frac{2}{5}\}$

103-K The number of students enrolled in summer classes last summer

103-L 31

103-M Yes. A per cent is a rate pair that represents a comparison and that has a second component of 100. * The comparison of the number of students that was the increase in enrollment with each 100 students enrolled in summer classes last summer

103-N It is the first component. * Since the first component of 31/100 refers

to the increase in enrollment, the first component of the rate pair you are to find must also refer to the increase in enrollment. * The number of students enrolled in summer classes last summer

104-O It is equivalent to 31/100.

104-P Yes * {500}

104-Q 500 students

104-A Yes

104-B No * A rate whose first component refers to the total length of the 3 portages and whose second component refers to the number of portages

104-C An average is the first component of a rate pair whose second component is 1. For the problem in D4, the average distance portaged per one portage is equivalent to the total distance portaged per 3 portages.

104-D y is a variable for the total length of the portages. The problem states that the lengths of the portages were 120 yd., 174 yd., and 212 yd. This means that the total length of the 3 portages is equal to the sum of 120, 174, and 212. * y, and not x, is the variable for the total length of the portages.

104-E In order to find the average length per portage, you must first find the total length of the three portages. This means that both of the simple conditions, $x/1 \sim y/3$ and $y = 120 + 174 + 212$, must be satisfied to obtain an answer to the problem. Therefore, the connective "and" is used in the compound condition for the problem.

104-F 506

104-G $168\frac{2}{3}$

104-H $\{(168\frac{2}{3}, 506)\}$

104-I $168\frac{2}{3}$ * $168\frac{2}{3}$ is the replacement for x that satisfies the condition $x/1 \sim 506/3$. The problem asks you to find the average length per portage. Since x, not y, is the variable for the average length, $168\frac{2}{3}$ is used to get the answer to the problem.

105-J The average length per portage was $168\frac{2}{3}$ yd.

105-K 1 * Yes

105-L Since the first component of 5/1 refers to a number of correct answers, the first component of each rate pair that is equivalent to 5/1 must also refer to a number of correct answers.

105-M The number of incorrect answers Linda had

105-N The rate pair 5/1 represents the comparison of the number of correct answers with the number of incorrect answers. Since the rate pair x/y represent the same comparison as the rate pair 5/1, x/y is equivalent to 5/1.

105-O The problem states that Linda had at most 20 correct answers. The words "at most 20" mean equal to 20 correct answers or less than 20 correct answers. * x refers to the number of correct answers Linda had. The problem states that Linda had at the most 20 correct answers. This means that x is equal to 20 or less than 20. * x, not y, refers to the number of correct answers Linda had.

105-P Yes. A proportional relation is a set of ordered pairs in which each member is equivalent to each of the other members. {5/1, ...} contains those members, and only those members, that are equivalent to 5/1. * An infinite set

105-Q There are infinitely many members of {5/1, ...}. Examples are 10/2, 15/3, 20/4, 25/5, and 30/6. * Yes

105-R 5/1, 10/2, 15/3, and 20/4

105-S {5/1, 10/2, 15/3, 20/4}

105-T The problem asks you to find the number of incorrect answers Linda had. y is the variable for the number of incorrect answers Linda had and is the second component of each solution of $5/1 \sim x/y \wedge x \leq 20$.

105-U 1, 2, 3, or 4 incorrect answers

106-A Yes

106-B Since the first component of $\frac{1}{12}/1$ refers to an amount of money the

monthly payment can be, the first component of each rate pair that is equivalent to $\frac{1}{12}/1$ must also refer to an amount of money that the monthly payment can be. * 126

106-C The problem asks what amount of $126 Mr. Owen will have to pay each month if he pays $\frac{1}{12}$ of the amount of the bill each month. The rate pair $\frac{1}{12}/1$ represents the comparison of the part of a dollar with each one dollar of the bill. The rate pair $x/126$ represents the same comparison and is equivalent to $\frac{1}{12}/1$.

106-D The comparison of Mr. Owen's monthly payment of $\frac{1}{3}$ of a dollar with each one dollar of the amount of the bill

106-E The rate pair $\frac{1}{3}/1$ represents the comparison of an amount of money Mr. Owen's monthly payment can be with each one dollar of his total bill. The total amount of Mr. Owen's monthly payment (x) is the first component of a rate pair that has a second component of 126 (the total amount of the bill) and that is equivalent to $\frac{1}{3}/1$.

106-F Yes * Mr. Owen has a choice of two payments. He can pay either $\frac{1}{12}$ of the total amount or he can pay $\frac{1}{3}$ of the total amount. He will make only one payment. Since each number that satisfies either $\frac{1}{12}/1 \sim x/126$ or $\frac{1}{3}/1 \sim x/126$ can be used to get an answer to the problem, the connective "or" is used in the compound condition for the problem in D7.

106-G The union

106-H $10\frac{1}{2}$, or 10.50 * 42

106-I $\{10\frac{1}{2}, 42\}$, or $\{10.50, 42\}$

106-J Mr. Owen's monthly payment can be either $10.50 or $42.

On your own

106-1 x, amount of money that was the regular price of the stove.
$25/100 \sim 94.75/x$.
$U = \{x \mid x \in Z\}$.
$\{379\}$. $379

106-2 x, number of times as many slices of bread in a loaf as is the daily requirement.
$26/3 \sim x/1$.
$U = \{x \mid x \in Z\}$.
$\{8\frac{2}{3}\}$. $8\frac{2}{3}$ times

106-3 x, number of Mr. Winthrop's commission.
$4/100 \sim x/750$.
$U = \{x \mid x \in Z\}$.
$\{30\}$. $30

106-4 x, fraction the number of science fiction books John read was of the number of these books Allen read.
$18/24 \sim x/1$.
$U = \{x \mid x \in Z\}$.
$\{\frac{3}{4}\}$. The number of science fiction books that John read was $\frac{3}{4}$ of the number of these books that Allen read.

107-5 x, number of golf balls Dave can buy.
$.75/1 \sim 5.00/x$.
$U = \{x \mid x \in Z\}$.
$\{6\frac{2}{3}\}$. 6 golf balls

107-6 x, number of hours it takes to travel between Elmwood and Greenvale.
$60/1 \sim 175/x \vee$
$75/1 \sim 175/x$.
$U = \{x \mid x \in Z\}$.
$\{2\frac{11}{20}, 2\frac{1}{3}\}$. It takes either $2\frac{11}{20}$ hr. or $2\frac{1}{3}$ hr. to travel from Elmwood to Greenvale by train.

107-7 x, first component of the per cent of decrease in the cost of Doug's automobile insurance.
y, amount of money the cost of his insurance decreased.
$x/100 \sim y/150 \wedge$
$150 - 120 = y$.
$U = \{(x,y) \mid x \in Z \wedge y \in Z\}$.
$\{(20,30)\}$. 20%

107-8 x, amount of discount Karen receives.
$25/100 \sim x/12 \vee$
$15/100 \sim x/12$.
$U = \{x \mid x \in Z\}$.
$\{3.00, 1.80\}$. It is possible for Karen to receive a discount of either $3.00 or $1.80.

107-9 x, amount of Mr. Rush's yearly take-home pay.
$523.45/1 \sim x/12$.
$U = \{x \mid x \in Z\}$.
$\{6281.40\}$. $6281.40

107-10 x, number of stamps Tom gave away.
y, number of stamps Tom gave Jim.
$\frac{1}{3}/1 \sim y/x \wedge y \leq 10$.
$U = \{(x,y) \mid x \in Z \wedge y \in Z\}$.
$\{(x,y) \mid \frac{1}{3}/1 \sim y/x \wedge$

$y \leqq 10\}$.

You cannot tabulate the solution set of $\frac{1}{3}/1 \sim y/x \wedge y \leqq 10$. You should assume that Jim did not give away a fraction of a stamp. Therefore, the only members of the universe that you can use to get sensible answers to the problem and that also satisfy the condition are (3,1), (6,2), (9,3), (12,4), (15,5), (18,6), (21,7), (24,8), (27,9), and (30,10). Each of these ordered pairs can be used to obtain an answer to the problem. The answer obtained from (18,6) is: Tom gave 18 stamps away and he gave 6 of these stamps to Jim.

107-11 x, length in inches of the curtains before she washed them.

y, length in inches of the curtains after she washed them.

$3/100 \sim 3\frac{1}{4}/x \wedge x - 3\frac{1}{4} = y$.

$U = \{(x,y) \mid x \in Z \wedge y \in Z\}$.

$\{(108\frac{1}{3}, 105\frac{1}{12})\}$. $105\frac{1}{12}$ in.

107-12 $3/5 \sim x/y \wedge y - x = 12$.

$U = \{(x,y) \mid x \in Z \wedge y \in Z\}$.

$\{18/30\}$

107-13 $20/100 \sim x/y \wedge x \leqq 12$.

$U = \{(x,y) \mid x \in Z \wedge y \in Z\}$.

Since you cannot tabulate the solution set of this condition, you can give a standard description of the solution set.

$\{(x,y) \mid 20/100 \sim x/y \wedge x \leqq 12\}$.

Keeping skilful

107-1 $\frac{1}{4}$

107-2 $\frac{42}{25}$

107-3 $\frac{103}{10}$

107-4 $\frac{6}{625}$

107-5 $\frac{3}{2000}$

107-6 $\frac{1}{1000}$

107-7 $\frac{51}{2}$

107-8 $\frac{7}{8}$

107-9 $\frac{302}{5}$

107-10 $\frac{17}{10000}$

107-11 $\frac{11}{200}$

107-12 $\frac{1}{16}$

107-13 $\frac{83}{100}$

107-14 $\frac{3}{40}$

107-15 $\frac{1}{80}$

107-16 $\frac{501}{5}$

107-17 .65

107-18 .4

107-19 5.2

107-20 13.5

107-21 .75

107-22 .375

107-23 7.25

107-24 .9

107-25 .36

107-26 .84

107-27 .013

107-28 10.5

107-29 .66

107-30 .15

107-31 .039

107-32 .002

107-33 93.6

107-34 5.467

107-35 .0325

107-36 1.8

107-37 1.05

107-38 .637

107-39 1.7390

107-40 .53600

107-41 20.395

107-42 .0289

107-43 18.95

107-44 7.2

107-45 2.95

107-46 .111

107-47 18.5711

107-48 .136620

107-49 2.778

107-50 153.3

107-51 .0937

107-52 4.186

107-53 72.58

107-54 667.1

107-55 $11\frac{3}{4}$

107-56 $11\frac{1}{8}$

107-57 51

107-58 $2\frac{13}{90}$

107-59 $17\frac{19}{26}$

107-60 $\frac{1}{34}$

107-61 $23\frac{7}{10}$

107-62 5

107-63 $2\frac{7}{15}$

107-64 17

107-65 $4\frac{9}{16}$

107-66 $18\frac{6}{11}$

107-67 $56\frac{14}{25}$

107-68 $5\frac{1}{39}$

Lesson 118

108-A No

108-B $22\frac{1}{2}$ lb. * The problem states that the total weight of the three fish was $22\frac{1}{2}$ lb. This means that the sum of the weights of the pike (x), of the bass (y), and of the catfish (z) is equal to $22\frac{1}{2}$.

108-C The problem states that the pike weighed $1\frac{1}{2}$ times as much as the bass. "$1\frac{1}{2}$ times" means $1\frac{1}{2}$ compared with 1. * Yes

108-D The first component of x/y refers to the number of pounds the pike weighed. The second component of x/y refers to the number of pounds the bass weighed.

108-E $1\frac{1}{2}/1$ represents the comparison of the number of pounds the pike weighed with the number of pounds the bass weighed. x/y represents the same comparison as $1\frac{1}{2}/1$ and is equivalent to $1\frac{1}{2}/1$.

108-F 15 lb. * The problem states that the pike weighed 15 lb. less than the total weight of the three fish. This means that when the weight of the pike (x) is subtracted from the total weight of the three fish ($22\frac{1}{2}$), the difference is equal to 15.

108-G Yes

108-H $22\frac{1}{2} - x = 15$ includes only one variable. Each of the other conditions includes at least two variables.

108-I $7\frac{1}{2}$

108-J The first component of each solution is $7\frac{1}{2}$. * Since the universe is given for each variable in (x,y,z), x is the first component of each ordered triple in the universe. $7\frac{1}{2}$ is the only replacement for x that satisfies $22\frac{1}{2} - x = 15$. Therefore, $7\frac{1}{2}$ is the first component of each solution.

108-K $1\frac{1}{2}/1 \sim 7\frac{1}{2}/y$. * 5

108-L 5

108-M 10

109-N (Column 1) 10

109-O (Column 1) $\{(7\frac{1}{2},5,10)\}$

109-P (Column 1) Yes * Yes * Yes

109-A No

109-B No

109-C The number of dollars it cost to rent both bicycles for 3 days

109-D No

109-E Both $x + y$ and z refer to the number of dollars it cost to rent both bicycles for 3 days.

109-F 2 times * The problem states that the rent for the tandem bicycle was 2 times the rent for the regular bicycle. This means that the rent for the tandem bicycle was \$2 for each \$1 that was the rent for the regular bicycle. The rate pair 2/1 represents this comparison.

109-G The first component of 2/1 refers to the rent for the tandem bicycle. Since the first component of each rate pair that is equivalent to 2/1 must also refer to the rent for the tandem bicycle, y, and not x, is the first component of the rate pair you are to find. The second component of 2/1 refers to the rent for the regular bicycle. Since the second component of each rate pair that is equivalent to 2/1 must also refer to the rent for the regular bicycle, x, and not y, is the second component of the rate pair you are to find.

109-H 2/1 represents the comparison of the rent for the tandem bicycle with the rent for the regular bicycle. y/x represents the same comparison as 2/1 and is equivalent to 2/1

109-I \$1.50 * The first component of 1.50/1 refers to the rent for the regular bicycle. The second component refers to a number of days.

109-J The problem states that the rent for one day was \$1.50 and asks you to find what the rent was for 3 days. This means that you must find a rate pair that is equivalent to 1.50/1 and that has a second component of 3. * The number of dollars that is the rent for the regular bicycle for 3 days

109-K Yes

109-L $1.50/1 \sim x/3$. * $1.50/1 \sim x/3$ includes only one variable. Each of the other conditions include at least two variables.

109-M 4.50 * 4.50

109-N (Column 2) 9.00

109-O (Column 2) 4.50 is the first component and 9.00 is the second component.

109-P (Column 2) 13.50

109-Q {(4.50,9.00,13.50)}

110-R \$13.50 * The third component

110-A The problem states that the sum of three numbers (r, s, and t) is less than 5.

110-B The problem states that the first number (r) is less than 3.

110-C $r - s = 2$. * The problem states that the difference of the first number and

the second number is 2. This means that when the second number (s) is subtracted from the first number (r), the difference is 2.

110-D Yes

110-E 0, 1, and 2

110-F Yes

110-G No * No * 0

110-H 2 * 0

110-I $2 + 0 + t < 5$. * 0, 1, and 2

110-J Three * {(2,0,0), (2,0,1), (2,0,2)}

110-K Yes

On your own

110-1 x, amount of money Rita spent for the bias tape.
y, amount of money Rita spent for the thread.
z, amount of money Rita spent in all.
$.12/1 \sim x/5 \wedge .15/2 \sim y/6 \wedge x + y = z$.
{(.60,.45,1.05)}. $1.05

110-2 x, number of nickels.
y, number of dimes.
z, number of pennies.
$x < y \wedge z + 3 = 15 \wedge z - y = 5$.
{(1,7,12), (2,7,12), (3,7,12), (4,7,12), (5,7,12), (6,7,12)}.
Bud has from 1 through 6 nickels, 7 dimes and 12 pennies.

110-3 x, Marjorie's age.
y, Sue's age.
z, Donna's age.
$x + y = z \wedge x - y = 2 \wedge x + 3 = 18$.
{(15,13,28)}. Marjorie is 15 years old, Sue is 13 years old, and Donna is 28 years old.

110-4 x, number of cans of beets.
y, number of cans of peas.
z, number of cans of carrots.
$x + y + z < 10 \wedge x < 3 \wedge x + y = 7$.
{(1,6,1), (1,6,2), (2,5,1), (2,5,2)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (1,6,2) is: Mrs. Olsen bought 1 can of beets, 6 cans of peas, and 2 cans of carrots.

111-5 x, amount of loan Mr. Angeletti paid the credit union at the end of one month.
y, amount of interest he paid at the end of one month.
z, total amount of money Mr. Angeletti paid the credit union at the end of one month.
$\frac{1}{12}/1 \sim x/600 \wedge 1/100 \sim y/600 \wedge x + y = z$.
{(50,6,56)}. $56

111-6 x, number of pounds Judy lost.
y, number of pounds Harriet lost.
z, number of pounds Carol lost.
$2/1 \sim x/y \wedge 2/1 \sim z/x \wedge y + x = 12$.
{(8,4,16)}. Judy lost 8 lb., Harriet lost 4 lb., and Carol lost 16 lb.

111-7 x, amount of money Mr. Harper's commission increased.
y, amount of his commission the second year.
z, amount of commission that he received for the two years.
$17/100 \sim x/700 \wedge 700 + x = y \wedge 700 + y = z$.
{(119,819,1519)}. $1519

111-8 $x > 8 \wedge x + y = 10 \wedge 7 - z = y$.
{(9,1,6), (10,0,7)}

111-9 $x < 4 \wedge y = 4 - x \wedge y + z < 3$.
{(2,2,0), (3,1,0), (3,1,1)}

111-10 There are 3 possible conditions for this problem. These are expressed below with their solution sets. Any one of the three is a correct response.
$x + y = 6 \wedge x = 2y \wedge x - y = z$.
{(4,2,2)}.
$x + z = 6 \wedge x = 2y \wedge x - y = z$.
{(4,2,2)}.
$y + z = 6 \wedge x = 2y \wedge x - y = z$.
{(6,3,3)}.

Applying mathematics

111-1 x, number of albums Marge loaned to Jane.
y, number of albums Marge had left.
$12 - x = y \wedge 0 < x < 12$.
{(1,11), (2,10), (3,9), (4,8), (5,7), (6,6), (7,5), (8,4), (9,3), (10,2), (11,1)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (7,5) is: Marge loaned Jane 7 albums and had 5 albums left.

111-2 x, number of stamps Tom had before he sold the 245 stamps.
$x > 300 \wedge x - 245 < 75$.
{301, 302, 303, ..., 319}.
From 301 through 319 stamps

111-3 x, number of new cars sold last month.
y, number of used cars sold last month.
$x + y = 14 \wedge x \leq 5$.
{(0,14), (1,13), (2,12), (3,11), (4,10), (5,9)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (2,12) is: Mr. Miller sold 2 new cars and 12 used cars last month.

111-4 $x + y + z = 15 \wedge x < 5 \wedge y = x + 1$.
{(0,1,14), (1,2,12), (2,3,10), (3,4,8), (4,5,6)}

111-5 x, number of seventh-grade students who are members of the Camera Club.
y, number of eighth-grade students who are members of the Camera Club.
$x + y = 22 \wedge x \geq 15$.
{(15,7), (16,6), (17,5), (18,4), (19,3), (20,2), (21,1)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (16,6) is: 16 seventh-grade students and 6 eighth-grade students belong to the Camera Club.

111-6 x, number of tennis rackets Mr. Hanley sold on Saturday.
y, number of tennis balls he gave away.
$4/1 \sim y/x \wedge x \leq 8$.
{(1,4), (2,8), (3,12), (4,16), (5,20), (6,24), (7,28), (8,32)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (4,16) is: Mr. Hanley sold 4 tennis rackets and gave away 16 tennis balls on Saturday.

112-7 x, total attendance for the three nights.
y, average attendance per night.
z, difference of the third night attendance and the average attendance.
$200 + 264 + 286 = x \wedge y/1 \sim x/3 \wedge 286 - y = z$.
{(750,250,36)}. 36

112-8 $14/15 \sim x/y \wedge y - x = 5$.
{70/75}

112-9 x, number of consecutive push-ups Jim can do.
$\frac{2}{5}/1 \sim x/20$.
{8}. 8 consecutive push-ups

112-10 x, amount of money that is Mr. Bond's monthly take-home pay.
$82.80/100 \sim x/420$.
{347.76}. $347.76

112-11 x, distance from Toronto to Saint John.
$2.5/1 \sim 1680/x$.
{672}. About 672 mi.

112-12 x, amount of money that was the price of 18 photographs.
$2.70/12 \sim x/18$.
{4.05}. $4.05

112-13 x, amount of money that was the Sanford family's income last year.
$25.5/100 \sim 1530/x$.
{6000}. $6000

112-14 x, number of weeks it will take Ed to save $36.00.
$1.50/1 \sim 36.00/x \vee 1.80/1 \sim 36.00/x$.
{24, 20}. It will take Ed either 24 weeks or 20 weeks to save $36.00.

112-15 x, number of hats sold during July.
y, number of hats left at the end of July.
z, number of hats sold during August.
$\frac{2}{3}/1 \sim x/1500 \wedge 1500 - x = y \wedge \frac{4}{5}/1 \sim z/y$.
{(1000,500,400)}.
1000 hats were sold during July and 400 hats were sold during August.

112-16 $90.9/121.2 \sim x/100$.
{75}. 75%

112-17 $12\frac{1}{2}/100 \sim x/84$, or $\frac{1}{8}/1 \sim x/84$.
$\{10\frac{1}{2}\}$

112-18 $66\frac{2}{3}/100 \sim 318\frac{2}{5}/x$, or $\frac{2}{3}/1 \sim 318\frac{2}{5}/x$.
$\{477\frac{3}{5}\}$

Checking up

Test 118

112-1 x, number of unused spaces for autographs.
y, number of autographs collected.
$x + y = 50 \wedge x \leq 12$.
{(1,49), (2,48), (3,47), (4,46), (5,45), (6,44), (7,43), (8,42), (9,41), (10,40), (11,39), (12,38)}.
From 38 through 49 autographs

112-2 x, number of fire extinguishers on the first floor.
y, number of fire extinguishers on the second floor.
$x + y = 28 \wedge x \leq 12$.
{(1,27), (2,26), (3,25), (4,24), (5,23), (6,22), (7,21), (8,20), (9,19), (10,18), (11,17), (12,16)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (11,17) is: There are 11 fire extinguishers on the first floor and 17 on the second floor.

112-3 x, number of minutes Jean spent washing dishes that week.
y, average number of minutes Jean spent washing dishes per day.
$17 + 9 + 12 + 14 + 10 + 18 + 26 = x \wedge x/7 \sim y/1$.
{(106,15$\frac{1}{7}$)}. 15$\frac{1}{7}$ min.

112-4 x, amount of interest added to Bill's account.
$64.00 + x > 64.50 \wedge 64.00 + x < 65.50$.
$\{x \mid .50 < x < 1.50\}$.
The amount of interest was between \$.50 and \$1.50

112-5 x, total number of runs the pitcher allowed.
y, total number of innings he pitched.
z, number of runs allowed for each 9 innings pitched.
$2 + 5 + 6 = x \wedge 9 + 12 + 4 = y \wedge x/y \sim z/9$.
{(13,25,4.68)}.
The pitcher allowed 4.68 runs for each 9 innings that he pitched.

112-6 $49/100 \sim x/23$.
{11.27}

112-7 $35/100 \sim 53\frac{1}{2}/x$.
{152$\frac{6}{7}$}

112-8 $24/38 \sim x/1$.
{$\frac{12}{19}$}

Lesson 113

113-A Yes * Yes * {E, F, G, H, I, J}

113-B E, F, and J

113-C No * Yes

113-D Since ℓ_1 contains points E, F, and G, which are in the plane represented in D1, ℓ_1 is included in this plane.

113-E Points contained in the same line are collinear points. Since points E, F, and G are contained in ℓ_1, they are collinear.

113-F No * Yes

113-G Point H * The set of all points is called space.

113-H Three noncollinear points determine a plane. The names of any three noncollinear points can be used to name the plane that they determine. Points S, T, and N are noncollinear because they are not contained in the same line. * Points V, F, and R are noncollinear points. * Points V, S, and T are collinear points. Collinear points do not determine a plane because they are in infinitely many planes.

114-I The set of points contained in plane STN

114-J Point F, together with point S, determine segment SF, which is congruent to segment ST. * $\overline{ST} \cong \overline{ST}$ is a true statement because each segment is congruent to itself.

114-K V, J, K, and G

114-L A circle * It is infinite.

114-M Point S

114-N No. S does not satisfy $\overline{SX} \cong \overline{ST}$. The centre of a circle is not a member of the circle. * Yes. $\overline{SV} \cong \overline{ST}$ is a true statement. Therefore, V satisfies $\overline{SX} \cong \overline{ST}$.

114-O A radius of a circle is a segment determined by the centre of the circle and a point in the circle. Point S is the centre of ⊙ S. Point T is in ⊙ S. Therefore, segment ST is a radius of circle S. * Segments SF, SV, and SG. Points J and K along with the centre of ⊙ S determine radii SJ and SK.

114-P Segment SR * Since R is between S and F, $\overline{SR} < \overline{SF}$. Segment SF is congruent to segment ST. Therefore, $\overline{SR} < \overline{ST}$.

114-Q D, R, S, H, and I * No * Yes

114-R Condition C requires that each point you are to find must, together with point S, determine a segment greater than a radius. * Since G is between S and M, $\overline{SM} > \overline{SG}$. Segment SG is congruent to segment ST. Therefore, $\overline{SM} > \overline{ST}$.

114-S O, E, N, M, and W * No * Yes

114-A {D, C} * {D, F, B, E, C}

115-B ℓ_2 and ⊙ K have points D and C as common members. Geometric figures that have common members are said to meet, or to intersect. * {D, C}

115-C A segment whose endpoints are in a circle is a chord of that circle. The endpoints of $\overline{CD}$ are in ⊙ K. * $\overline{EF}$ is a chord of ⊙ K that is represented in D4.

115-D One. Two points determine just one line. * ℓ_2 * Yes * Each point in chord CD is also a member of ℓ_2.

115-E Secant ℓ_3 * {E, F} * Chord EF

115-F A chord that contains the centre of a circle is a diameter of the circle. * Each point in chord EF is also in ℓ_3.

115-G One. The diameter of ⊙ K that contains E must also contain K. Points E and K determine exactly one line. This line includes just one diameter of ⊙ K. * One. Points C and K determine exactly one line, which includes exactly one diameter of ⊙ K.

115-H Yes. Each secant of a circle intersects the circle in two points. These two points determine a chord of the circle. * No. Only those secants that contain the centre of the circle include a diameter of that circle.

115-I {B, D} * {B, C}

115-J ℓ_4 and ⊙ A have point B in common. * {B}

115-K ℓ_5 does not intersect ⊙ A. * { }

On your own

116-1 Point N

116-2 $\overline{QU}$

116-3 $\overline{NQ}$, $\overline{NR}$, $\overline{NU}$, and $\overline{NT}$

116-4 $\overline{TQ}$, $\overline{TR}$, $\overline{TU}$, $\overline{QR}$, $\overline{QU}$, and $\overline{RU}$

116-5 Points J, M, and N

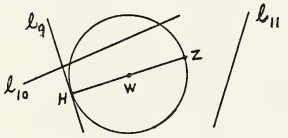
116-6 Points C, G, K, B, and S

116-7 ℓ_7

116-8 ℓ_8

116-9 { } * { } * The set of points between T and U

116-10 See the sketch below.



116-11 See the sketch for 58-10.

116-12 See the sketch for 58-10.

116-13 See the sketch for 58-10.

Lesson 120

116-A $\overrightarrow{AC}$ * Point A

116-B Vertex A * Side AB and side AC

116-C Points B and C * One

116-D $\overline{EF}$ is a chord of ⊙ D that contains the centre of the circle. * The union of rays DE and DF is a line and does not form an angle.

117-E No * Two points in a circle determine a central angle only if the two points are not endpoints of a diameter.

117-F Points A, B, and C * Points H, I, G, N, and R * Points A, B, C, H, I, G, N, and R

117-G Points B, C, G, and H

117-H Points J, K, L, S, and V * Points A, B, C, J, K, L, S, and V

117-I Points B, C, J, and V

117-J Two. A line separates a plane that includes the line into two half-planes. * Line EF. A line is the boundary of each half-plane into which it separates the plane. The boundary is not included in either half-plane.

117-K The points in the same half-plane as L are H, G, T, and N. * Points B, E, D, and F * Points B, D, E, F, G, H, N, L, and T

118-L Points E, H, N, and F

118-M Diameter EF

118-N Two * Point E, S, and F * Points E and F

118-O Yes * Line KJ includes a diameter of ⊙ G. $\overset{\frown}{KIJ}$ is the subset of ⊙ G that is included in the union of $\overleftrightarrow{KJ}$ and one of the half-planes bounded by $\overleftrightarrow{KJ}$. * ∠HGJ is a central angle of ⊙ G. $\overset{\frown}{HDJ}$ is the subset of ⊙ G that is included in the union of ∠HGJ and the interior of ∠HGJ. * ∠IGJ is a central angle of ⊙ G. $\overset{\frown}{IHJ}$ is the subset of ⊙ G that is included in the union of ∠IGJ and the exterior of ∠IGJ.

118-P Yes * $\overset{\frown}{IDJ}$

118-Q Two

118-R $\overset{\frown}{BCD}$ and $\overset{\frown}{BLD}$ are two different semicircles of the circle.

119-S A major arc

119-A 60 * 120 * Find the abso-

lute difference of 60 and 120. This is 120 − 60, or 60. Therefore, the measure of ∠LCQ is 60.

119-B 30 * 90 * An angle whose measure in degrees is 90 is a right angle.

119-C Minor $\overset{\frown}{PR}$, or $\overset{\frown}{PTR}$, and major $\overset{\frown}{PR}$, or $\overset{\frown}{PSR}$ * ∠PKR

119-D 70

120-E 45

120-F One * {R}

120-G $\overset{\frown}{SLM}$ and $\overset{\frown}{MRT}$ are two minor arcs of the same circle that intersect in exactly one point. * No. $\overset{\frown}{SLR}$ and $\overset{\frown}{MRT}$ intersect in more than one point. * No. $\overset{\frown}{SLM}$ and $\overset{\frown}{PTR}$ do not intersect.

120-H Minor $\overset{\frown}{LP}$. Examples of other names for this arc are $\overset{\frown}{LRP}$ and $\overset{\frown}{PML}$. * Yes

120-I 45 * 70 * 115

120-J 115 * Yes * Yes

120-K 90 * Each angle formed by two intersecting perpendicular lines is a right angle. A right angle has a measure in degrees of 90.

120-L 180. Semicircle BCD is the union of the two adjacent arcs, minor $\overset{\frown}{BC}$ and minor $\overset{\frown}{CD}$. Since each of these arcs has a measure in degrees of 90, the measure of semicircle BCD in degrees is equal to 90 + 90, or 180. * 180. $\overset{\frown}{CDE}^{\circ}$ = minor $\overset{\frown}{CD}^{\circ}$ + minor $\overset{\frown}{DE}^{\circ}$.

120-M Two * {B, D}

121-N Yes * Semicircle CDE and semicircle CBE have exactly two points in common. Therefore, $\overset{\frown}{CDE} \cup \overset{\frown}{CBE}$, or ⊙ A, has a measure in degrees of 180 + 180, or 360.

121-O Yes * {N, P} * Circle M

121-P 75 * 360 − 75, or 285

121-Q 360 − 27, or 333

your own

121-1 ∠APD

121-2 80

121-3 $\overset{\frown}{AMD}$

121-4 Each of the symbols $\overset{\frown}{AND}$, $\overset{\frown}{ABD}$, $\overset{\frown}{ACD}$, and $\overset{\frown}{AED}$ may be used to name major $\overset{\frown}{AD}$.

121-5 80

121-6 360 − 80, or 280

121-7 $\overset{\frown}{AC}$

121-8 Semicircle ABC and semicircle AEC

121-9 180 − 30, or 150

121-10 360 − 120, or 240

121-11 180

121-12 180

121-13 40

121-14 18

121-15 360 − 18, or 342

121-16 22

121-17 180 − 40, or 140

121-18 140

121-19 Minor $\overset{\frown}{SH}$

121-20 Major $\overset{\frown}{SK}$

Lesson 121

122-A Sides UV, VW, WX, XY, YZ, and ZU * Vertices U, V, W, X, Y, and Z * Six

122-B Polygon HIJKL

122-C Polygon PQRS. A quadrilateral is a simple polygon that has four sides. * △ABC. A triangle is the union of three segments determined by any three noncollinear points. The symbol △ means triangle.

122-D Side WZ * Side WX

122-E Yes. Parallel segments are segments that are included in parallel lines. * { }. The intersection of two parallel lines is the empty set.

122-F Yes

122-G No

123-H No. A trapezoid has two, and only two, parallel sides. * Yes. A quadrilateral in which opposite sides are parallel is a parallelogram.

123-I Side AD * Yes * Side CD * Yes

123-J ∠A * ∠D

123-K Opposite sides of a parallelogram are congruent. If two segments are congruent, then their measures are equal. * Opposite angles of a parallelogram are congruent. If two angles are congruent, then their measures are equal.

123-L 70 * 70

123-M No * If a polygon is a parallelogram, it has more than two parallel sides. If a polygon is a trapezoid, one pair of opposite sides are not parallel.

123-A $m(\overline{EF}) = 1\frac{1}{4}$. $m(\overline{FG}) = 2\frac{5}{8}$. $m(\overline{GH}) = 1\frac{1}{4}$. $m(\overline{HE}) = 2\frac{5}{8}$. * If one angle of a parallelogram is a right angle, then each of the angles of the parallelogram is a right angle.

123-B A rectangle is a parallelogram in which each angle is a right angle. * Two lines that intersect to form a right angle are perpendicular. Two

segments are perpendicular if they intersect and if they are subsets of lines that are perpendicular.

123–C $\overline{EF}$ and $\overline{EH}$ meet in point E and $\angle FEH$ is a right angle. * $\overline{FG}$ and $\overline{GH}$ meet in point G and $\angle FGH$ is a right angle. * $\overline{EH}$ and $\overline{GH}$ meet in point H and $\angle EHG$ is a right angle.

124–D A quadrilateral in which the opposite sides are parallel is a parallelogram.

124–E 90

124–F A parallelogram in which each angle is a right angle is a rectangle. * Each side of IJKL is congruent to each of the other sides.

124–G Sides IJ and LK * Yes * Yes

124–H Opposite sides of a parallelogram are congruent and congruent segments have the same measure. Therefore, each side of parallelogram RSTU has a measure of $\frac{7}{8}$. Since each side has the same measure, each side of RSTU is congruent to each of the other sides.

124–I Yes * In a square, each angle is a right angle and has a measure of 90.

124–J Yes * IJKL is a parallelogram in which each of the sides is congruent to each of the other sides.

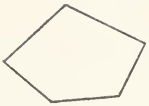
124–K Yes * No * Yes * No

your own

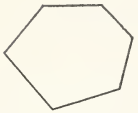
124–I

124–2 Three

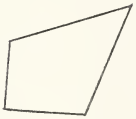
124–3



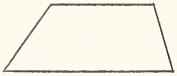
124–4



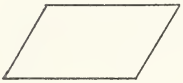
124–5



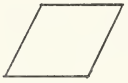
124–6



124–7



124–8



*

124–9

124–10 $\overline{EH}$ is not parallel to $\overline{FG}$. * A trapezoid

125–11 ABCD and VWXY

125–12 ABCD and QRST

125–13 ABCD

125–14 Sides AB, BC, CD, DA, RQ, ST, VW, WX, XY, and YV

125–15 Sides RS and TQ

125–16 Angles A, B, C, D, E, H, Q, R, S, and T

125–17 70 * 110

Keeping skilful

125–1 {80}

125–2 {1, 2, 3, ..., 126}

125–3 {156, 157, 158, ...}

125–4 {3}

125–5 {1, 2, 3, ..., 28}

125–6 {10}

125–7 {220}

125–8 {142}

125–9 {(1,1), (1,2), (1,3)}

125–10 {(4,96), (3,97), (2,98), (1,99)}

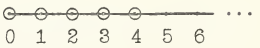
125–11 {(3,1), (4,2)}

125–12 {(3,1), (6,2), (9,3), (12,4)}

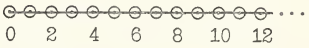
125–13 {(9,5), (18,10)}

125–14 {(7,8), (14,16), (21,24)}

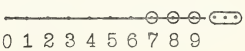
125–15 {0, 1, 2, 3, 4}



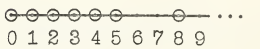
125–16 {0, 1, 2, ..., 12}



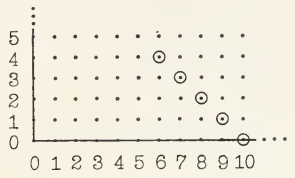
125–17 {7, 8, 9, ...}



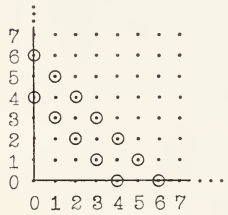
125–18 {0, 1, 2, 3, 4, 5, 8}



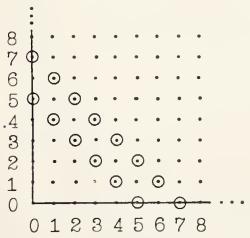
125–19 {(6,4), (7,3), (8,2), (9,1), (10,0)}



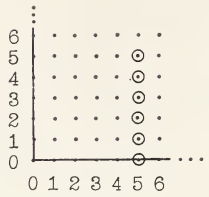
125–20 {(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0), (0,4), (1,3), (2,2), (3,1), (4,0)}



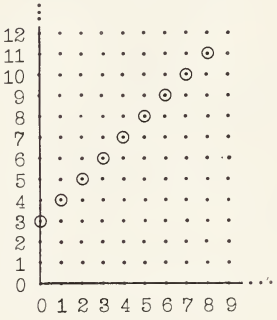
125–21 {(7,0), (6,1), (5,2), (4,3), (3,4), (2,5), (1,6), (0,7), (0,5), (1,4), (2,3), (3,2), (4,1), (5,0)}



125–22 {(5,0), (5,1), (5,2), (5,3), (5,4), (5,5)}
The graph of $\{(a,b) \mid a + 10 = 15 \wedge b + 1 < 7\}$ is at the top of the next column.



125–23 {(0,3), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (8,11)}



Lesson 122

125–A Sides AB, BC, and CA. Vertices A, B, and C

125–B Side AB and side BC

125–C $\overline{CA} > \overline{BC}$.

125–D $\angle C * \angle A$

125–E 45 * 45. The angles opposite the congruent sides of an isosceles triangle are congruent. * 180. For any triangle, the sum of the measures of the angles in degrees is 180. * 90. $\angle A^\circ + \angle C^\circ = 45 + 45$, or 90. Therefore, $90 + \angle B^\circ = 180$, or $\angle B^\circ = 90$.

126–F Three

126–G $\angle F * \angle D * \angle E$

126–H 60 * 60 * Yes

126–I In an equilateral triangle, any two angles are opposite congruent sides. Therefore, each of the angles is congruent to each of the other angles. Since the sum of the measures of the three angles in degrees is 180, each of the angles must have a measure in degrees of 60.

126–J $m(\overline{HI}) > m(\overline{GH})$. Therefore, $\overline{HI} > \overline{GH}$. * Yes. If one segment is greater than a second segment, and the second segment is greater than a third segment, then the first segment is greater than the third segment.

126–K If two angles of ΔGHI were congruent, the two opposite sides would be congruent. No two sides of ΔGHI are congruent. You also know that only one angle of a triangle can have a measure in degrees equal to 90 or greater than 90.

126–L ΔGHI. A scalene triangle is a triangle with no two of its sides congruent. * ΔDEF. An equilateral triangle is a triangle with its three sides congruent. * ΔDEF and ΔABC. An isosceles triangle is a triangle with two of its sides congruent.

126–M $\angle A$, $\angle C$, $\angle D$, $\angle E$, $\angle F$, $\angle G$, and $\angle I * \angle B * \angle H$

126–N ΔDEF. An acute triangle is a triangle that has three acute angles. * ΔABC. A right triangle

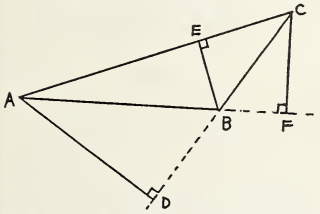
is a triangle with one right angle. * $\triangle GHI$.
An obtuse triangle is a triangle in which one angle is an obtuse angle. * $\triangle DEF$

126–A Sides JK, KL, and LJ.
Angles LJK, JKL, and KLJ

126–B An altitude of a triangle is a segment determined by a vertex of the triangle and a point in the line that includes the side opposite the vertex. The altitude is perpendicular to the line that includes the side opposite the vertex. $\overline{JM}$ is determined by vertex J and a point in ℓ_1 that includes side KL. $\overline{JM}$ is perpendicular to ℓ_1. * Side KL.
When $\overline{JM}$ is used as an altitude, the side opposite vertex J is the base.

126–C Three

126–D Obtuse triangle ABC is represented below. The altitudes are $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.



126–E Yes * $m(\overline{QN}) = m(\overline{NR})$.

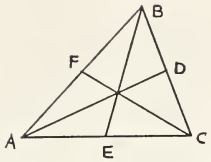
127–F One

127–G P is a vertex of $\triangle PQR$.
N is the midpoint of the side opposite vertex P.

127–H $\overline{QV}$

127–I Three

127–J Acute $\triangle ABC$ is represented at the top of the next column. The medians are $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.



127–K Rays ST, SU, and SW

127–L Ray SU

127–M $\angle TSU$ * $\angle USW$

127–N $m\angle TSU = m\angle USW$.

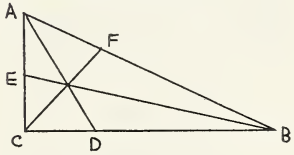
127–O One

127–P Sides LM, MN, and NL.
Angles NLM, LMN, and MNL

127–Q Since the measure of $\angle LMG$ in degrees is equal to the measure of $\angle NMG$ in degrees, angle LMG is congruent to angle NMG. * Yes

128–R No

128–S Right $\triangle ABC$ is represented below. The bisectors of the angles of $\triangle ABC$ are $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.



On your own

128–1 $\angle ACB$

128–2 Side BC

128–3 35

128–4 35

128–5 $\overline{BJ}$ is a median of $\triangle ABC$.

128–6 90

128–7 60

128–8 30

128–9 $\overline{FK}$ is an angle bisector of $\triangle DFE$.

128–10 120

128–11 Side ED

128–12 60

128–13 90 * 30

128–14 $\triangle ABC$, $\triangle ABJ$, $\triangle BCJ$, $\triangle DEF$, $\triangle DFK$, $\triangle EFK$, $\triangle GHI$, $\triangle GHL$, and $\triangle GIL$

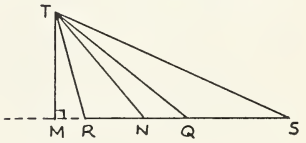
128–15 $\triangle ABC$, $\triangle GHI$, and $\triangle DFK$ are isosceles triangles.

128–16 $\triangle DEF$, $\triangle GHL$, $\triangle GIL$, and $\triangle FEK$ are right triangles.

128–17 $\triangle ABC$, $\triangle ABJ$, $\triangle BCJ$, and $\triangle DFK$ are obtuse triangles.

128–18 $\triangle GHI$

128–19 See the sketch below.



Keeping skilful

128–1 {78, 79, 80, ...}

128–2 { }

128–3 {11, 12, 13, ...}

128–4 {0, 1, 2, 3, 4, 5, 6, 7, 8, 26}

128–5 {26}

128–6 {11, 12, 13, ...}

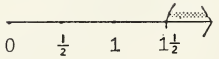
128–7 $\{\frac{4}{9}\}$

128–8 {15.25}

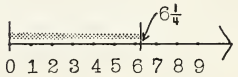
128–9 {9}

128–10 {40.5}

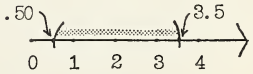
128–11 $\{a \mid a > 1\frac{1}{2}\}$



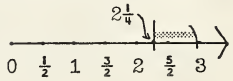
128–12 $\{x \mid x \leq 6\frac{1}{4}\}$



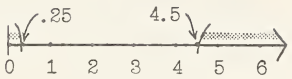
128–13 $\{y \mid .50 < y < 3.5\}$



128–14 $\{z \mid 2\frac{1}{4} \leq x < 3\}$



128–I5 $\{c \mid c > 4.5 \vee c < .25\}$



Lesson 123

130–A Infinitely many * Infinitely many

130–B {N} * Three noncollinear points determine a plane. Since points T, N, and S are noncollinear, these points determine plane TNS. If a line contains two points of a plane, the line is included in the plane. Since line MN contains points N and T, line MN is included in plane TNS. Since line RS contains points S and N, line RS is included in plane TNS. * Examples of names of the plane that include $\overleftrightarrow{MN}$ and $\overleftrightarrow{RS}$ are plane TNS, plane MNR, plane MNS, and plane TNR.

130–C Infinitely many

130–D 75

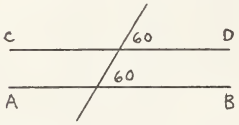
130–E 75

130–F A transversal is a line that intersects one of two coplanar lines in a point and also intersects the other line in a different point. Since $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{TU}$ in point T and intersects $\overleftrightarrow{RS}$ in point N, $\overleftrightarrow{MN}$ is a transversal of $\overleftrightarrow{TU}$ and $\overleftrightarrow{RS}$.

131–G Yes * If two angles have the same measure, then they are congruent. From exercise E on page 72, $\angle UTM^{\circ} = \angle MNS^{\circ}$. Therefore, $\angle MNS \cong \angle UTM$.

131–H If two corresponding angles are congruent, then the two lines are parallel.

131–I Your picture should be similar to the one below.



131–J $\overline{AB}$ * Point C

131–K On dot B

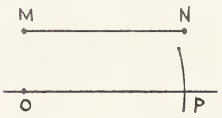
131–L On dot C

131–M The measure of each segment determined by C and a point in the curve is equal to the measure of $\overline{AB}$. * Circle C

132–N Radius EF

132–O $\overline{GH} \cong \overline{EF}$.

132–P Your drawing should be similar to the one below.



132–Q Yes

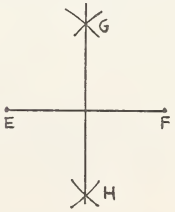
133–A $\overline{PQ}$

133–B Points R and S

133–C {T}

133–D Yes

133–E Your drawing should be similar to the one below. $\overleftrightarrow{GH}$ is the perpendicular bisector of $\overline{EF}$.



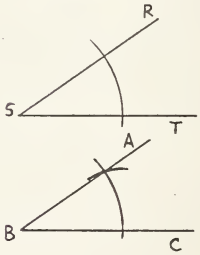
133–F Use your ruler to find the measures of the segments determined by the midpoint and each of the endpoints of $\overline{EF}$. The measures that you find should be approximately equal. Use your protractor to measure the angles formed by line EF and the perpendicular bisector of $\overline{EF}$. The measures that you find should be approximately equal to 90.

133–G $\angle UVW$ * {L, D}

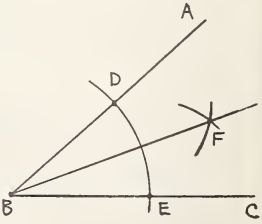
133–H Point J

133–I $\angle XYZ$

133–J Your drawing should be similar to the one below. $\angle ABC$ is constructed congruent to $\angle RST$. Therefore these angles have the same measure.

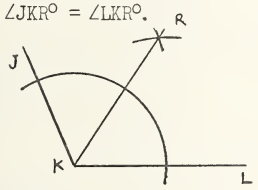


134–K In the drawing below, certain dots are labeled to help explain the construction.



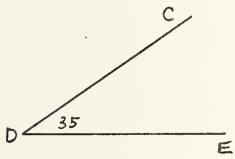
With the compasses set to a convenient radius and the tip on dot B, arc DE is constructed. With the compasses set to a convenient radius and the tip first on dot E and then on dot D, arcs are constructed that intersect in point F. A straightedge is used to construct $\overrightarrow{BF}$, which bisects ∠ABC.

134–L Your drawing should be similar to the one below. ∠JKR° = ∠LKR°.

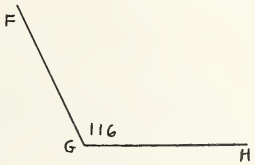


On your own

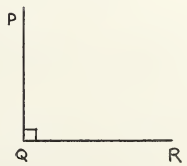
134–1



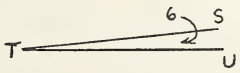
134–2



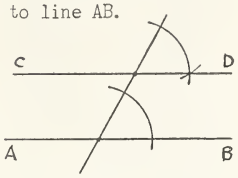
134–3



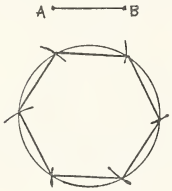
134–4



134–5 Given line AB. Line CD is constructed parallel to line AB.



134–6 Given segment is $\overline{AB}$.



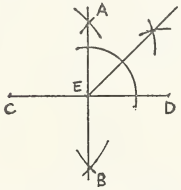
134–7 See the drawing for 76–6.

134–8 Hexagon * It has 6 sides.

134–9 Yes

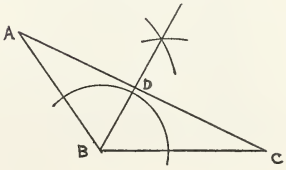
134–10 Yes

134–11 $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{CD}$. Each of angles AEC, AED, BEC, and BED is a right angle.

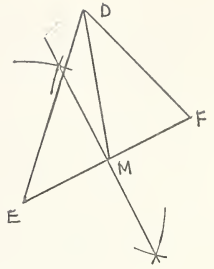


134–12 See the drawing for 76–11. $\overrightarrow{EF}$ bisects ∠AED. ∠AEF° = 45. ∠DEF° = 45.

134–13 ΔABC is an obtuse triangle. ∠B is the obtuse angle. $\overline{BD}$ is the bisector of ∠B of ΔABC.



134–14



Lesson 124

135–A Angles R, S, and T.
Sides RS, ST, and TR *
Angles U, V, and W.
Sides UV, VW and WU

135–B S is mapped onto V.
T is mapped onto W.

135–C $\overline{RS}$ and $\overline{UV}$. $\overline{ST}$ and $\overline{VW}$. $\overline{RT}$ and $\overline{UW}$ * ∠R and ∠U. ∠S and ∠V. ∠T and ∠W

135–D R is mapped onto W.
S is mapped onto V.
T is mapped onto U.

135–E Corresponding sides are $\overline{RS}$ and $\overline{WV}$, $\overline{ST}$ and $\overline{VU}$, and $\overline{RT}$ and $\overline{WU}$. Corresponding angles are ∠R and ∠W, ∠S and ∠V, and ∠T and ∠U.

135–F Yes. The corresponding sides and the corresponding angles represented by diagram D are congruent. Therefore, diagram D represents a congruence correspondence. * If there is a congruence correspondence between the vertices of two triangles, then the triangles are congruent.

135–G RST ⟷ UWV.
RST ⟷ VUW.
RST ⟷ VWU.
RST ⟷ WUV.

136–H None * For each of the correspondences repre-

sented by the four diagrams, all pairs of corresponding parts are not congruent.

136–I Corresponding sides are $\overline{GH}$ and $\overline{JK}$, $\overline{HI}$ and $\overline{KL}$, and $\overline{GI}$ and $\overline{JL}$. Corresponding angles are ∠G and ∠J, ∠H and ∠K, and ∠I and ∠L.

136–J In a congruence correspondence, corresponding sides are congruent. * In a congruence correspondence, corresponding angles are congruent.

136–A Any point J was located in ℓ_1. Then with the tip of the compasses on dot J and with the compasses set to radius FH, point K was located in ℓ_1.

136–B Any segment determined by a point in this arc and point J is congruent to $\overline{FG}$.

136–C Point L

136–D Point L is in the arc that was constructed with the compasses set to radius FG and the tip on dot J. * Point L is in the arc that was constructed with the compasses set to radius GH and the tip on dot K.

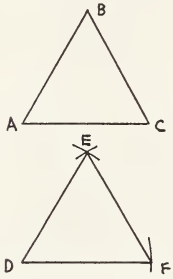
137–E The corresponding sides are $\overline{JL}$ and $\overline{FG}$, $\overline{LK}$ and $\overline{GH}$, and $\overline{JK}$ and $\overline{FH}$. The corresponding angles are ∠J and ∠F, ∠L and ∠G, and ∠K and ∠H.

137–F Yes

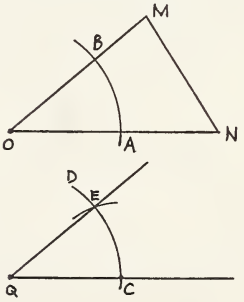
137–G The measures should appear to be equal.

137–H Yes * In the correspondence between the vertices of ΔJLK and ΔFGH, the three pairs of corresponding sides are congruent, and the three pairs of corresponding angles are congruent. Therefore, the correspondence is a congruence correspondence, and ΔJLK is congruent to ΔFGH.

137–I See the drawings below.



137–A ΔOMN * Refer to the drawings below.



With the compasses set to a convenient radius and the tip on dot O, construct arc AB. With the compasses set to the same radius and the tip on dot Q, construct arc CD. With the compasses set to radius AB and the tip on dot C, locate point E in arc CD. Using a straightedge, construct ray QE.

137–B Radius ON

137–C With the compasses set to radius OM and the tip on dot Q, point S was located in ray QS. * After point S was located, a straightedge was used to construct segment SR.

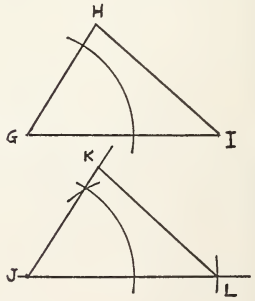
137–D The corresponding sides are $\overline{QS}$ and $\overline{OM}$, $\overline{SR}$ and $\overline{MN}$, and $\overline{QR}$ and $\overline{ON}$. The corresponding angles are ∠Q and ∠O, ∠S and ∠M, and ∠R and ∠N.

138–E (Column 1) Sides OM and QS. Sides ON and QR

138–F (Column 1) Sides MN and SR * Yes

138–G (Column 1) For any two triangles, if the corresponding sides are congruent, then the two triangles are congruent. (s-s-s property of congruence for triangles)

138–H See the drawings below.



138–A Yes

138–B Yes

138–C {W} * Point W was located by constructing rays QS and XW.

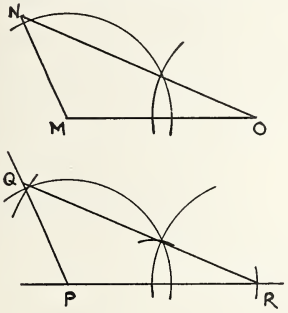
138–D Yes * Yes

38–E (Column 2) Corresponding sides are $\overline{VW}$ and $\overline{ST}$, $\overline{WX}$ and $\overline{TU}$, and $\overline{VX}$ and $\overline{SU}$. Corresponding angles are $\angle V$ and $\angle S$, $\angle W$ and $\angle T$, and $\angle X$ and $\angle U$.

38–F (Column 2) Yes

38–G (Column 2) Yes * The s–a–s property of congruence for triangles

39–H See the drawings below.

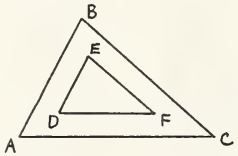


39–I The sum of the measures of the angles of a triangle in degrees is 180. You know that $\angle A^o + \angle B^o = \angle D^o + \angle E^o$. Therefore, $180 - (\angle A^o + \angle B^o) = 180 - (\angle D^o + \angle E^o)$ and $\angle C^o = \angle F^o$. This means that $\angle C \cong \angle F$.

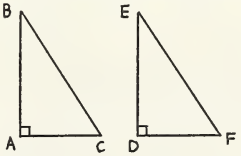
39–J The a–s–a property of congruence for triangles * Since you know that $\angle H \cong \angle K$ and $\angle I \cong \angle L$, you also know that $\angle G \cong \angle J$. Therefore, in $\triangle GHI$, two angles ($\angle G$ and $\angle H$) and the side that is a subset of each of these angles ($\overline{GH}$) are congruent to the corresponding parts in $\triangle JKL$ ($\angle J$, $\angle K$, and $\overline{JK}$).

39–K Yes

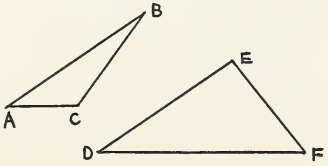
39–L Yes. The idea expressed by the "if" clause and the idea expressed by the "then" clause in "If two triangles are congruent, then the corresponding angles of the two triangles are congruent" have been interchanged. * No * See the sketch below. $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. However, $\triangle ABC$ is not congruent to $\triangle DEF$.



139–M Yes. Since all right angles have a measure in degrees of 90, all right angles are congruent. Therefore, you can use the s–a–s property of congruence for triangles. * See the sketch below. $\angle A$ and $\angle D$ are right angles. $\overline{AB} \cong \overline{DE}$. $\overline{AC} \cong \overline{DF}$. Therefore, $\triangle ABC \cong \triangle DEF$.

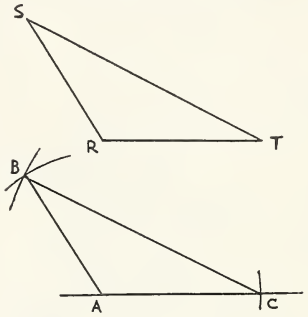


139–N No * See the sketch below. $\angle A \cong \angle D$. $\overline{AB} \cong \overline{DE}$. $\overline{BC} \cong \overline{EF}$. $\triangle ABC$ is not congruent, in this case, to $\triangle DEF$.

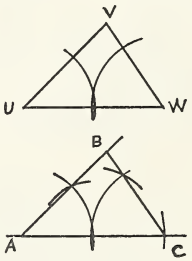


On your own

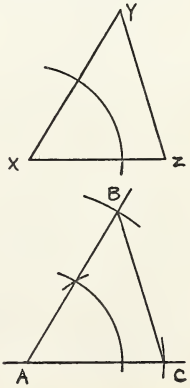
139–1 See the drawing below. $\triangle ABC$ is constructed congruent to $\triangle RST$.



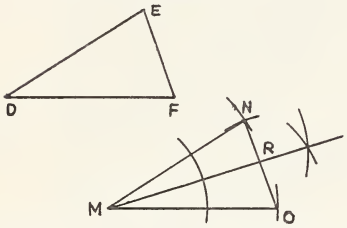
139–2 See the drawing below. $\triangle ABC$ is constructed congruent to $\triangle UVW$.



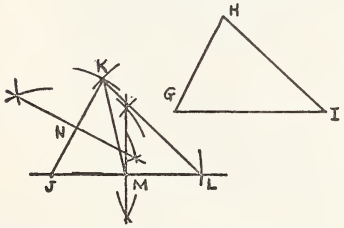
139–3 See the drawing below. $\triangle ABC$ is constructed congruent to $\triangle XYZ$.



139–4 See the drawing below. ΔMNO is constructed congruent to ΔDEF. Segment MR is the bisector of ∠M of ΔMNO.

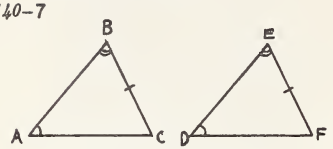


139–5 See the drawing below. ΔJKL is constructed congruent to ΔGHI. Point N is the midpoint of $\overline{JK}$.



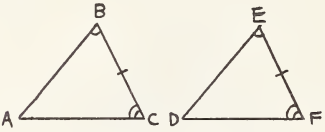
140–6 Picture A: Congruent. The s-a-s property.
Picture B: Congruent. The a-s-a property.
Picture C: You cannot tell if these triangles are congruent.
Picture D: Congruent. The a-s-a property.
Picture E: You cannot tell if these triangles are congruent.
Picture F: Congruent. The s-s-s property.
Picture G: Congruent. The a-s-a property.
Picture H: You cannot tell if these triangles are congruent.
Picture I: You cannot tell if these triangles are congruent.

140–7



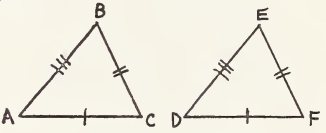
Congruent. The a-s-a property

140–8



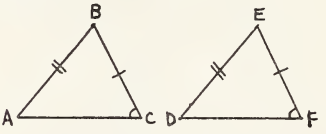
Congruent. The a-s-a property

140–9



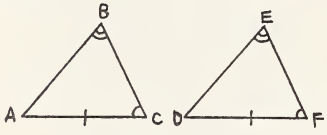
Congruent. The s-s-s property

140–10



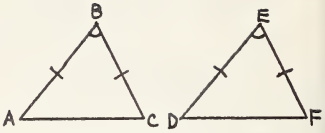
You cannot tell if these triangles are congruent.

140–11



Congruent. The a-s-a property

140–12



Congruent. The s-a-s property

Lesson 125

141–A Yes. Rectangle ABCD is a simple closed curve. A closed region is the union of a simple closed curve and its interior. * Yes. A closed region formed by the union of a rectangle and its interior is called a rectangular region.

141–B 1

141–C 4

141–D 3 * 3

141–E The rate pair $x/3$ represents the fact that x square units are included in 3 rows of rectangular region ABCD.

141–F Yes

141–G 12

141–H Yes * Since the measures of the sides are equal, each of the sides is congruent to each of the other sides. A square is a rectangle in which each side is congruent to each of the other sides. * $4.6/1 \sim x/4.6$.

141–I 21.16

142–J The rate pair 56.5/1 represents the fact that 56.5 square units are included in 1 row of the rectangular region.

142–K Yes * The rate pair $m/7.25$ represents the fact that m square units are included in 7.25 rows of the rectangular region.

142–L Yes

142–M {409.625}

142–N The area of the drive-way is 410 sq. ft.

142–A $\overline{FI}$ and $\overline{GJ}$. An altitude of a parallelogram is a segment that is perpendicular to both of the lines that include a pair of opposite sides of the parallelogram. The endpoints of the altitude are in these lines. * $\overline{FG}$

142–B $2\frac{1}{2}$ * $1\frac{1}{4}$ * $2\frac{1}{2}/1 \sim x/1\frac{1}{4}$.

142–C Yes * If a parallelogram that is a rectangle and a parallelogram that is not a rectangle have a common base and a common altitude, then the two parallelograms have the same area.

142–D $3\frac{1}{8}$

142–E The measure of each of the other sides of rhombus STUV is 3.5.

142–F 3.5

142–G The area of rhombus STUV in square units

142–H 10.5

143–I 18 * The first component of 12/1 refers to the number of inches in one foot. The second component of 12/1 refers to 1 foot. * Since the first component of 12/1 refers to a number of inches, the first component of the rate pair you are to find must also refer to inches. Since the second component of 12/1 refers to a number of feet, the second component of the rate pair you are to find must also refer to feet. * The number of feet that is the same length as 18 inches

143–J The rate pairs 12/1 and $18/x$ represent the same rate and are equivalent to each other.

143–K The rate pair 4/1 represents the fact that 4 square units are included in 1 row. * The rate pair y/x refers to the number of square units included in x rows.

143–L Yes

143–M $12/1 \sim 18/x \wedge 4/1 \sim y/x$. * Each solution must satisfy both simple conditions. You must find the solution of $12/1 \sim 18/x$ before you can find the ordered pair that satisfies $4/1 \sim y/x$.

143–N $\{(1\frac{1}{2}, 6)\}$ * The area of the piece of plywood is 6 sq. ft.

143–A $\overline{KN}$

143–B A parallelogram and either of its diagonals form two congruent triangles. * Yes. If two triangles are congruent, then they have the same area.

143–C If a parallelogram and a triangle have a common altitude and a common base, then the area of the parallelogram is 2 times the area of the triangle.

143–D $4\frac{1}{2}$ * $2\frac{1}{2}$

143–E The area of parallelogram KLMN is 2 times the area of ΔKLN.

143–F The rate pair $4\frac{1}{4}/1$ represents the fact that $4\frac{1}{4}$ square units are included in 1 row. The rate pair $2x/2\frac{1}{2}$ represents the fact that $2x$ square units are included in $2\frac{1}{2}$ rows. Since both $4\frac{1}{4}/1$ and $2x/2\frac{1}{2}$ represent the same rate, $4\frac{1}{4}/1$ and $2x/2\frac{1}{2}$ are equivalent rate pairs.

143–G $\{5\frac{5}{16}\}$ * $5\frac{5}{16}$

144–H The rate pair $1\frac{1}{4}/1$ represents the fact that $1\frac{1}{4}$ square units are included in 1 row of the closed region. * The rate pair $2x/\frac{3}{4}$ represents the fact that $2x$ square units are included in $\frac{3}{4}$ rows of the closed region. * The area of the window in square yards

144–I Since $1\frac{1}{4}/1$ and $2x/\frac{3}{4}$ represent the same rate, they are equivalent rate pairs.

144–J 9

144–K The rate pair 9/1 represents the comparison of the number of square feet in one square yard with one square yard. The rate pair y/x represents the same comparison as 9/1 and is equivalent to 9/1.

144–L $1\frac{1}{4}/1 \sim 2x/\frac{3}{4} \wedge 9/1 \sim y/x$.

144–M $\{(\frac{15}{32}, 4\frac{7}{32})\}$ * The area of the window is $4\frac{7}{32}$ sq. ft.

On your own

144–I x, area of the lot in square feet.
$75/1 \sim x/50\frac{1}{2}$.

{3787.5}.
3787.5 sq. ft.

144–2 x, area of the sign in square feet.
$12\frac{1}{2}/1 \sim x/8$.
{100}. 100 sq. ft.

144–3 x, area of the shelf in square inches.
$28/1 \sim 2x/14\frac{1}{6}$.
{$197\frac{3}{4}$}. $197\frac{3}{4}$ sq. in.

144–4 x, area of the baseball diamond in square feet.
$90/1 \sim x/90$.
{8100}. 8100 sq. ft.

144–5 x, length of the base in yards.
$x/1 \sim 4/1\frac{1}{3}$.
{3}. 3 yd.

144–6 x, area of the rhombus in square feet.
y, area of the rhombus in square inches.
$4.8/1 \sim x/2.5 \wedge$
$144/1 \sim y/x$.
{(12,1728)}.
1728 sq. in.

144–7 x, length of the base of ΔIJK.
y, area of ΔIJK in square inches.
$4/1 \sim x/8.5 \wedge$
$x/1 \sim 2y/8.5$.
{(34,144.5)}.
144.5 sq. in.

144–8 x, width of the rectangle in feet.
$13.5/1 \sim 162/x$.
{12}. 12 ft.

144–9 x, area of '()} in square yards.
y, area in square feet.
$6/1 \sim x/2.75 \wedge$
$9/1 \sim y/x$.
{(16.5,148.5)}.
148.5 sq. ft.

144–10 x, length of the altitude of ΔDEF in inches.
y, length of base in inches.
$12/1 \sim x/1\frac{1}{4} \wedge$
$y/1 \sim 2(27)/x$.
{(15,$3\frac{3}{5}$)}. $3\frac{3}{5}$ in.

Applying mathematics

145–1 x, area of the ceiling in square feet.
$180/1 \sim x/90\frac{1}{2}$.
{16290}. 16,290 sq. ft.

145–2 x, perimeter of the patio in feet.
$x = 24\frac{1}{2} + 15 + 24\frac{1}{2} + 15$.
{79}. 79 ft.

145–3 x, perimeter in inches.
y, perimeter in feet.
$x = 35 + 18 + 35 \wedge$
$12/1 \sim x/y$.
{(88,$7\frac{1}{3}$)}. $7\frac{1}{3}$ ft.

145–4 x, width of the bulletin board in feet.
$12/1 \sim 50/x$.
{$4\frac{1}{6}$}. $4\frac{1}{6}$ ft.

145–5 x, area of the sail in square feet.
$6/1 \sim 2x/10\frac{1}{2}$.
{$31\frac{1}{2}$}. $31\frac{1}{2}$ square ft.

145–6 x, area of the piece of canvas in square yards.
y, area in square feet.
$3\frac{1}{2}/1 \sim x/3\frac{1}{2} \wedge 9/1 \sim y/x$.
{($12\frac{1}{4}$,$110\frac{1}{4}$)}.
$110\frac{1}{4}$ sq. ft.

145–7 x, area of the field in square rods.
y, area of the field in acres.
z, amount of money Mr. Alessandro paid for the field.
$75/1 \sim x/40 \wedge 160/1 \sim x/y \wedge 310/1 \sim z/y$.
{(3000,18.75,5812.50)}.
$5812.50

145–8 x, length of the base in feet.
$x/1 \sim 5\frac{1}{4}/1\frac{7}{8}$.
{$2\frac{4}{5}$}. $2\frac{4}{5}$ ft.

145–9 x, length of the base of ΔQRS in inches.
$x/1 \sim 2(39)/6\frac{1}{2}$.
{12}. 12 in.

145–10 x, length of side AB in inches.
y, length of the base of LMNO in inches.
$4\frac{1}{2}/1 \sim x/6\frac{2}{3} \wedge y/1 \sim x/4$.
{(30,$7\frac{1}{2}$)}. $7\frac{1}{2}$ in.

145–11 x, length of side AB in feet.
y, length of side BC in feet.
z, perimeter of ΔABC in feet.
$1\frac{1}{3}/1 \sim x/4\frac{1}{2} \wedge 1\frac{2}{3}/1 \sim y/4\frac{1}{2} \wedge z = 4\frac{1}{2} + x + y$.
{(6,$7\frac{1}{2}$,18)}. 18 ft.

145–12 x, measure of angle R in degrees.
y, measure of angle S in degrees.
z, measure of angle T in degrees.
$90 - x = 10 \wedge 5/1 \sim x/y \wedge x + y + z = 180$.
{(80,16,84)}. $\angle R^o = 80$.
$\angle S^o = 16$. $\angle T^o = 84$.

145–13 x, measure of angle I in degrees.
y, measure of angle M in degrees.
$90 - x = 29 \wedge 90 + x + y = 180$.
{(61,29)}. $\angle K^o = 90$.
$\angle L^o = 61$. $\angle M^o = 29$.

145–14 x, measure of angle X in degrees.
y, measure of angle Y in degrees.
z, measure of angle Z

in degrees.
$3/1 \sim x/y \wedge x + 17 = 132 \wedge x + y + z = 180$.
$\{(115, 38\frac{1}{3}, 26\frac{2}{3})\}$.
$\angle X^\circ = 115$. $\angle Y^\circ = 38\frac{1}{3}$.
$\angle Z^\circ = 26\frac{2}{3}$.

Checking up

Test 119

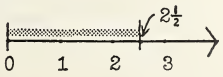
146–1 $\{x \mid x \leq \frac{5}{12}\}$
146–2 $\{x \mid 1\frac{1}{12} > x\}$
146–3 $\{x \mid x > 2.5\}$
146–4 $\{\ \}$
146–5 $\{x \mid x > 7\frac{7}{12}\}$
146–6 $\{x \mid x \leq 1.7\}$

Test 120

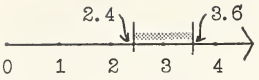
146–7 triple
146–8 $\leq$
146–9 union
146–10 360
146–11 coördinate
146–12 five
146–13 $\perp$
146–14 universe
146–15 trapezoid
146–16 a half-open
146–17 2
146–18 perpendicular bisector

t 121

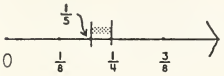
146–19 180
146–20 {(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (2,0), (2,1), (3,0)}
146–21 {0, 1, 2, 3, 4, 5}
146–22 {(2,6,0), (2,6,5), (2,7,0), (2,7,5), (3,6,0), (3,6,5), (3,7,0), (3,7,5), (4,6,0), (4,6,5), (4,7,0), (4,7,5)}
146–23 $\{s \mid 11.2 \leq s \leq 12.8\}$
146–24 80
146–25 75
146–26



146–27



146–28



Test 122

146–29 x, area of the parking lot in square feet.
$128/1 \sim x/48\frac{1}{4}$.
$\{6176\}$. 6176 sq. ft.
147–30 x, number of miles per hour object moves.
$24900/24 \sim x/1$.
$\{1037\frac{1}{2}\}$. About 1038 miles per hour
147–31 x, increase in the number of tokens sold.
y, first component of the per cent of increase.
$294 - 260 = x \wedge x/260 \sim y/100$.
$\{(84, 13\frac{1}{13})\}$. 13.1%
147–32 x, area of the window in square feet.
$2\frac{2}{3}/1 \sim x/1\frac{1}{4}$.
$\{3\frac{1}{3}\}$. $3\frac{1}{3}$ sq. ft.
147–33 x, age of Brad's mother.
y, age of Brad's father.
$y - x = 2 \wedge 41 \leq y < 47$.
{(39,41), (40,42), (41,43), (42,44), (43,45), (44,46)}.
Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (40,42) is: Brad's mother is 40 and his father is 42.
147–34 $x + y < 6 \wedge x - z = 1 \wedge z > 2$.
{(4,0,3), (4,1,3), (5,0,4)}
147–35 $8/3 \sim x/y \wedge \sim(x + y > 44)$.
{8/3, 16/6, 24/9, 32/12}
147–36 $18/12 \sim x/y \wedge xy = 20$.
$\{\ \}$
147–37 $x \geq 32\frac{1}{2} \wedge \sim(x > 412\frac{1}{4} - 286\frac{3}{4})$.
$\{x \mid 32\frac{1}{2} \leq x \leq 125\frac{1}{2}\}$

Lesson 126

148–A {0, 1, 2, ...} * No * No * Yes
148–B Since 648 $\in$ N and 5984 $\in$ N, 648 + 5984 $\in$ N because N is closed under addition. * N is closed under multiplication.
148–C Yes
148–D There is no natural number that you can add to 28 to get 13.
148–E Yes * N is closed under multiplication.
148–F No. $a - b$ is an element of N only if a is greater than b. * No. If you can find at least one example for which it is not true that $a - b \in$ N, then it is not true that, for each a and b, $a - b \in$ N. Such an example was given in exercise D. 13 − 28 is not a member of N.
148–G 5 * Yes
148–H There is no natural number by which 10 can be multiplied to get 18.
148–I No. $a \div b$ is an element of N only if a is a multiple of b. * No. If you can find at least one example for which it is not true that $a \div b \in$ N, then it is not true that, for each a and b, $a \div$

$b \in N$. Such an example was given in exercise H. $18 \div 10$ is not a member of N.

148–J Yes * By using the definition of equivalent ordered pairs

149–K No * No * No * No

149–L Yes * No * No

149–A 30 * 30 * Yes

149–B $\frac{3}{5} \sim \frac{9}{15}$ because $3 \times 15 = 9 \times 5$. * $\frac{6}{10} \sim \frac{9}{15}$ because $6 \times 15 = 9 \times 10$.

149–C The only common factor of 3 and 5 is 1. * No * Both 1 and 2 are common factors of 6 and 10. Both 1 and 3 are common factors of 9 and 15.

149–D A rational number of arithmetic is a set of equivalent fractions. A rational number of arithmetic contains a basic fraction and all fractions equivalent to the basic fraction.

149–E Examples are $\frac{12}{16}$ and $\frac{30}{40}$.

149–F Examples are $\frac{25}{40}$ and $\frac{500}{800}$.

149–G Examples are $\frac{54}{48}$ and $\frac{81}{72}$.

149–H Examples are $\frac{50}{35}$ and $\frac{80}{56}$.

150–I The universe for a and c is N. The universe for b and d is C.

150–J $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$. * $\frac{7}{3} \times \frac{9}{14} = \frac{63}{42}$. * $\frac{0}{4} \times \frac{11}{8} = \frac{0}{32}$. * Yes

150–K Yes * From the closure property of multiplication of natural numbers, $ac \in N$ and, since bd is not 0, $bd \in C$. Therefore, $\frac{ac}{bd} \in R_a$.

150–L The universe for a and c is N. The universe for b and d is C.

150–M $\frac{3}{4} + \frac{2}{5} = \frac{15 + 8}{20}$. $\frac{15 + 8}{20} = \frac{23}{20}$, or $1\frac{3}{20}$. * $\frac{7}{6} + \frac{8}{9} = \frac{63 + 48}{54}$. $\frac{63 + 48}{54} = \frac{111}{54}$, or $2\frac{1}{18}$. * $\frac{13}{4} + \frac{5}{16} = \frac{208 + 20}{64}$. $\frac{208 + 20}{64} = \frac{228}{64}$, or $3\frac{9}{16}$. * Yes

150–N Yes * From the closure property of multiplication of natural numbers, $ad \in N$ and $bc \in N$. Since bd is not 0, $bd \in C$. From the closure property of addition of natural numbers, $ad + bc \in N$. Therefore, $\frac{ad + bc}{bd} \in R_a$.

150–O $\frac{15}{15}$, or 1 * Yes * The product of $\frac{12}{16}$ and $\frac{4}{3}$ is $\frac{48}{48}$, or 1.

150–P $1/\frac{9}{10}$, or $\frac{10}{9}$ * $1/3\frac{2}{3}$, or $1/\frac{11}{3}$, or $\frac{3}{11}$ * $1/7$, or $\frac{1}{7}$

150–Q Yes * Yes * $\frac{5}{2} \div \frac{1}{4} = \frac{5}{2} \times \frac{4}{1}$, or 10. * Yes

150–R $\frac{11}{9} \div \frac{8}{15} = \frac{11}{9} \times \frac{15}{8}$. $\frac{11}{9} \times \frac{15}{8} = \frac{165}{72}$, or $2\frac{7}{24}$. * $\frac{13}{4} \div \frac{25}{26} = \frac{13}{4} \times \frac{26}{25}$. $\frac{13}{4} \times \frac{26}{25} = \frac{338}{100}$, or $3\frac{19}{50}$. * Yes

150–S Yes * Yes * Yes * Jerry sailed an average of $\frac{19}{3}$, or $6\frac{1}{3}$, miles per hour.

151–T There is no rational number of arithmetic that you can add to $\frac{7}{8}$ to get $\frac{3}{8}$.

151–U No * No

151–V Yes * No * There is no rational number of arithmetic that you can add to 15 to get 10. * No

151–A The universe for x, y, and z is S.

151–B Addition and multiplication * $x + y \in S$. $xy \in S$.

152–C $x + y = y + x$. * $xy = yx$.

152–D $(x + y) + z = x + (y + z)$. $(xy)z = x(yz)$.

152–E Distributive property

152–F Yes

152–G If the universe for x, y, and z is N, then for each x, y, and z, $x + y \in N$, $xy \in N$, and true statements are obtained from the other conditions expressed in D5.

152–H If the universe for x, y, and z is R_a, then for each x, y, and z, $x + y \in R_a$, $xy \in R_a$, and true statements are obtained from the other conditions expressed in D5.

On your own

152–1 Yes * Yes

152–2 No. There is no natural number that you can add to 753 to get 752. * No. There is no rational number of arithmetic that you can add to 753 to get 752.

152–3 Yes * Yes

152–4 No. There is no natural number by which you can multiply 2 to get 269. * Yes

152–5 Yes * Yes

152–6 Yes * Yes

152–7 No. There is no natural number that you can add to 1 to get 0. * No. There is no rational number of arithmetic that you can add to 1 to get 0

152–8 Yes * Yes

152–9 No. There is no natural number by which you can multiply 75 to get 250. * Yes

152–10 Yes * Yes

152–11 $\frac{4}{9}$
152–12 $\frac{15}{14}$
152–13 $\frac{4}{6}$
152–14 $\frac{5}{7}$
152–15 $\frac{2}{5}$
152–16 True statement. Commutative property of addition
152–17 False statement
152–18 True statement. Commutative property of multiplication
152–19 True statement. The quotient property and the closure property of multiplication
152–20 False statement
152–21 True statement. Distributive property of multiplication over addition
152–22 False statement
152–23 True statement. Associative property of multiplication
152–24 False statement
152–25 True statement. Associative property of addition
152–26 True statement. Commutative property of addition
152–27 True statement. Closure property of addition

Lesson 127

153–A $\frac{1}{2}$ * $\frac{5}{2}$ * 0
153–B $\frac{3}{2}$ and 3 * 3 – $\frac{3}{2}$, or $\frac{3}{2}$
153–C $\frac{5}{2}$ – 1, or $\frac{3}{2}$ * $\frac{9}{2}$ – $\frac{5}{2}$, or 2
153–D $\overline{AG}$, $\overline{BH}$, $\overline{CJ}$, and $\overline{DL}$
153–E M * $\frac{15}{4}$ * $\frac{15}{4}$ – $\frac{1}{2}$, or $\frac{13}{4}$
153–F Yes * $\frac{15}{4}$ – $\frac{1}{2}$, or $\frac{13}{4}$
153–G The coördinate of R is $\frac{3}{2}$ and the coördinate of S is 2. The absolute difference of $\frac{3}{2}$ and 2 is 2 – $\frac{3}{2}$, or $\frac{1}{2}$. * $\frac{13}{3}$ – 3, or $\frac{4}{3}$
153–H To the left * To the right * To the right

154–I The symbol for a directed segment has an arrow in the middle of the bar over the letters. The symbol for a segment does not have this arrow.
154–J The starting point is D and the terminal point is L.
154–K The direction from the starting point S to the terminal point R is to the left. * The direction from the starting point T to the terminal point U is to the right.
154–L No. The starting point of $\overrightarrow{GW}$ is a different point from the starting point of $\overrightarrow{WG}$. The terminal points are also different. * $\overrightarrow{GW}$ and $\overrightarrow{WG}$ are the same set of points. * $\overrightarrow{GW}$ and $\overrightarrow{WG}$ are directed in opposite directions.
154–M $\overrightarrow{KA}$ * $\overrightarrow{ZL}$
154–N $\overrightarrow{AB}$, $\overrightarrow{JD}$, $\overrightarrow{EF}$, and $\overrightarrow{HC}$
154–O The coördinate of A is 0. The coördinate of B is $\frac{1}{2}$. The absolute difference of 0 and $\frac{1}{2}$ is $\frac{1}{2}$ – 0, or $\frac{1}{2}$. * $\frac{5}{2}$ – 2, or $\frac{1}{2}$ * Yes. The measure of each of these directed segments is $\frac{1}{2}$. * Yes. Both $\overrightarrow{AB}$ and $\overrightarrow{EF}$ are directed to the right.
154–P $\overrightarrow{HC}$ and $\overrightarrow{AB}$ have the same direction (to the right) and the same measure ($\frac{1}{2}$) * $\overrightarrow{HC}$ and $\overrightarrow{EF}$ have the same direction (to the right) and the same measure ($\frac{1}{2}$).

155–Q To the right. Since $\frac{5}{2}$ > 25, terminal point V is to the right of starting point W. * $\frac{51}{2}$ – 25, or $\frac{1}{2}$. m($\overrightarrow{WV}$) is the absolute difference of 25 and $\frac{51}{2}$, which are the coordinates of the endpoints of WV.
155–R $\overrightarrow{AB}$, $\overrightarrow{EF}$, and $\overrightarrow{HC}$
155–A The second component, 2 * The first component, $\frac{5}{2}$
155–B ($\frac{1}{2}$,0) * The coördinate of the starting point of $\overrightarrow{AB}$ is 0. This is the second component of the ordered pair that determines $\overrightarrow{AB}$. The coördinate of the terminal point of $\overrightarrow{AB}$ is $\frac{1}{2}$. This is the first component of the ordered pair that determines $\overrightarrow{AB}$.
155–C $\frac{1}{2}$ + 2, or $\frac{5}{2}$ * $\frac{5}{2}$ + 0, or $\frac{5}{2}$ * Yes
155–D A true statement
155–E (4,$\frac{7}{2}$) * Since 4 + 0 = $\frac{1}{2}$ + $\frac{7}{2}$, (4,$\frac{7}{2}$) ~ ($\frac{1}{2}$,0). * Since 4 + 2 = $\frac{5}{2}$ + $\frac{7}{2}$, (4,$\frac{7}{2}$) ~ ($\frac{5}{2}$,2).
155–F No * (3,$\frac{3}{2}$) * No * No
156–G They are equivalent directed segments and, therefore, have the same measure and the same direction.
156–H They are not equivalent ordered pairs for directed segments.
156–I The first component
156–J (x,y) ~ ($\frac{1}{2}$,0).
156–K Yes. (3,$\frac{5}{2}$) ~ ($\frac{1}{2}$,0) because 3 + 0 = $\frac{1}{2}$ + $\frac{5}{2}$. * Yes. ($\frac{11}{2}$,5) ~ ($\frac{1}{2}$,0) because $\frac{11}{2}$ + 0 = $\frac{1}{2}$ + 5. * $\frac{1}{4}$ + 0 is not equal to $\frac{1}{2}$ + $\frac{3}{4}$.

156–L Yes. $(\frac{1}{2},0) \sim (\frac{1}{2},0)$ because $\frac{1}{2} + 0 = \frac{1}{2} + 0$. * Yes. $\frac{2}{4}$ is the same member of R_a as $\frac{1}{2}$. * No. When you replace y in $(x,y) \sim (\frac{1}{2},0)$, by 0, you obtain $(x,0) \sim (\frac{1}{2},0)$. If $(x,0) \sim (\frac{1}{2},0)$, then $x + 0 = \frac{1}{2} + 0$. Since $\frac{1}{2}$ is the only member of R_a that satisfies $x + 0 = \frac{1}{2} + 0$, $\frac{1}{2}$ is the only member of R_a that satisfies $(x,0) \sim (\frac{1}{2},0)$. Therefore, the only member of $R_a \times R_a$ that has a second component of 0 and that satisfies $(x,y) \sim (\frac{1}{2},0)$ is $(\frac{1}{2},0)$.

156–M An infinite set

157–N $^{+}\frac{1}{4}$

157–O $(\frac{29}{5},0)$

157–P Yes * Since $\frac{1}{4}$ is the same member of R_a as $\frac{2}{8}$, $(\frac{1}{4},0)$ is the same ordered pair as $(\frac{2}{8},0)$. Therefore, $^{+}\frac{1}{4} = {}^{+}\frac{2}{8}$.

157–Q $\frac{37}{8}$ * They are directed to the right.

157–R $^{+}\frac{7}{8}$

157–S $(\frac{2}{3},0)$ * Any member of a positive rational number can be used to indicate the number. Since $\frac{5}{3} + 0 = \frac{2}{3} + 1$, $(\frac{5}{3},1) \sim (\frac{2}{3},0)$. Therefore, $(\frac{5}{3},1)$ is a member of the same positive rational number as $(\frac{2}{3},0)$ and can be used to indicate $^{+}\frac{2}{3}$.

157–T $^{+}6$

157–A $(\frac{1}{3},0)$ * $(\frac{4}{3},1)$ * $\frac{1}{3} + 1 = \frac{4}{3} + 0$.

157–B Both $(\frac{1}{3},0)$ and $(\frac{4}{3},1)$ satisfy $(x,y) \sim (\frac{1}{3},0)$.

157–C $(\frac{1}{3},0)$ * $^{+}\frac{1}{3}$

157–D Yes * Yes

157–E $(3,2)$ * No * $(3,2)$ does not have a component of 0.

157–F Replace b by 0, c by 3, and d by 2.

157–G Because of the definition of equivalent ordered pairs for directed segments * 1

157–H $(1,0)$ * Yes

157–I $^{+}1$

158–J 1 * The first component

158–K $\frac{1}{4}$ * $(\frac{1}{4},0)$

158–L The first component

158–M An infinite set * Yes. If a and b are different members of Z, then $(a,0)$ and $(b,0)$ indicate different positive rational numbers. In mathematical terms: if $a \neq b$, then $a + 0 \neq b + 0$ and, therefore, $(a,0)$ is not equivalent to $(b,0)$.

158–N An infinite set. Since Z is an infinite set, the set of ordered pairs whose first component is a member of Z and whose second component is 0 is also an infinite set.

158–O $^{+}\frac{7}{9}$

158–P $^{+}\frac{2}{5}$

158–Q $^{+}25$

158–R $^{+}\frac{29}{4}$

158–S $^{+}85$

On your own

159–1 $(\frac{5}{3},0)$

159–2 $\{(x,y) \mid (x,y) \sim (\frac{5}{3},0)\}$

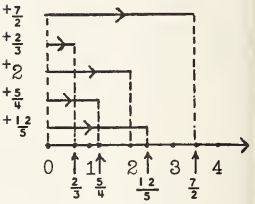
159–3 $^{+}\frac{5}{3}$

159–4 $(2,\frac{1}{2})$ determines $\overrightarrow{CD}$.
$(\frac{7}{3},1)$ determines $\overrightarrow{EF}$.
$(3,\frac{1}{4})$ determines $\overrightarrow{GH}$.

159–5 $\{(x,y) \mid (x,y) \sim (2,\frac{1}{2})\}$, or $\{(x,y) \mid (x,y) \sim (\frac{3}{2},0)\}$. $\{(x,y) \mid (x,y) \sim (\frac{7}{3},1)\}$, or $\{(x,y) \mid (x,y) \sim (\frac{4}{3},0)\}$. $\{(x,y) \mid (x,y) \sim (3,\frac{1}{4})\}$, or $\{(x,y) \mid (x,y) \sim (\frac{1}{4},0)\}$.

159–6 $^{+}\frac{3}{2}$. $^{+}\frac{4}{3}$. $^{+}\frac{1}{4}$

159–7 See the picture below. Only one of infinitely many segments is pictured



159–8 See the picture for 101–7.

159–9 See the picture for 101–7.

159–10 See the picture for 101–7.

159–11 See the picture for 101–7.

159–12 $^{+}\frac{4}{3}$

159–13 $^{+}\frac{1}{12}$

159–14 $^{+}\frac{5}{6}$

159–15 $^{+}\frac{1}{2}$

159–16 $^{+}\frac{13}{12}$

159–17 $^{+}\frac{3}{2}$

159–18 $(\frac{7}{2},0)$ * $^{+}\frac{7}{2}$

159–19 $(\frac{7}{20},0)$ * $^{+}\frac{7}{20}$

159–20 $(\frac{39}{2},0)$ * $^{+}\frac{39}{2}$

159–21 $(\frac{3}{40},0)$ * $^{+}\frac{3}{40}$

159–22 $(\frac{19}{3},0)$ * $^{+}\frac{19}{3}$

159–23 $(\frac{23}{6},0)$ * $^{+}\frac{23}{6}$

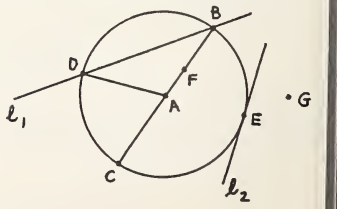
159–24 $(159,0)$ * $^{+}159$

159–25 $(\frac{3}{10},0)$ * $^{+}\frac{3}{10}$

159–26 $(\frac{3}{2},0)$ * $^{+}\frac{3}{2}$

Keeping skilful

159–1 See the sketch below.



159–2 See the sketch given for 101–1.

159–3 See the sketch given for 101–1.

159–4 See the sketch given for 101–1.

159–5 See the sketch given for 101–1.

159–6 See the sketch given for 101–1.

159–7 See the sketch given for 101–1.

159–8 See the sketch given for 101–1.

159–9 Chord DB and chord BC

159–10 Radii AB, AC, and AD

159–11 An example is $\{X \mid \overline{AX} \cong \overline{AB}\}$.

159–12 An example is $\{X \mid \overline{AX} < \overline{AB}\}$. * An example is $\{X \mid \overline{AX} > \overline{AB}\}$.

159–13 $\angle BAD$

159–14 $\{\ \}$

159–15 The set of all points between B and D

159–16 The set of all points in either ⊙ A or $\overline{AG}$ or in both ⊙ A and $\overline{AG}$

159–17 $\overline{AF} < \overline{AB}$, or $\overline{AB} > \overline{AF}$.

159–18 $\overline{FC} > \overline{AC}$, or $\overline{AC} < \overline{FC}$.

Lesson 128

160–A Q * P * The symbol $\overrightarrow{QP}$, and not $\overrightarrow{PQ}$, names the directed segment whose starting point is Q and whose terminal point is P. $\overrightarrow{PQ}$ names the directed segment whose starting point is P and whose terminal point is Q.

160–B $\frac{3}{5}$ * 0 * $\frac{3}{5} - 0$, or $\frac{3}{5}$

160–C The coördinate of the starting point

160–D The absolute difference of 2 and $\frac{7}{5}$ is $\frac{3}{5}$.

160–E Both $\overrightarrow{QP}$ and $\overrightarrow{UT}$ have a measure of $\frac{3}{5}$ and are directed to the left.

160–F $\frac{3}{2} - 1$, or $\frac{1}{2}$ * No * No

160–G $\frac{3}{5}$ * Yes * Yes

160–H An infinite set

160–I The first component, 0 * The second component, $\frac{3}{5}$

160–J $(\frac{7}{5},2)$ * Since $\frac{7}{5} + \frac{3}{5} = 0 + 2$ is a true statement, $(\frac{7}{5},2) \sim (0,\frac{3}{5})$.

160–K $(\frac{12}{5},3)$ * Yes. Since $\frac{12}{5} + \frac{3}{5} = 0 + 3$ is a true statement, $(\frac{12}{5},3) \sim (0,\frac{3}{5})$. * Yes. Since $\frac{12}{5} + 2 = \frac{7}{5} + 3$ is a true statement, $(\frac{12}{5},3) \sim (\frac{7}{5},2)$.

160–L $(1,\frac{3}{2})$ * Since $1 + \frac{3}{5}$ does not equal $0 + \frac{3}{2}$, $(1,\frac{3}{2})$ is not equivalent to $(0,\frac{3}{5})$.

160–M No. Since $1 + \frac{3}{5}$ does not equal $0 + \frac{3}{2}$, $(1,\frac{3}{2})$ is not equivalent to $(0,\frac{3}{5})$. * No. Since $\frac{3}{5} + \frac{3}{5}$ does not equal $0 + 0$, $(\frac{3}{5},0)$ is not equivalent to $(0,\frac{3}{5})$. * Yes. Since $5 + \frac{3}{5} = 0 + \frac{28}{5}$ is a true statement, $(5,\frac{28}{5}) \sim (0,\frac{3}{5})$.

160–N An infinite set

160–O $(0,\frac{3}{5})$ has a component of 0. * $\frac{9}{15}$ is the same member of R_a as $\frac{3}{5}$.

160–P One * When you replace x in $(x,y) \sim (0,\frac{3}{5})$ by 0, you obtain $(0,y) \sim (0,\frac{3}{5})$. If $(0,y) \sim (0,\frac{3}{5})$, then $0 + \frac{3}{5} = 0 + y$. Since $\frac{3}{5}$ is the only member of R_a that satisfies $0 + \frac{3}{5} = 0 + y$, $\frac{3}{5}$ is the only member of R_a that satisfies $(0,y) \sim (0,\frac{3}{5})$. Therefore, the only member of $R_a \times R_a$ that has a first component of 0 and that satisfies $(x,y) \sim (0,\frac{3}{5})$ is $(0,\frac{3}{5})$.

161–Q $-\frac{9}{8}$

161–R (0,11)

161–S $\frac{2}{3}$ and $\frac{4}{6}$ are the same member of R_a. Therefore, $(0,\frac{2}{3})$ is the same basic pair as $(0,\frac{4}{6})$ and the same negative rational number is indicated by each of these basic pairs.

161–T $\frac{17}{10}$ * They are directed to the left.

161–U $-\frac{7}{4}$

161–V $(0,\frac{1}{3})$ * Since $2 + \frac{1}{3} = 0 + \frac{7}{3}$ is a true statement, $(2,\frac{7}{3}) \sim (0,\frac{1}{3})$.

161–W $-\frac{11}{12}$

162–A $(0,\frac{3}{4})$ * $(\frac{3}{2},\frac{9}{4})$ * Since $0 + \frac{9}{4} = \frac{3}{2} + \frac{3}{4}$ is a true statement, $(0,\frac{3}{4}) \sim (\frac{3}{2},\frac{9}{4})$.

162–B Both $(0,\frac{3}{4})$ and $(\frac{3}{2},\frac{9}{4})$ satisfy $(x,y) \sim (0,\frac{3}{4})$. * $-\frac{3}{4}$

162–C (1,3) * No * (1,3) does not have a component of 0.

162–D (1,3) determines a directed segment that is directed to the left, not to the right. * Yes. Each member of the infinite set of ordered pairs that are equivalent to (1,3) indicates -2.

162–E Replace a by 0, c by 1, and d by 3. * The basic pair that is equivalent to (1,3) * The basic pair that is equivalent to (1,3) has a first component, not a second component, of 0.

162–F Because of the definition of equivalent ordered

pairs for directed segments * 2

162–G (0,2) * Yes

162–H $^{-}2$

162–I 2 * The second component

162–J $(2,\frac{5}{2})$

162–K The absolute difference of 2 and $\frac{5}{2}$ is the second component of the basic pair that is equivalent to $(2,\frac{5}{2})$. The first component of this basic pair is 0.

162–L A negative rational number * The second component of $(\frac{3}{16},\frac{5}{16})$ is greater than the first component.

162–M The second component

162–N An infinite set * Yes. If a and b are two different members of Z, then $(0,a)$ and $(0,b)$ indicate different negative rational numbers.

163–O $-\frac{5}{6}$

163–P $^{-}15$

163–Q $-\frac{1}{4}$

163–R $^{-}4$

163–S $-\frac{1}{12}$

163–A Since $\frac{1}{2} + 0 = 0 + \frac{1}{2}$ is a true statement, $(\frac{1}{2},\frac{1}{2}) \sim (0,0)$. * Since $3 + 0 = 0 + 3$ is a true statement, $(3,3) \sim (0,0)$. * Since $\frac{6}{7} + 0 = 0 + \frac{6}{7}$ is a true statement, $(\frac{6}{7},\frac{6}{7}) \sim (0,0)$.

163–B They are the same.

163–C An infinite set * To obtain a member of $\{(x,y) \mid (x,y) \sim (0,0)\}$, x and y are replaced by the same member of R_a. R_a has infinitely many members.

163–D $(0,0)$ satisfies $\{(x,y) \mid (x,y) \sim (0,0)\}$, and $(0,0)$ has a component of 0.

163–E No. The basic pair of each member of R_p has a non-zero first component. * No. The basic pair of each member of R_n has a non-zero second component.

163–F Yes * Yes * Yes * An infinite set

163–G An infinite set

163–H Yes * { }

164–I The coördinates of points to the right of point 0 are positive rational numbers. The coördinates of points to the left of point 0 are negative rational numbers.

164–J To the right of point 0 * To the left of point 0

164–K No * Yes. A point is associated with each member of R and R is an infinite set.

164–L The arrow at the right tells you that the points associated with positive rational numbers go on and on. The arrow at the left tells you that the points associated with negative rational numbers go on and on.

164–M $^{+}\frac{1}{2}$ * 0

164–N The coördinate of point E is $^{-}\frac{1}{2}$.

164–O $^{-}2$

164–P $-\frac{5}{2}$

164–Q $\overrightarrow{FK}$

164–R The directed segments associated with $^{-}3$ are directed to the left. The directed segments associated with $^{+}3$ are directed to the right.

On your own

164–1 (0,1)

164–2 $\{(x,y) \mid (x,y) \sim (0,1)\}$

164–3 $^{-}1$

164–4 $(\frac{1}{2},\frac{5}{3})$ determines $\overrightarrow{DC}$. $(2,\frac{1}{4})$ determines $\overrightarrow{FE}$. $(\frac{5}{2},3)$ determines $\overrightarrow{HG}$.

164–5 $\{(x,y) \mid (x,y) \sim (\frac{1}{2},\frac{5}{3})\}$, or $\{(x,y) \mid (x,y) \sim (0,\frac{7}{6})\}$. $\{(x,y) \mid (x,y) \sim (2,\frac{1}{4})\}$, or $\{(x,y) \mid (x,y) \sim (0,\frac{3}{4})\}$. $\{(x,y) \mid (x,y) \sim (\frac{5}{2},3)\}$, or $\{(x,y) \mid (x,y) \sim (0,\frac{1}{2})\}$

164–6 $-\frac{7}{6}$. $-\frac{3}{4}$. $-\frac{1}{2}$

164–7 $^{-}10$

165–8 $-\frac{4}{9}$

165–9 0

165–10 $^{+}21$

165–11 $^{+}1\frac{3}{7}$

165–12 $^{-}8$

165–13 $(0,\frac{1}{4}) * -\frac{1}{4}$

165–14 $(3,0) * ^{+}3$

165–15 $(0,7) * ^{-}7$

165–16 $(0,\frac{2}{5}) * -\frac{2}{5}$

165–17 $(0,0) * 0$

165–18 $(7,0) * ^{+}7$

165–19 $(0,\frac{3}{40}) * -\frac{3}{40}$

165–20 $(0,0) * 0$

165–21 $^{-}1 * ^{+}\frac{1}{4} * ^{+}\frac{2}{3}$

165–22 Point H * Point D * Point A

165–23 $^{+}2$

165–24 $-\frac{1}{4}$

165–25 $^{+}\frac{2}{3} * -\frac{5}{4} * ^{+}2$

165–26 $\overrightarrow{DH} * \overrightarrow{DC} * \overrightarrow{DG}$

165–27 $^{+}3720$ could represent 3720 ft. above sea level. $^{-}39$ could represent 39 ft. below sea

level.

165–28 $^{-}15$ * $^{+}212$

165–29 $^{-}5$ * $^{+}10$

Lesson 129

166–A Yes * No * To the left of point 0

166–B No * Yes

166–C $(0,\frac{3}{4})$ * Since $\frac{1}{2} + \frac{3}{4} = 0 + \frac{5}{4}$ is a true statement, $(\frac{1}{2},\frac{5}{4}) \sim (0,\frac{3}{4})$. Therefore, both $(\frac{1}{2},\frac{5}{4})$ and $(0,\frac{3}{4})$ indicate the rational number $^{-}\frac{3}{4}$.

166–D Yes. The ordered pair $(0,\frac{3}{4})$ indicates $^{-}\frac{3}{4}$. Therefore, the symbol $(0,\frac{3}{4})$ names $^{-}\frac{3}{4}$. * Yes. The ordered pair $(\frac{1}{2},\frac{5}{4})$ indicated $^{-}\frac{3}{4}$. Therefore, the symbol $(\frac{1}{2},\frac{5}{4})$ names $^{-}\frac{3}{4}$. * Yes. Both $(0,\frac{3}{4})$ and $(\frac{1}{2},\frac{5}{4})$ indicate the rational number $^{-}\frac{3}{4}$.

166–E Yes. Point $^{-}1$ is to the left of point $^{-}\frac{1}{4}$. * Yes. $(0,\frac{1}{2})$ indicates $^{-}\frac{1}{2}$ and $(1,0)$ indicates $^{+}1$. Point $^{-}\frac{1}{2}$ is to the left of point $^{+}1$.

166–F Point 0 is to the left of each point associated with a positive rational number. * Each point associated with a negative rational number is to the left of point 0.

166–G Yes

166–H 0

166–I The first component of $(\frac{1}{4},0)$ and the second component of $(0,\frac{7}{8})$ * $\frac{9}{8}$

166–J R_a is closed under the operation of addition. * Yes. 0 is less than $\frac{9}{8}$.

166–K (Column 1) Yes * $^{+}\frac{2}{5} = (\frac{2}{5},0)$. $^{+}\frac{4}{15} = (\frac{4}{15},0)$.

167–L (Column 1) The first component of $(\frac{2}{5},0)$ and the second component of $(\frac{4}{15},0)$ * The first component of $(\frac{4}{15},0)$ and the second component of $(\frac{2}{5},0)$ * No

167–A R_a

167–B a was replaced by 0, b by $\frac{2}{3}$, c by 0, and d by $\frac{1}{2}$. * a was replaced by 0, b by $\frac{2}{3}$, c by 0, and d by $\frac{1}{2}$.

167–C Yes. The sum of $0 + \frac{1}{2}$, or $\frac{1}{2}$, is less than the sum of $0 + \frac{2}{3}$, or $\frac{2}{3}$. * The sum of the first component of $(0,\frac{2}{3})$ and the second component of $(0,\frac{1}{2})$ is less than the sum of the first component of $(0,\frac{1}{2})$ and the second component of $(0,\frac{2}{3})$. Therefore, the rational number $(0,\frac{2}{3})$, or $^{-}\frac{2}{3}$, is less than the rational number $(0,\frac{1}{2})$, or $^{-}\frac{1}{2}$.

167–D Replace a by 0, b by $\frac{3}{4}$, c by 0, and d by $\frac{5}{8}$. * Yes

167–E Yes

167–F False statement * Since $4 + 0$ is not less than $3 + 0$, $(4,0)$, or $^{+}4$, is not less than $(3,0)$, or $^{+}3$.

167–G True statement * Since $0 + \frac{2}{3}$ is less than $0 + \frac{5}{6}$, $(0,\frac{5}{6})$, or $^{-}\frac{5}{6}$, is less than $(0,\frac{2}{3})$, or $^{-}\frac{2}{3}$.

167–H True statement * Since $0 + 0$ is less than $\frac{1}{2} + 9$, $(0,9)$, or $^{-}9$, is less than $(\frac{1}{2},0)$, or $^{+}\frac{1}{2}$.

167–I False statement * Since $0 + 8$ is not less than $0 + 4$, $(0,4)$, or $^{-}4$, is not less than $(0,8)$, or $^{-}8$.

167–J False statement * Since $\frac{3}{4} + \frac{3}{2}$ is not less than $0 + 0$, $(\frac{3}{4},0)$, or $^{+}\frac{3}{4}$, is not less than $(0,\frac{3}{2})$, or $^{-}\frac{3}{2}$.

167–K (Column 2) True statement * Since $\frac{8}{3} + 0$ is less than $5 + 0$, $(\frac{8}{3},0)$, or $^{+}\frac{8}{3}$, is less than $(5,0)$, or $^{+}5$.

167–L (Column 2) $^{+}\frac{2}{5} > ^{+}\frac{1}{3}$.

167–M $(0,\frac{1}{2}) < (0,\frac{1}{4})$. * Yes. Since $0 + \frac{1}{4}$ is less than $0 + \frac{1}{2}$, $(0,\frac{1}{2})$, or $^{-}\frac{1}{2}$, is less than $(0,\frac{1}{4})$, or $^{-}\frac{1}{4}$. * Yes. Since $^{-}\frac{1}{2} < ^{-}\frac{1}{4}$ is a true statement, $^{-}\frac{1}{4} > ^{-}\frac{1}{2}$ is also a true statement.

167–N True statement * $^{+}\frac{3}{4} > ^{+}\frac{1}{2}$ gives the same information as $^{+}\frac{1}{2} < ^{+}\frac{3}{4}$, and $\frac{1}{2} + 0 < \frac{3}{4} + 0$ is a true statement.

167–O False statement * $^{-}\frac{5}{8} > ^{-}\frac{1}{4}$ gives the same information as $^{-}\frac{1}{4} < ^{-}\frac{5}{8}$, and $0 + \frac{5}{8} < 0 + \frac{1}{4}$ is a false statement.

167–P False statement * $^{-}\frac{7}{8} > ^{+}\frac{5}{6}$ gives the same information as $^{+}\frac{5}{6} < ^{-}\frac{7}{8}$ and $\frac{5}{6} + \frac{7}{8} < 0 + 0$ is a false statement.

168–Q The universe for a and b is Z. a, or $(0,a)$, represents any negative rational number. b, or $(b,0)$, represents any positive rational number. Since, for each a and b, $0 + 0$ is less than $b + a$,

$(0,a)$, or a, is less than $(b,0)$, or b.

168–A $5 * \frac{7}{1}$

168–B Yes. (0,5) is the basic pair of $^{-}5$. Both 0 and 5 are mates of natural numbers. * Yes. Both 16 and 0 are mates of natiral numbers. * No. $(0,\frac{7}{8})$ is the basic pair of $\frac{-7}{8}$. $\frac{7}{8}$ is not the mate of a natural number. * Yes. 0 is the mate of a natural number.

168–C Yes * Yes * Yes * Examples are $-\frac{1}{2}$, $+\frac{7}{2}$, and $-\frac{4}{5}$.

168–D Each member of I is also a member of R.

168–E Yes * Yes

168–F No * No * Yes

168–G No * Each member of I_p is also a member of R. * Each member of I_p is also a member of R_p. * Each member of I_p is also a member of I.

168–H $^{+}1$ * No

168–I Yes * Yes

168–J Yes. $^{-}83$ is a negative integer. * No. $^{+}6$ is not a negative integer. * No. $-\frac{7}{2}$ is not an integer.

168–K Each member of I_n is also a member of R. * Each member of I_n is also a member of R_n. * Each member of I_n is also a member of I.

169–L No * $^{-}1$

169–M 0

169–N I * { }

169–O The three dots indicate that the positive integers go on and on and that there is not a greatest positive integer. * The three dots indicate that the negative integers go on and on and that there is not a least negative integer.

169–P No * No

On your own

169–1 $^{-}2$

169–2 $+\frac{15}{16}$

169–3 $^{+}3$

169–4 $+\frac{3}{8}$

169–5 (0,3)

169–6 (0,0)

169–7 $(\frac{2}{3},0)$

169–8 $(0,\frac{3}{4})$

169–9 $-\frac{5}{8}$, $+\frac{2}{3}$, $+\frac{5}{6}$, $+\frac{7}{8}$

169–10 $^{-}2$, $-\frac{3}{2}$, $^{-}1$, $-\frac{1}{2}$, 0

169–11 $-\frac{5}{3}$, $-\frac{4}{3}$, 0, $+\frac{1}{3}$, $^{+}1$, $+\frac{4}{3}$

169–12 $^{-}3$, $^{-}2$, $^{-}1$, $^{+}1$, $^{+}2$, $^{+}3$

169–13 False statement

169–14 True statement

169–15 True statement

169–16 False statement

169–17 True statement

169–18 False statement

169–19 $\{^{-}2, \frac{-6}{2}\}$

169–20 $\{^{-}2, \frac{-6}{2}, 0, ^{+}3\}$

169–21 $\{^{+}3\}$

169–22 $\{^{-}2, \frac{+8}{9}, ^{+}3, -\frac{2}{3}, -\frac{3}{4}, 0, \frac{-6}{2}\}$

169–23 $\{\frac{+8}{9}, ^{+}3\}$

169–24 $\{^{-}2, -\frac{2}{3}, -\frac{3}{4}, \frac{-6}{2}\}$

Keeping skilful

170–1 2388

170–2 227.3360

170–3 $34\frac{5}{24}$

170–4 958

170–5 784

170–6 1,198,800

170–7 27.993

170–8 1155

170–9 62,946

170–10 $\frac{1986}{343}$

170–11 .00036

170–12 $12\frac{5}{8}$

170–13 $232\frac{5}{6}$

170–14 $51\frac{11}{16}$

170–15 $6\frac{1}{3}$

170–16 $69\frac{1}{3}$

170–17 .303

170–18 23.156

170–19 $42\frac{1}{4}$

170–20 261.0

170–21 27

170–22 118.5

170–23 4.05

170–24 128.294

170–25 7.7

Checking up

Test 123

170–1 Distributive property of multiplication over addition

170–2 Commutative property of multiplication

170–3 Identity-element property of multiplication

170–4 Associative property of addition

170–5 $^{-}2$, $-\frac{7}{4}$, $-\frac{3}{2}$, $-\frac{1}{2}$, 0, $+\frac{1}{4}$, $+\frac{1}{2}$

170–6 $^{-}5$, $-\frac{2}{3}$, $-\frac{3}{5}$, 0, $+\frac{1}{10}$, $+\frac{2}{5}$, $+\frac{5}{2}$

170–7 $+\frac{9}{2}$

170–8 $-\frac{31}{20}$

170–9 0

170–10 $^{-}1$

170–11 0 and $^{-}1$

170–12 $+\frac{3}{8}$

170–13 Point B

170–14 $+\frac{3}{4}$

170–15 $-\frac{1}{2}$

170–16 $^{-}1$

Lesson 130

171–A Point A. $^{-}3$ is a trip of 3 units to the left. * Point G. $^{+}3$ is a trip of 3 units to the right.

171–B Point E * $^{+}2$ is a trip of 2 units to the right. * Yes

171–C Point F * 3 units * Yes

171–D Point $^{+}2$ * Point $^{+}2$ * Point $^{+}5$

171–E Point $^{+}5$ * Yes

171–F Point $^{+}\frac{1}{4}$ * If you take a trip of $\frac{5}{4}$ units to the right from point $^{+}\frac{1}{4}$, you go to point $^{+}\frac{6}{4}$.

171–G Point $^{+}\frac{6}{4}$ * Yes

171–H Yes

171–I $^{+}12$. A trip of $^{+}5$ takes you from point 0 to point $^{+}5$. From point $^{+}5$, a trip of $^{+}7$ takes you to point $^{+}12$. A single trip of $^{+}12$ takes you from point 0 to point $^{+}12$. Therefore, the sum of $^{+}5$ and $^{+}7$ should be $^{+}12$. * $^{+}22$. A trip of $^{+}13$ takes you from point 0 to point $^{+}13$. From point $^{+}13$, a trip of $^{+}9$ takes you to point $^{+}22$. A single trip of $^{+}22$ takes you from point 0 to point $^{+}22$. Therefore, the sum of $^{+}13$ and $^{+}9$ should be $^{+}22$. * $^{+}\frac{9}{4}$. A trip of $^{+}\frac{3}{2}$ takes you from point 0 to point $^{+}\frac{3}{2}$. From point $^{+}\frac{3}{2}$, a trip of $^{+}\frac{3}{4}$ takes you to point $^{+}\frac{9}{4}$. A single trip of $^{+}\frac{9}{4}$ takes you from point 0 to point $^{+}\frac{9}{4}$. Therefore, the sum of $^{+}\frac{3}{2}$ and $^{+}\frac{3}{4}$ should be $^{+}\frac{9}{4}$.

171–J If you start at point 0 and take any two successive trips to the right, you will always end up at a point to the right of point 0.

172–K Point $^{-}1$ * Point $^{-}1$ * Point $^{-}3$

172–L Point $^{-}3$ * Yes

172–M Yes

172–N $^{-}11$. A trip of $^{-}5$ takes you from point 0 to point $^{-}5$. From point $^{-}5$, a trip of $^{-}6$ takes you to point $^{-}11$. A single trip of $^{-}11$ takes you from point 0 to point $^{-}11$. Therefore, the sum of $^{-}5$ and $^{-}6$ should be $^{-}11$. * $^{-}\frac{7}{9}$. A trip of $^{-}\frac{4}{9}$ takes you from point 0 to point $^{-}\frac{4}{9}$. From point $^{-}\frac{4}{9}$, a trip of $^{-}\frac{1}{3}$ takes you to point $^{-}\frac{7}{9}$. A single trip of $^{-}\frac{7}{9}$ takes you from point 0 to point $^{-}\frac{7}{9}$. Therefore, the sum of $^{-}\frac{4}{9}$ and $^{-}\frac{1}{3}$ should be $^{-}\frac{7}{9}$. * $^{-}20$. A trip of $^{-}13$ takes you from point 0 to point $^{-}13$. From point $^{-}13$, a trip of $^{-}7$ takes you to $^{-}20$. A single trip of $^{-}20$ takes you from point 0 to point $^{-}20$. Therefore, the sum of $^{-}13$ and $^{-}7$ should be $^{-}20$.

172–O If you start at point 0 and take any two successive trips to the left, you will always end up at a point to the left of point 0.

172–A Point $^{-}3$ * Point $^{-}3$ * A trip of $^{+}1$ is a trip of 1 unit to the right. * Point $^{-}2$

172–B Point $^{-}2$ * Yes

172–C $^{-}2$

172–D A trip of $^{-}3$ * The same as a trip to the left from point 0

172–E $^{-}3$. A trip of $^{-}5$ takes you from point 0 to point $^{-}5$. From point $^{-}5$, a trip of $^{+}2$ takes you to point $^{-}3$. A single trip of $^{-}3$ takes you from point 0 to point $^{-}3$. Therefore, the sum of $^{-}5$ and $^{+}2$ should be $^{-}3$. * $^{-}8$. A trip of $^{-}14$ takes you from point 0 to point $^{-}14$. From point $^{-}14$, a trip of $^{+}6$ takes you to point $^{-}8$. A single trip of $^{-}8$ takes you from point 0 to point $^{-}8$. Therefore, the sum of $^{-}14$ and $^{+}6$ should be $^{-}8$. * $^{-}\frac{4}{2}$. A trip of $^{-}\frac{5}{2}$ takes you from point 0 to point $^{-}\frac{5}{2}$. From point $^{-}\frac{5}{2}$, a trip of $^{+}\frac{1}{2}$ takes you to point $^{-}\frac{4}{2}$. A single trip of $^{-}\frac{4}{2}$ takes you from point 0 to point $^{-}\frac{4}{2}$. Therefore, the sum of $^{-}\frac{5}{2}$ and $^{+}\frac{1}{2}$ should be $^{-}\frac{4}{2}$.

172–F Point $^{+}3$ * Point $^{+}2$

172–G Yes

173–H The sum of $^{+}3$ and $^{-}1$ should be $^{+}2$.

173–I A trip of $^{+}3$ * The same as a trip to the right from point 0

173–J $^{+}6$. A trip of $^{+}9$ takes you from point 0 to point $^{+}9$. From point $^{+}9$, a trip of $^{-}3$ takes you to point $^{+}6$. A single trip of $^{+}6$ takes you from point 0 to point $^{+}6$. Therefore, the sum of $^{+}9$ and $^{-}3$ should be $^{+}6$. * $^{+}6$. A trip of $^{+}16$ takes you

from point 0 to point $^+16$. From point $^+16$, a trip of $^-10$ takes you to point $^+6$. A single trip of $^+6$ takes you from point 0 to point $^+6$. Therefore, the sum of $^+16$ and $^-10$ should be $^+6$. * $^+\frac{4}{4}$. A trip of $^+\frac{7}{4}$ takes you from point 0 to point $^+\frac{7}{4}$. From point $^+\frac{7}{4}$, a trip of $^-\frac{3}{4}$ takes you to point $^+\frac{4}{4}$. A single trip of $^+\frac{4}{4}$ takes you from point 0 to point $^+\frac{4}{4}$. Therefore, the sum of $^+\frac{7}{4}$ and $^-\frac{3}{4}$ should be $^+\frac{4}{4}$.

173–K If the trip represented by the positive rational number is longer than the trip represented by the negative rational number, then the sum of the two numbers should be positive. If the trip represented by the negative rational number is longer than the trip represented by the positive rational number, then the sum of the two numbers should be negative.

173–L When the trip to the right is the same number of units as the trip to the left * When the trip represented by the positive rational number is the same number of units as the trip represented by the negative rational number.

173–M Yes. A trip to the left followed by a trip to the right of the same number of units takes you to point 0. * A trip of $^-3$ takes you from point 0 to point $^-3$. From point $^-3$, a trip of $^+3$ takes you back to point 0.

173–N No * No

173–O For a trip represented by the rational number 0, you stay at point 0. Therefore, when you take two successive trips and the first trip is 0, you start the second trip from point 0. This is the same as taking just the second trip. Therefore, the sum of 0 and any given rational number should be the given number.

On your own

173–1 Point $^+3$ * Point $^+7$

173–2 $^+7$

173–3 Point $^+2$

173–4 $^+2$

173–5 Point $^-2$ * Point $^-4$

173–6 $^-4$

173–7 Point $^-2$

173–8 $^-2$

173–9 A trip of $^+6$ followed by a trip of $^-2$ takes you from point 0 to point $^+4$. A trip of $^-2$ followed by a trip of $^+6$ also takes you from point 0 to point $^+4$.

173–10 A trip of $^-5$ takes you from point 0 to point $^-5$. From point $^-5$, a trip of $^+5$ takes you back to point 0.

173–11 For a trip of 0, you stay at point 0. So a trip of 0 followed by a trip of $^-7$ is the same as taking just the one trip $^-7$. A trip of $^-7$ takes you from point 0 to point $^-7$.

173–12 From point 0, a trip of $^+5$ followed by a trip of $^-6$ takes you to point $^-1$. From point 0, a trip of $^-4$ followed by a trip of $^+3$ also takes you to point $^-1$.

Lesson 131

174–A A trip of $^+2$ takes you from point 0 to point $^+2$. From point $^+2$, a trip of $^+3$ takes you to point $^+5$. This is the same as taking a single trip of $^+5$. * Yes * $^+2 = (2,0)$. $^+3 = (3,0)$, $^+5 = (5,0)$.

174–B A trip of $^-1$ takes you from point 0 to point $^-1$. From point $^-1$, a trip of $^-2$ takes you to point $^-3$. This is the same as taking a single trip of $^-3$. * $^-1 = (0,1)$. $^-2 = (0,2)$. $^-3 = (0,3)$.

174–C 5 * Yes

174–D 3 * Yes

174–E Yes. $^+\frac{3}{4} = (\frac{3}{4},0)$, and $^+\frac{1}{2} = (\frac{1}{2},0)$. * $\frac{5}{4}$ * $^+\frac{5}{4}$. If you are finding the sum of two positive rational numbers, the component you obtain is a first component and the sum is positive.

174–F Yes. $^-2 = (0,2)$, and $^-\frac{2}{3} = (0,\frac{2}{3})$. $2 + \frac{2}{3} = \frac{8}{3}$. * Second component * $(0,\frac{8}{3})$, or $^-\frac{8}{3}$. If you are finding the sum of

two negative rational numbers, the component you obtain is the second component, and the sum is negative.

174–G $^{-}4 = (0,4)$. $^{+}1 = (1,0)$. $^{-}3 = (0,3)$. * $^{+}4 = (4,0)$. $^{-}1 = (0,1)$. $^{+}3 = (3,0)$. * $^{-}2 = (0,2)$. $^{+}2 = (2,0)$. $0 = (0,0)$.

175–H 3 * Yes. $4 - 1 = 3$.

175–I 3 * Yes. $4 - 1 = 3$.

175–J No * 0. $2 - 2 = 0$.

175–K Yes * $^{-}\frac{1}{2}$

175–L Yes * $^{+}\frac{1}{2}$

175–M The non-zero component of the basic pair for $^{-}\frac{4}{3}$ is $\frac{4}{3}$. The non-zero component of the basic pair for $^{+}\frac{4}{3}$ is $\frac{4}{3}$. Find the difference of these non-zero components. Since this difference is 0, the basic pair for the sum of $^{-}\frac{4}{3}$ and $^{+}\frac{4}{3}$ is $(0,0)$. * 0

175–N 0. $3 - 3 = 0$.

175–O $^{-}21$. $12 + 9 = 21$.

175–P $^{-}\frac{6}{7}$. $2 - \frac{8}{7} = \frac{6}{7}$.

175–Q $^{-}1$. $3 - 2 = 1$.

175–R 0. $6 - 6 = 0$.

175–S $^{-}\frac{3}{2}$. $\frac{2}{3} + \frac{5}{6} = \frac{3}{2}$.

175–T $^{+}\frac{11}{2}$. $8 - \frac{5}{2} = \frac{11}{2}$.

175–U $^{-}\frac{16}{3}$. $7 - \frac{5}{3} = \frac{16}{3}$.

175–V $^{+}\frac{1}{40}$. $\frac{5}{8} - \frac{3}{5} = \frac{1}{40}$.

175–W $^{+}\frac{16}{9}$. $1 + \frac{7}{9} = \frac{16}{9}$.

175–X $^{+}\frac{1}{3}$. $\frac{5}{4} - \frac{11}{12} = \frac{1}{3}$.

175–Y $^{+}\frac{2}{9}$. $\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$.

176–A Yes * $^{+}3$ * $^{+}2$ and $^{+}3$

176–B 11 * Yes * 6 * Yes

176–C $^{+}5$ * Yes. $(11,6) = {}^{+}5$, $(7,5) = {}^{+}2$ and $(4,1) = {}^{+}3$.

176–D $(1,2)$ and $(8,10)$ * $(1,2) = {}^{-}1$. $(8,10) = {}^{-}2$.

176–E 9 * Yes * 12 * Yes

176–F $^{-}3$ * Yes. $(1,2) + (8,10) = (9,12)$ is the same statement as $^{-}1 + {}^{-}2 = {}^{-}3$.

176–G $^{-}4$ * $^{+}1$

176–H 7 * 10 * Yes

176–I From your work with trips, you know that the sum of $^{-}4$ and $^{+}1$ should be $^{-}3$. $(7,10) = {}^{-}3$.

176–J From your work with trips, you know that the sum of $^{+}4$, or $(8,4)$, and $^{-}1$, or $(3,4)$, should be $^{+}3$. The sum of the first components of $(8,4)$ and $(3,4)$ is 11. The sum of the second components of $(8,4)$ and $(3,4)$ is 8. You can form the ordered pair $(11,8)$ from these sums. $(11,8) = {}^{+}3$.

176–K Yes * Yes

177–L Yes * Because of the definition of the sum of two rational numbers

177–M a was replaced by $\frac{1}{3}$, b by 8, c by 2, and d by $\frac{7}{2}$. * a was replaced by 2, b by 3, c by 10, and d by 5. * a was replaced by $\frac{1}{2}$, b by $\frac{1}{3}$, c by $\frac{7}{8}$, and d by 6. * Yes

177–N True statement

177–O False statement

177–P False statement

177–Q True statement

177–R True statement

177–A Closure property of addition

177–B From the definition of the sum of two rational numbers, if (a,b) and (c,d) are any two rational numbers, $(a,b) + (c,d) = (a + c, b + d)$. From the closure property of addition of rational numbers of arithmetic, $(a + c, b + d)$ is an ordered pair of rational numbers of arithmetic. Therefore, $(a + c, b + d)$ is a rational number and the sum of any two rational numbers is always a rational number.

177–C Yes. From your response to exercise B, you know that if $x \in R$ and $y \in R$, then $x + y \in R$.

177–D Yes * $(\frac{3}{4},0)$, or $^{+}\frac{3}{4}$, and $(0,2)$, or $^{-}2$

177–E Yes. $(\frac{3}{4},0) = {}^{+}\frac{3}{4}$ and $(\frac{3}{2},\frac{3}{4}) = {}^{+}\frac{3}{4}$. * Yes. $(1,3) = {}^{-}2$ and $(0,2) = {}^{-}2$.

177–F (3,5) * Yes

177–G $(\frac{7}{4},1)$ * Yes

177–H Yes * Each of the ordered pairs $(\frac{3}{4},2)$, $(\frac{5}{2},\frac{15}{4})$, $(\frac{15}{4},5)$, and $(\frac{1}{4},3)$ indicates the rational number $^{-}\frac{5}{4}$.

177–I Yes

178–A Yes * Yes

178–B Yes. The definition of the sum of two rational numbers * Yes. $0 + 0 = 0$. * A positive rational number. The first component of the basic pair $(m + n, 0)$ is non-zero.

178–C Yes

178–D Negative * The second components of both $(0,m)$ and $(0,n)$ are non-zero.

178–E Yes. The definition of

the sum of two rational numbers * Yes. $0 + 0 = 0$. * Negative. The second component of the basic pair $(0, m + n)$ is non-zero.

178–F Yes

178–G A positive rational number * A negative rational number

178–H Yes. The definition of the sum of two rational numbers * Yes. $m + 0 = m$ and $0 + n = n$. * Negative. If $m < n$, then the second component of (m,n) is greater than the first component.

178–I Positive. If $m > n$, then the first component of (m,n) is greater than the second component. * 0. If $m = n$, then the first component of (m,n) is the same as the second component.

178–J No. If the non-zero component of the basic pair for the positive rational number is not greater than the non-zero component of the basic pair for the negative rational number, then the sum of the two numbers is not positive. * No. If the non-zero component of the basic pair for the negative rational number is not greater than the non-zero component of the basic pair for the positive rational number, then the sum is not negative. * When the non-zero component of the basic pair for the positive rational number is the same as the non-zero component of the basic pair for the negative rational number.

178–K Yes. From the definition of the sum of two rational numbers, $(0,0) + (c,d) = (0 + c, 0 + d)$ and $(0 + c, 0 + d) = (c,d)$. * The given rational number

On your own

178–1 False statement

178–2 True statement

178–3 True statement

178–4 False statement

178–5 False statement

178–6 True statement

179–7 Examples are $(2,5) + (\frac{1}{6},\frac{1}{2}) = (\frac{13}{6},\frac{11}{2})$ and $(\frac{1}{2},\frac{7}{2}) + (1,\frac{4}{3}) = (\frac{3}{2},\frac{29}{6})$.

179–8 Examples are $(2,\frac{1}{2}) + (7,2) = (9,\frac{5}{2})$ and $(\frac{5}{2},1) + (\frac{11}{2},\frac{1}{2}) = (8,\frac{3}{2})$.

179–9 Examples are $(9,2) + (3,24) = (12,26)$ and $(\frac{15}{2},\frac{1}{2}) + (1,22) = (\frac{17}{2},\frac{45}{2})$.

179–10 Examples are $(\frac{3}{4},1) + (\frac{2}{9},0) = (\frac{35}{36},1)$ and $(1,\frac{5}{4}) + (1,\frac{5}{9}) = (2,\frac{73}{36})$.

179–11 Examples are $(18,2) + (18,4) = (36,6)$ and $(16,0) + (24,10) = (40,10)$.

179–12 $^{+}37$

179–13 $^{+}8$

179–14 $^{+}32$

179–15 0

179–16 $^{-}13$

179–17 $^{-}6$

179–18 $^{-}3$

179–19 $^{+}27$

179–20 $^{+}21$

179–21 $^{+}219$

179–22 $^{-}89$

179–23 $^{-}26$

179–24 $^{+}\frac{17}{15}$

179–25 $^{-}17$

179–26 $^{+}9$

179–27 0

179–28 $^{-}464$

179–29 0

179–30 $^{+}89$

179–31 $^{+}28$

179–32 0

179–33 $^{-}\frac{1}{12}$

179–34 $^{+}\frac{4}{3}$

179–35 $^{-}\frac{22}{3}$

179–36 $^{-}9 * ^{+}9 * ^{-}13 * ^{+}13$

Keeping skilful

179–1 $\{6\frac{2}{3}\}$

179–2 $\{4\frac{3}{4}\}$

179–3 $\{11,1\}$

179–4 $\{78\frac{3}{4}\}$

179–5 $\{1.9\}$

179–6 $\{\frac{2}{17}\}$

179–7 $\{.093\}$

179–8 $\{25\}$

179–9 $\{70.7\}$

179–10 $\{95.18\}$

179–11 $\{\frac{5}{16}, 1\}$

179–12 Since the solution set of $\frac{2}{5} + \frac{3}{4} > y \wedge \frac{1}{3} + \frac{1}{8} < y$ is an infinite set, you should give a standard description of this solution set. $\{y \mid \frac{11}{24} < y < \frac{23}{20}\}$

179–13 $\{17\frac{1}{4}, 19\frac{1}{4}\}$

Lesson 132

179–A $\frac{3}{4} * ^{+}\frac{3}{4}$

179–B $6 * ^{+}6$

179–C $^{+}\frac{6}{5} * ^{+}\frac{4}{21}$ * The product of any two positive rational numbers should be a positive rational number.

180–D 0

180–E 0 * 0 * 0

180–F 0 * Yes

180–G 0 * Yes

180–H Statement A tells us that $^{+}3 + {}^{-}3$ is the same rational number as 0. Since we want multiplication of rational numbers to be well defined, we should be able to replace 0 in the product $^{+}2(0)$ by $^{+}3 + {}^{-}3$ without affecting the product. Statement B tells us that the product $^{+}2(0)$ is 0. If we replace 0 in $^{+}2(0)$ by $^{+}3 + {}^{-}3$, the product $^{+}2(^{+}3 + {}^{-}3)$ should be 0.

180–I Yes

180–J Yes

180–K The transitive property of equality

180–L Yes * $^{-}6$ is the number that must be added to $^{+}6$ to obtain the sum 0.

180–M $^{-}6$. If multiplication of rational numbers has the commutative property, then the product of $^{-}3$ and $^{+}2$ should be the same as the product of $^{+}2$ and $^{-}3$.

180–N $^{-}\frac{14}{15}$ * $^{-}\frac{14}{15}$ * Negative

180–O 0 * Yes

180–P We want 0 to be the product of any rational number and 0.

180–Q The well-defined property of multiplication of rational numbers * The rational number 0 in the product $^{-}3(0)$ was replaced by $^{+}2 + {}^{-}2$.

180–R Distributive property

181–S The transitive property of equality

181–T Yes * $^{+}6$ is the number that must be added to $^{-}6$ to obtain the sum 0.

181–U We want multiplication of rational numbers to have the commutative property.

181–V $^{+}60$ * $^{+}\frac{7}{24}$ * Positive

On your own

181–1 $^{+}30$ * $^{+}2$ * $^{+}\frac{4}{7}$

181–2 Because of the definition of the sum of two rational numbers

181–3 Zero property of multiplication of rational numbers

181–4 Statement A tells you that $^{+}6 + {}^{-}6$ is equal to 0. To obtain statement C, you can replace 0 in the product $^{+}5(0)$ by the equal rational number $^{+}6 + {}^{-}6$.

181–5 Distributive property

181–6 Use the transitive property of equality.

181–7 $^{-}30$

181–8 If multiplication of rational numbers is to have the commutative property, then $^{-}6 \cdot {}^{+}5$ must be the same as $^{+}5 \cdot {}^{-}6$. If $^{-}30$ is a standard name of $^{+}5 \cdot {}^{-}6$, then $^{-}30$ should also be a standard name of $^{-}6 \cdot {}^{+}5$.

181–9 $^{+}14$ * 0 *
$^{+}2 + {}^{-}2 = 0$.
$^{+}7(0) = 0$.
$^{+}7(^{+}2 + {}^{-}2) = 0$.
$^{+}7(^{+}2 + {}^{-}2) = (^{+}7 \cdot {}^{+}2) + (^{+}7 \cdot {}^{-}2)$.
$(^{+}7 \cdot {}^{+}2) + (^{+}7 \cdot {}^{-}2) = 0$.

181–10 If $^{+}7 \cdot {}^{-}2 = {}^{-}14$ and we want multiplication of rational numbers to have the commutative property, then $^{-}2 \cdot {}^{+}7$ must be the same as $^{+}7 \cdot {}^{-}2$, or $^{-}14$.

181–11 0

181–12 $^{+}7 + {}^{-}7 = 0$.
$^{-}2(0) = 0$.
$^{-}2(^{+}7 + {}^{-}7) = 0$.
$^{-}2(^{+}7 + {}^{-}7) = (^{-}2 \cdot {}^{+}7) + (^{-}2 \cdot {}^{-}7)$.
$(^{-}2 \cdot {}^{+}7) + (^{-}2 \cdot {}^{-}7) = 0$.

181–13 $^{+}14$

Lesson 133

182–A $^{+}2 = (3,1)$, $^{+}3 = (5,2)$, and $^{+}6 = (17,11)$. * $^{+}2 = (4,2)$, $0 = (3,3)$, and $0 = (18,18)$. * $^{+}2 = (5,3)$, $^{-}3 = (1,4)$, and $^{-}6 = (17,23)$. * $^{-}3 = (2,5)$, $^{-}2 = (2,4)$, and $^{+}6 = (24,18)$.

182–B 15 * 2 * Yes

182–C 6 * 5 * Yes

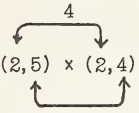
182–D By finding the product of the first components of (4,2) and (3,3) * By finding the product of the second components of (4,2) and (3,3) * Yes

182–E By finding the product of the first component of (4,2) and the second component of (3,3) * By finding the product of the second component of (4,2) and the first component of (3,3) * Yes

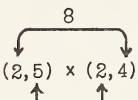
182–F Find the product of the first components of (5,3) and (1,4). Next find the product of the second components of (5,3) and (1,4). Then find the sum of these two products.

183–G Find the product of the first component of (5,3) and the second component of (1,4). Next find the product of the second component of (5,3) and the first component of (1,4). Then find the sum of these two products.

183–H



4 + 20 = 24. *



8 + 10 = 18.

183–I R_a

183–J $ac + bd * ad + bc$

183–K Find the product of the first components of (a, b) and (c, d). Next find the product of the second components of (a, b) and (c, d). Then find the sum of these two products. * Find the product of the first component of (a, b) and the second component of (c, d). Next find the product of the second component of (a, b) and the first component of (c, d). Then find the sum of these two products.

183–L Yes. To simplify an expression like $3 \cdot 0 + 1 \cdot \frac{2}{3}$, you always perform the operation of multiplication before you perform the operation of addition. Since $3 \cdot 0 = 0$ and $1 \cdot \frac{2}{3} = \frac{2}{3}$, $3 \cdot 0 + 1 \frac{2}{3} = 0 + \frac{2}{3}$. * Yes. Since $3 \cdot \frac{2}{3} = 2$ and $1 \cdot 0 = 0$, $3 \cdot \frac{2}{3} + 1 \cdot 0 = 2 + 0$.

184–M Because of the definition of the product of two rational numbers

184–N a was replaced by $\frac{1}{2}$ b by 2, c by 2, and d by $\frac{2}{3}$. * Yes

184–O Since $1 + \frac{4}{3} = \frac{7}{3}$ and $\frac{1}{3} + 4 = 1\frac{3}{3}$, $(1 + \frac{4}{3}, \frac{1}{3} + 4) = (\frac{7}{3}, 1\frac{3}{3})$.

184–P a was replaced by $\frac{3}{5}$, b by 1, c by $\frac{1}{2}$, and d by $\frac{5}{2}$. * Yes. $(\frac{3}{5}, 1) \times (\frac{1}{2}, \frac{5}{2}) = (\frac{3}{10} + \frac{5}{2}, \frac{3}{2} + \frac{1}{2})$. $(\frac{3}{10} + \frac{5}{2}, \frac{3}{2} + \frac{1}{2}) = (1\frac{4}{5}, 2)$.

184–Q False statement

184–R True statement

184–S True statement

184–T True statement

184–A From the closure property of multiplication of rational numbers of arithmetic, $ac \in R_a$, $bd \in R_a$, $ad \in R_a$, and $bc \in R_a$. From the closure property of addition of rational numbers of arithmetic, $ac + bd \in R_a$ and $ad + bc \in R_a$. Therefore, $(ac + bd, ad + bc)$ is an ordered pair of rational numbers of arithmetic.

184–B From the definition of the product of two rational numbers, $(a, b) \times (c, d) = (ac + bd, ad + bc)$. From your response to exercise A, $(ac + bd, ad + bc)$ is an ordered pair of rational numbers of arithmetic. Therefore, $(ac + bd, ad + bc) \in R$, and the product of any two rational numbers is always a rational number.

184–C Yes. The operation of multiplication of rational numbers is a binary operation because it is defined for any two members of R.

184–D Yes * Because of the definition of the product of two rational numbers

184–E (1,6)

184–F (0,5) * Yes * Yes

184–G $(\frac{4}{3}, 1\frac{9}{3})$ * Yes

184–H (20,18) * Yes

184–I Yes * Each of the ordered pairs $(\frac{5}{2}, \frac{25}{2})$, $(\frac{54}{3}, \frac{84}{3})$, and (128,128) indicates the same rational number as (16,26).

184–J Yes

185–A (Column 1) Z * A basic pair that has a non-zero first component indicates a positive rational number.

185–B (Column 1) By the definition of the product of two rational numbers, $(m,0) \times (n,0) = (mn + 0, 0 + 0)$. $mn + 0 = mn$ and $0 + 0 = 0$.

185–C (Column 1) Positive * The product of any two positive numbers is positive.

185–D Negative * $(m,0) \times (0,n) = (0 + 0, mn + 0)$. $0 + 0 = 0$ and $mn + 0 = mn$. * Negative

185–E The product of any positive number and any negative number is negative.

185–F Use the definition of the product of two rational numbers. * The product of any negative number and any positive number is negative.

185–G Use the definition of the product of two rational numbers. * The product of any two negative numbers is positive.

185–H (0,0) * (0,0)

185–I 0 * 0

185–J Negative number

185–K 0

185–L Positive number

185–M Negative number

185–N Positive number

185–O Positive number

185–A (Column 2) $(\frac{2}{7},0) \cdot (14,0)$ $(14,0) = (4 + 0, 0 + 0)$. $4 + 0 = 4$ and $0 + 0 = 0$. * Yes * $^{+}14$ * $^{+}4$

185–B (Column 2) Find the product of $\frac{2}{7}$ and 14.

185–C (Column 2) Positive

186–D Yes * $^{-}\frac{3}{2}$ * $^{+}\frac{5}{8}$

186–E $\frac{15}{16}$ * Negative * $^{-}\frac{15}{16}$

186–F Yes * $^{-}10$ * $^{-}7$

186–G 70 * Positive * $^{+}70$

186–H $^{+}\frac{7}{45}$

186–I $^{-}\frac{9}{2}$

186–J $^{+}54$

186–K 0

186–L $^{-}\frac{3}{32}$

186–M 0

186–N $^{-}16$

186–O $^{+}54$

On your own

186–1 False statement

186–2 True statement

186–3 True statement. $(20,20) = (10,10)$.

186–4 True statement

186–5 False statement

186–6 True statement

186–7 True statement. Since $(2,3) = (\frac{1}{3},\frac{4}{3})$, you can replace $(2,3)$ by $(\frac{1}{3},\frac{4}{3})$.

186–8 False statement

186–9 False statement

186–10 False statement

186–11 True statement. Since $(\frac{11}{9},3) = (0,\frac{16}{9})$ and $(0,\frac{10}{7}) = (4,\frac{38}{7})$, you can replace $(\frac{11}{9},3)$ by $(0,\frac{16}{9})$ and $(0,\frac{10}{7})$ by $(4,\frac{38}{7})$.

186–12 $^{-}27$

186–13 $^{+}48$

186–14 $^{+}36$

186–15 $^{-}52$

186–16 $^{+}55$

186–17 $^{+}144$

186–18 $^{-}100$

186–19 $^{-}\frac{35}{24}$

186–20 $^{+}\frac{3}{20}$

Keeping skilful

186–1 $^{+}\frac{5}{2}$

186–2 $^{-}\frac{83}{16}$

186–3 $^{-}\frac{64}{5}$

186–4 $^{-}\frac{1}{10}$

186–5 $^{-}\frac{3}{7}$

186–6 $^{-}\frac{347}{14}$

186–7 $^{-}\frac{57}{7}$

186–8 $^{-}\frac{91}{51}$

186–9 $^{-}\frac{129}{35}$

186–10 $^{+}1\frac{17}{5}$

186–11 $^{-}\frac{105}{16}$

186–12 $^{+}\frac{36}{5}$

186–13 $^{-}50$

186–14 $^{-}\frac{140}{9}$

186–15 $^{+}\frac{72}{7}$

186–16 $^{+}\frac{20}{13}$

186–17 0

186–18 $^{-}\frac{27}{56}$

Lesson 134

187–A $^{+}\frac{3}{2}$ * 3

187–B None * $^{-}\frac{1}{2}$, $^{-}1$, and $^{-}\frac{3}{2}$

187–C $^{+}\frac{5}{8}$ * $^{+}\frac{15}{3}$, or $^{+}5$

187–D Yes

187–E Yes * None

187–F Since 0 is a rational number but is not a negative number, 0 is a non-negative rational number. * A proper subset of a set is a subset that does not contain all the members of the set. Each member of $\{0\} \cup R_p$ is also a member of R, and $\{0\} \cup R_p$ does not contain all the members of R.

187–G Yes * $\{0\} \cup R_p$ contains all those members of R, and only those members of R, that do not belong to R_n.

187–H Each member of R_a is mapped onto just one member of R_n, and each member of R_n is mapped onto just one member of R_a. * Sets that can be put in one-to-one correspondence are equivalent sets. R_a and R_n can be put in one-to-one correspondence.

187–I $^{+}\frac{1}{2}$ * $\frac{1}{2}$

188–J $^{+}\frac{10}{12}$ is the same rational number as $^{+}\frac{5}{6}$. $\frac{5}{6}$ and $^{+}\frac{5}{6}$ are mapped onto each other. * $\frac{5}{6}$ and $^{+}\frac{2}{3}$ are not mapped onto each other. * In the one-to-one correspondence that we have been discussing, the negative rational numbers are not used in the mapping.

188–K One * One

188–L $(\frac{8}{9},0)$ * Yes

188–M a

188–N Yes * Yes * Yes

188–O $\frac{5}{4}$ * $^{+}\frac{5}{4}$

188-P Yes

188-Q $\frac{2}{5} * \frac{1}{5} * \frac{2}{5} + \frac{1}{5}$, or $\frac{3}{5}$

188-R By the definition of the sum of two rational numbers, $(a,0) + (b,0) = (a + b,0)$, or ${}^{+}a + {}^{+}b = {}^{+}(a + b)$. Since $a + b$ is the mate of ${}^{+}(a + b)$, $a + b$ is also the mate of ${}^{+}a + {}^{+}b$.

188-S $\frac{8}{27} * \frac{+8}{27}$

188-T Yes

188-U $\frac{1}{6} * \frac{1}{2} * \frac{1}{6} \cdot \frac{1}{2}$, or $\frac{1}{12}$

188-V By the definition of the product of two rational numbers, $(a,0) \cdot (b,0) = (a \cdot b,0)$, or ${}^{+}a \cdot {}^{+}b = {}^{+}(a \cdot b)$. Since $a \cdot b$ is the mate of ${}^{+}(a \cdot b)$, $a \cdot b$ is also the mate of ${}^{+}a \cdot {}^{+}b$.

On your own

188-1 $\frac{7}{8}$

188-2 $\frac{11}{7}$

188-3 $\frac{5}{2}$

188-4 $\frac{3}{19}$

188-5 $\frac{9}{10}$

188-6 0

188-7 ${}^{+}\frac{1}{4}$

188-8 $\frac{+13}{6}$

188-9 $\frac{+4}{15}$

188-10 $\frac{+7}{100}$

189-11 $\frac{5}{9} + \frac{7}{3} = \frac{26}{9}$.
${}^{+}\frac{5}{9} + {}^{+}\frac{7}{3} = {}^{+}\frac{26}{9}$.

189-12 $\frac{11}{10} + \frac{3}{10} = \frac{7}{5}$.
${}^{+}\frac{11}{10} + {}^{+}\frac{3}{10} = {}^{+}\frac{7}{5}$.

189-13 $\frac{3}{2} + 2 = \frac{7}{2}$.
${}^{+}\frac{3}{2} + {}^{+}2 = {}^{+}\frac{7}{2}$.

189-14 $\frac{1}{4} + \frac{5}{8} = \frac{7}{8}$.
${}^{+}\frac{1}{4} + {}^{+}\frac{5}{8} = {}^{+}\frac{7}{8}$.

189-15 $\frac{1}{3} + \frac{4}{5} = \frac{17}{15}$.
${}^{+}\frac{1}{3} + {}^{+}\frac{4}{5} = {}^{+}\frac{17}{15}$.

189-16 $9 + 4 = 13$.
${}^{+}9 + {}^{+}4 = {}^{+}13$.

189-17 $6 + \frac{4}{5} = \frac{34}{5}$.
${}^{+}6 + {}^{+}\frac{4}{5} = {}^{+}\frac{34}{5}$.

189-18 $\frac{2}{9} + 7 = \frac{65}{9}$.
${}^{+}\frac{2}{9} + {}^{+}7 = {}^{+}\frac{65}{9}$.

189-19 $\frac{5}{3} + \frac{10}{12} = \frac{5}{2}$.
${}^{+}\frac{5}{3} + {}^{+}\frac{10}{12} = {}^{+}\frac{5}{2}$.

189-20 $\frac{6}{7} + \frac{3}{10} = \frac{81}{70}$.
${}^{+}\frac{6}{7} + {}^{+}\frac{3}{10} = {}^{+}\frac{81}{70}$.

189-21 $0 \times {}^{+}\frac{1}{2} = 0$.
$0 \times \frac{1}{2} = 0$.

189-22 ${}^{+}5 \times {}^{+}\frac{1}{4} = {}^{+}\frac{5}{4}$.
$5 \times \frac{1}{4} = \frac{5}{4}$.

189-23 ${}^{+}\frac{7}{10} \times {}^{+}1 = {}^{+}\frac{7}{10}$.
$\frac{7}{10} \times 1 = \frac{7}{10}$.

189-24 ${}^{+}\frac{5}{6} \times {}^{+}\frac{5}{12} = {}^{+}\frac{25}{72}$.
$\frac{5}{6} \times \frac{5}{12} = \frac{25}{72}$.

189-25 ${}^{+}\frac{3}{5} \times {}^{+}\frac{4}{15} = {}^{+}\frac{4}{25}$.
$\frac{3}{5} \times \frac{4}{15} = \frac{4}{25}$.

189-26 ${}^{+}\frac{7}{1} \times {}^{+}\frac{8}{1} = {}^{+}\frac{56}{1}$.
$\frac{7}{1} \times \frac{8}{1} = \frac{56}{1}$.

189-27 True statement

189-28 False statement

189-29 False statement

189-30 True statement

189-31 True statement

189-32 False statement. 0 is a member of I but is not a member of $I_p \cup I_n$.

Keeping skilful

189-1 $\{s \mid 1.8 < s < 5.0\}$

189-2 $\{n \mid n \leq 5\}$

189-3 {.8, .7}

189-4 $\{r \mid 1\frac{4}{5} < r < 3\frac{7}{10}\}$

189-5 {7}

189-6 $\{q \mid q > \frac{5}{6}\}$

189-7 $\{z \mid z \geq 14\frac{1}{2}\}$

189-8 $\{m \mid m < 3.1\}$

189-9 $\{w \mid .8 \leq w < 3.4\}$

Checking up

Test 124

189-1 ${}^{+}\frac{5}{6}$

189-2 ${}^{+}\frac{8}{9}$

189-3 $\frac{+47}{10}$

189-4 0

189-5 ${}^{+}67$

189-6 $\frac{4}{9}$

189-7 0

189-8 81

189-9 $\frac{17}{8}$

189-10 $\frac{1}{100}$

189-11 R_p

189-12 R_n

189-13 R_n

189-14 R_n

189-15 R_p

189-16 R_p

189-17 R_p

189-18 {0}

189-19 R_n

Test 125

189-20 ${}^{+}55$

189-21 ${}^{-}22$

189-22 $\frac{-19}{10}$

189-23 ${}^{-}1$

189-24 ${}^{+}26$

189-25 $\frac{+5}{8}$

189-26 $\frac{+4}{5}$

189-27 $\frac{-23}{14}$

189-28 $\frac{-7}{6}$

189-29 ${}^{+}546$

189-30 ${}^{+}14$

189-31 $\frac{-69}{4}$

189-32 $\frac{+5}{12}$

189-33 $\frac{-2}{3}$

189-34 $\frac{+297}{10}$

189-35 $\frac{-2}{15}$

189-36 $\frac{+17}{2}$

189-37 $\frac{-5}{18}$

Lesson 135

190-A Yes * ${}^{+}\frac{1}{4}$ * $\frac{+13}{12}$

190-B Yes

190-C The order in which you add $\frac{5}{6}$ and $\frac{1}{4}$ does not affect the sum.

190-D From statements C and D * Yes * The order in which you add $\frac{-3}{8}$ and $\frac{-1}{2}$ does not affect the sum.

190-E $\frac{2}{5} + \frac{-1}{3} = \frac{-1}{3} + \frac{2}{5}$. * The order in which you add $\frac{2}{5}$ and $\frac{-1}{3}$ does not affect the sum.

190-F R_a

190–G Yes * Because of the definition of the sum of two rational numbers * $(c + a, d + b)$

191–H Yes * Yes * The commutative property of addition * Yes

191–I Conditions H and J

191–J Since true statements are obtained from conditions H and J for each a, b, c, and d, a true statement is also obtained from condition K.

191–K True statement

191–L False statement

191–M True statement

191–N True statement

191–A Yes

191–B Yes * Yes

191–C $\frac{-3}{5} \times \frac{-2}{9} = \frac{-2}{9} \times \frac{-3}{5}$. * $\frac{2}{3} \times \frac{-4}{7} = \frac{-4}{7} \times \frac{2}{3}$.

191–D The order in which you multiply $\frac{1}{3}$ and $\frac{4}{5}$ does not affect the product. * The order in which you multiply $\frac{-3}{5}$ and $\frac{-2}{9}$ does not aaffect the product. * The order in which you multiply $\frac{-4}{7}$ and $\frac{2}{5}$ does not affect the product.

191–E R_a

192–F $(ac + bd, ad + bc)$ * $(ca + db, cb + da)$ * Because of the definition of the product of two rational numbers

192–G A: $(a, b) \cdot (c, d) = (ac + bd, ad + bc)$. Definition of the product of two rational numbers. B: $(c, d) \cdot (a, b) = (ca + db, cb + da)$. Definition of the product of two rational numbers. C: $(ca + db, cb + da) = (ac + bd, bc + ad)$. Commutative property of multiplication of rational numbers of arithmetic. D: $(c, d) \cdot (a, b) = (ac + bd, bc + ad)$. Conditions B and C and the transitive property of equality. E: $(ac + bd, bc + ad) = (ac + bd, ad + bc)$. Commutative property of addition of rational numbers of arithmetic. F: $(c, d) \cdot (a, b) = (ac + bd, ad + bc)$. Conditions D and E and the transitive property of equality. G: $(a, b) \cdot (c, d) = (c, d) \cdot (a, b)$. Conditions A and F and the transitive property of equality

192–H False statement

192–I True statement

192–J True statement

192–K False statement

On your own

192–1 True statement

192–2 False statement

192–3 False statement

192–4 False statement

192–5 True statement

192–6 False statement

192–7 True statement

192–8 True statement. However, this is true, not because of the commutative property of multiplication, but because the product of two negative numbers, as well as the product of two positive numbers, is always positive.

192–9 $\frac{7}{10} + \frac{-3}{4} = \frac{-1}{20}$. $\frac{-3}{4} + \frac{7}{10} = \frac{-1}{20}$.

192–10 $\frac{5}{8} + \frac{5}{6} = \frac{35}{24}$. $\frac{5}{6} + \frac{5}{8} = \frac{35}{24}$.

192–11 $\frac{-1}{2} + \frac{-3}{7} = \frac{-13}{14}$. $\frac{-3}{7} + \frac{-1}{2} = \frac{-13}{14}$.

192–12 $\frac{2}{3} + \frac{-7}{12} = \frac{1}{12}$. $\frac{-7}{12} + \frac{2}{3} = \frac{1}{12}$.

192–13 $\frac{-3}{5} + \frac{2}{11} = \frac{-23}{55}$. $\frac{2}{11} + \frac{-3}{5} = \frac{-23}{55}$.

192–14 $\frac{3}{16} + \frac{5}{12} = \frac{29}{48}$. $\frac{5}{12} + \frac{3}{16} = \frac{29}{48}$.

192–15 $\frac{-3}{10} + \frac{-7}{8} = \frac{-47}{40}$. $\frac{-7}{8} + \frac{-3}{10} = \frac{-47}{40}$.

192–16 $\frac{1}{4} + \frac{-5}{9} = \frac{-11}{36}$. $\frac{-5}{9} + \frac{1}{4} = \frac{-11}{36}$.

192–17 $\frac{-7}{15} + \frac{-3}{15} = \frac{-2}{3}$. $\frac{-3}{15} + \frac{-7}{15} = \frac{-2}{3}$.

192–18 $\frac{-3}{8} \times \frac{3}{4} = \frac{-9}{32}$. $\frac{3}{4} \times \frac{-3}{8} = \frac{-9}{32}$.

192–19 $\frac{-7}{9} \times \frac{-2}{15} = \frac{14}{135}$. $\frac{-2}{15} \times \frac{-7}{9} = \frac{14}{135}$.

192–20 $\frac{+1}{6} \times \frac{-5}{12} = \frac{-5}{72}$. $\frac{-5}{12} \times \frac{+1}{6} = \frac{-5}{72}$.

192–21 $\frac{3}{10} \times \frac{5}{9} = \frac{1}{6}$. $\frac{5}{9} \times \frac{3}{10} = \frac{1}{6}$.

192–22 $\frac{2}{3} \times \frac{-1}{2} = \frac{-1}{3}$. $\frac{-1}{2} \times \frac{2}{3} = \frac{-1}{3}$.

192–23 $\frac{-7}{20} \times \frac{-4}{7} = \frac{1}{5}$. $\frac{-4}{7} \times \frac{-7}{20} = \frac{1}{5}$.

192–24 $\frac{-2}{13} \times \frac{3}{4} = \frac{-3}{26}$. $\frac{3}{4} \times \frac{-2}{13} = \frac{-3}{26}$.

192–25 $\frac{3}{16} \times \frac{4}{5} = \frac{3}{20}$. $\frac{4}{5} \times \frac{3}{16} = \frac{3}{20}$.

192–26 $\frac{1}{10} \times \frac{1}{7} = \frac{1}{70}$. $\frac{1}{7} \times \frac{1}{10} = \frac{1}{70}$.

192–27 $\frac{-35}{8} \times \frac{+7}{8} = \frac{-35}{8}$. $\frac{+7}{8} \times \frac{-35}{8} = \frac{-35}{8}$.

192–28 $-12 \times -6 = 72$. $-6 \times -12 = 72$.

192–29 $\frac{4}{3} \times \frac{1}{16} = \frac{1}{12}$. $\frac{1}{16} \times \frac{4}{3} = \frac{1}{12}$.

Keeping skilful

192–1 $2\frac{3}{8}$

192–2 $12\frac{1}{2}$

192–3 $7\frac{1}{2}$

192–4 $9\frac{4}{11}$

192–5 $9\frac{8}{9}$

192–6 $41\frac{2}{3}$

192–7 $33\frac{4}{7}$

192–8 $12\frac{7}{11}$

192–9 .7

192–10 .375

192–11 .416. $\frac{5}{12}$ is expressed by a repeating decimal. The digit 6 in the decimal for $\frac{5}{12}$ keeps repeating without end. The digit 6 is called the repetend. The bar over the digit 6 tells you that the digit 6 is the repetend of the decimal.

192–12 .28

192–13 9.25

192–14 15.6

192–15 31.5

192–16 8.75

192–17 $\{1\frac{1}{4}\}$

192–18 {64}

192–19 {45}

192–20 $\{5\frac{1}{2}\}$

192–21 $\{4\frac{4}{5}\}$

192–22 {80.5}

192–23 {13}

192–24 {36}

192–25 {34}

192–26 $\{2\frac{29}{108}\}$

Lesson 136

193–A (Column 1) $\frac{2}{5}$ and $^{-}\frac{1}{3}$ * Yes

193–B $\frac{17}{30} * \frac{17}{30}$

193–C $^{-}\frac{1}{3}$ and $\frac{1}{2} * \frac{1}{6}$

193–D $\frac{17}{30} * \frac{17}{30}$

193–E Yes * No

193–F $=\frac{19}{12} * -\frac{53}{24}$

193–G $\frac{1}{24} * -\frac{53}{24}$

193–H Yes * No

193–I $-\frac{20}{6}$, or $-\frac{10}{3} * -\frac{47}{12}$

193–J $-\frac{17}{12} * -\frac{47}{12}$

193–K Yes * No

193–A (Column 2) R_a

194–B Yes

194–C Yes * The sum of the first components of $(a + c, b + d)$ and (e, f) is $(a + c) + e$. The sum of the second components is $(b + d) + f$.

194–D Yes

194–E Yes * The sum of the first components of (a, b) and $(c + e, d + f)$ is $a + (c + e)$. The sum of the second components if $b + (d + f)$.

194–F Yes * The associative property of addition

194–G Yes * Because of the associative property of addition of rational numbers of arithmetic

194–H The first component of $\big((a + c) + e, (b + d) + f\big)$ is the same as the first component of $\big(a + (c + e), b + (d + f)\big)$. The second component of $\big((a + c) + e, (b + d) (b + d) + f\big)$ is the same as the second component of $\big(a + (c + e), b + (d + f)\big)$. * Yes

194–I From conditions E and G and the transitive property of equality, you can obtain the condition $[(a, b) + (c, d)] + (e, f) = \big(a + (c + e), b + (d + f)\big)$. From the condition just expressed and condition F, you can use the transitive property of equality to obtain condition H.

195–J (Column 1) True statement

195–K (Column 1) False statement

195–L (Column 1) True statement

195–A $^{-}\frac{7}{5}$ and $^{-}\frac{3}{5}$ * Yes

195–B $^{-}\frac{21}{100} * ^{-}\frac{21}{100}$

195–C $^{-}\frac{3}{5}$ and $^{-}\frac{1}{4} * \frac{3}{20}$

195–D $^{-}\frac{21}{100}$ * Yes

195–E Yes * No

195–F $^{-}\frac{9}{40} * \frac{63}{80}$

195–G $^{-}\frac{21}{16} * \frac{63}{80}$

195–H Yes * No

195–I $(\frac{4}{3} \cdot -\frac{2}{9})^{-}5 = \frac{40}{27}$.
$\frac{4}{3}(-\frac{2}{9} \cdot ^{-}5) = \frac{40}{27}$. * No

195–J (Column 2) True statement

195–K (Column 2) True statement

195–L (Column 2) False statement

195–M False statement

195–N True statement

195–O False statement

On your own

196–1 True * Associative property of multiplication

196–2 True * Associative property of addition

196–3 True * Commutative property of addition

196–4 True * Commutative property of multiplication

196–5 False

196–6 False

196–7 False

196–8 True * Commutative property of addition and commutative property of multiplication

196–9 $(^{-}\frac{6}{7} + ^{-}\frac{3}{2}) + ^{-}\frac{3}{2}$.
$^{-}\frac{6}{7} + (^{-}\frac{3}{2} + ^{-}\frac{3}{2}) * ^{-}\frac{27}{7}$

196–10 $(\frac{4}{5} + ^{-}\frac{7}{16}) + 2$.
$\frac{4}{5} + (^{-}\frac{7}{16} + 2) * \frac{189}{80}$

196–11 $(^{-}\frac{1}{9} + 6) + \frac{11}{18}$.
$^{-}\frac{1}{9} + (6 + \frac{11}{18}) * \frac{13}{2}$

196–12 $(\frac{7}{10} + 0) + {}^{-}\frac{7}{10}$.
$\frac{7}{10} + (0 + {}^{-}\frac{7}{10}) * 0$

196–13 $({}^{-}\frac{5}{3} + {}^{-}\frac{7}{6}) + \frac{1}{4}$.
${}^{-}\frac{5}{3} + ({}^{-}\frac{7}{6} + \frac{1}{4}) * {}^{-}\frac{31}{12}$

196–14 $({}^{-}\frac{6}{7} + {}^{-}\frac{21}{12}) + {}^{-}7$.
${}^{-}\frac{6}{7} + ({}^{-}\frac{21}{12} + {}^{-}7) * {}^{-}\frac{269}{28}$

196–15 $({}^{-}4 \cdot {}^{-}9){}^{-}1$. ${}^{-}4({}^{-}9 \cdot {}^{-}1)$ * ${}^{-}36$

196–16 $({}^{-}\frac{9}{10} \cdot {}^{-}\frac{2}{3})\frac{4}{5}$. ${}^{-}\frac{9}{10}({}^{-}\frac{2}{3} \cdot \frac{4}{5})$ * $\frac{12}{25}$

196–17 $(\frac{5}{6} \cdot \frac{7}{8})\frac{3}{2}$. $\frac{5}{6}(\frac{7}{8} \cdot \frac{3}{2})$ * $\frac{35}{32}$

196–18 $({}^{-}\frac{8}{9} \cdot 0)\frac{35}{44}$. ${}^{-}\frac{8}{9}(0 \cdot \frac{35}{44})$ * 0

196–19 $(\frac{1}{6} : {}^{-}\frac{3}{8}){}^{-}\frac{12}{7}$. $\frac{1}{6}({}^{-}\frac{3}{8} \cdot {}^{-}\frac{12}{7})$ * $\frac{3}{28}$

196–20 $(\frac{3}{10} \cdot {}^{-}1)\frac{1}{3}$. $\frac{3}{10}({}^{-}1 \cdot \frac{1}{3})$ * ${}^{-}\frac{1}{10}$

196–21 $({}^{-}6 \cdot 19)\frac{1}{4}$. ${}^{-}6(19 \cdot \frac{1}{4})$ * ${}^{-}\frac{57}{2}$

196–22 $(7 \cdot {}^{-}\frac{2}{3})\frac{2}{3}$. $7({}^{-}\frac{2}{3} \cdot \frac{2}{3})$ * ${}^{-}\frac{28}{9}$

196–23 $({}^{-}\frac{1}{9} \cdot {}^{-}8)7$. ${}^{-}\frac{1}{9}({}^{-}8 \cdot 7)$ * $\frac{56}{9}$

Keeping skilful

196–1 ${}^{+}\frac{9}{4}$

196–2 ${}^{-}\frac{46}{5}$

196–3 ${}^{+}\frac{81}{68}$

196–4 ${}^{-}\frac{181}{9}$

196–5 ${}^{-}\frac{47}{8}$

196–6 ${}^{+}\frac{5}{21}$

196–7 ${}^{-}\frac{4}{21}$

196–8 ${}^{-}\frac{85}{7}$

196–9 ${}^{-}\frac{3}{10}$

196–10 ${}^{+}\frac{488}{15}$

196–11 ${}^{-}\frac{209}{36}$

196–12 ${}^{+}\frac{34}{25}$

196–13 ${}^{+}\frac{112}{5}$

196–14 ${}^{-}\frac{40}{147}$

196–15 ${}^{-}\frac{10}{13}$

196–16 ${}^{+}130$

196–17 ${}^{+}4$

196–18 ${}^{+}\frac{6}{5}$

196–19 ${}^{-}\frac{80}{63}$

196–20 ${}^{+}\frac{156}{7}$

196–21 ${}^{-}\frac{25}{2}$

Lesson 137

196–A The product of $\frac{3}{8}$ and $(\frac{3}{2} + {}^{-}2)$ * The sum of $\frac{3}{8}(\frac{3}{2})$ and $\frac{3}{8}({}^{-}2)$

196–B Yes * Yes

196–C ${}^{-}\frac{3}{16}$ * Yes

196–D Using the transitive property of equality, you can obtain statement D from statements B and C. Since statements B and C are true, statement D is also true.

196–E Yes * Yes * Yes

197–F ${}^{-}\frac{3}{16}$ * Yes

197–G Using the transitive property of equality, you can obtain statement G from statements E and F. Since statements E and F are true, statement G is also true.

197–H Yes

197–I Sentence H: $\frac{25}{24}$ * $\frac{25}{24}$ * True statement.
Sentence I: ${}^{-}\frac{5}{3}$ * ${}^{-}\frac{5}{3}$ * True statement.
Sentence J: ${}^{-}\frac{39}{2}$ * ${}^{-}\frac{39}{2}$ * True statement.
Sentence K: ${}^{+}1$ * ${}^{+}1$ * True statement

197–J To obtain statement L, x was replaced by ${}^{+}\frac{12}{5}$, y by ${}^{-}\frac{1}{7}$, and z by ${}^{+}\frac{5}{6}$. To obtain statement M, x was replaced by ${}^{-}\frac{4}{9}$, y by ${}^{+}\frac{3}{2}$, and z by ${}^{+}\frac{1}{3}$. To obtain statement N, x was replaced by ${}^{-}\frac{3}{4}$, y by ${}^{-}\frac{11}{12}$, and z by ${}^{-}\frac{7}{10}$. * From the distributive property, you know that for each x, y, and z, $x(y + z) = xy + xz$.

197–A $x + (yz) = (x + y)(x + y)$. * R

197–B x was replaced by ${}^{+}\frac{4}{5}$, y by ${}^{-}\frac{2}{3}$, and z by ${}^{+}\frac{3}{5}$.

197–C No. ${}^{+}\frac{4}{5} + ({}^{-}\frac{2}{3} \cdot {}^{+}\frac{3}{5}) = {}^{-}\frac{8}{25}$ and $({}^{+}\frac{4}{5} + {}^{-}\frac{2}{3})({}^{+}\frac{4}{5} + {}^{+}\frac{3}{5}) = {}^{+}\frac{14}{25}$.

198–D One. Any replacement for which you obtain a false statement shows that it is not the case that a true statement is obtained for each replacement.

198–E No

198–F Yes

198–G $x + y = y + x$. * $xy = yx$.

198–H $(x + y) + z = x + (y + z)$. * $(xy)z = x(yz)$. * $x(y + z) = xy + xz$.

198–I Yes. The set of rational numbers are closed, commutative and associative under both addition and multiplication and multiplication distributes over addition.

198–J Yes. Each member of R_p is also a member of R. * Yes. R_p is closed, commutative, and associative under both addition and multiplication and multiplication distributes over addition.

198–K Yes. Each member of R_n is also a member of R. * No. R_n is not closed under multiplication. For example, the product of ${}^{-}4$ and ${}^{-}2$ is ${}^{+}8$, and ${}^{+}8$ is not a member of R_n.

On your own

198–1 True statement * Distributive property

198–2 False statement

198–3 True statement * Associative property of

multiplication

198–4 False statement

198–5 True statement * Associative property of addition

198–6 True statement * Commutative property of addition

198–7 True statement * Associative property of addition

198–8 False statement

199–9 True statement * Commutative property of multiplication

199–10 True statement * Distributive property

199–11 $-\frac{3}{2}$

199–12 -3

199–13 $\frac{7}{10}$

199–14 $-\frac{3}{4}$

199–15 $-\frac{8}{10}$

199–16 $\frac{3}{4}$

199–17 $\frac{39}{8}$

199–18 $-\frac{11}{20}$

199–19 Yes

Applying mathematics

200–1 x, average depth of the Atlantic Ocean.
$14048 - x = 1168$.
{12880}. 12,880 ft.

200–2 x, perimeter of the triangle in feet.
$x = 12\frac{7}{8} + 12\frac{7}{8} + 12\frac{7}{8}$.
$\{38\frac{5}{8}\}$. $38\frac{5}{8}$ ft.

200–3 x, number of miles the train traveled in 1 min.
$41/29 \sim x/1$.
$\{1\frac{12}{29}\}$. 1.4 mi.

200–4 x, number of questions Catherine answered correctly.
$87\frac{1}{2}/100 \sim x/24$, or $\frac{7}{8}/1 \sim x/24$.
{21}. 21 questions

200–5 x, amount of Miss Larson's salary last year.
$22.5/100 \sim 1125/x$.
{5000}. $5000

200–6 x, amount of discount.
y, per cent of discount.
$340 - x = 272 \wedge x/340 \sim y/100$.
{(68,20)}. 20%

200–7 x, number of cherry trees.
y, number of apple trees.
z, number of trees in all.
$\frac{2}{3}/1 \sim x/y \wedge y + 10 = 55 \wedge x + y = z$.
{(30,45,75)}. 30 cherry trees, 45 apple trees, and 75 trees in all

200–8 x, area of the runway in square feet.
$6855/1 \sim x/200$.
{1371000}.
1,371,000 sq. ft.

200–9 x, number of coins.
$1/.25 \sim x/1.50 \vee 1/.10 \sim x/1.50$.
{6, 15}. 6 quarters or 15 dimes

200–10 x, number of teacups Mrs. James bought in foreign countries.
y, number of teacups she received as gifts.
$23 < x < 30 \wedge x + y = 40$.
{(24,16), (25,15), (26,14), (27,13), (28,12), (29,11)}.
From 11 through 16 teacups

200–11 x, number of girls enrolled.
y, number of boys enrolled.
$x - y = 47 \wedge y + 16 = 500$.
{(531,484)}.
531 girls and 484 boys

200–12 x, number of adults who attended the pet show.
y, total attendance.
$4/1 \sim 476/x \wedge 476 + x = y$.
{(119,595)}. 119 adults and 595 persons in all

200–13 x, number of light bulbs given to tenants.
y, number of light bulbs left.
$200 - x = y \wedge y \leq 12$.
{(199,1), (198,2), (197,3), (196,4), (195,5), (194,6), (193,7), (192,8), (191,9), (190,10), (189,11), (188,12)}.
From 188 through 199 light bulbs
Another way in which you could get the answer to the problem is given below.
x, number of light bulbs given to tenants.
$200 - x \leq 12 \wedge x < 200$.
{188, 189, 190, ..., 199}.
From 188 through 199 light bulbs

200–14 x, number of dimes.
y, number of nickels.
z, number of quarters.
$3/1 \sim x/y \wedge x = z \wedge 5 < z < 10$.
{(6,2,6), (9,3,9)}.
He could have 6 dimes, 2 nickels and 6 quarters.
He could have 9 dimes, 3 nickels and 9 quarters.

200–15 $x/y \sim 2/5 \wedge x \leq 10$.
{2/5, 4/10, 6/15, 8/20, 10/25}

200–16 $x > 43.14 \wedge 120.56 + x < 180$.
$\{x \mid 43.14 < x < 59.44\}$

200–17 $\frac{1}{3}/1 \sim x/y \wedge \frac{1}{2}/1 \sim y/90$.
{(15,45)}. $\angle A^{\circ} = 15$.

$\angle B^{\circ} = 45$.

Checking up

Test 126

201–1 Associative property of multiplication

201–2 Distributive property

201–3 Commutative property of addition

201–4 Closure property of multiplication

201–5 Commutative property of multiplication

201–6 Associative property of addition

201–7 Forms a number system

201–8 Does not form a number system

201–9 Does not form a number system

201–10 Does not form a number system

201–11 Forms a number system

201–12 Forms a number system

Test 127

201–13 $\{\frac{-5}{8}\}$

201–14 $\{-3\}$

201–15 $\{\frac{-4}{5}\}$

201–16 $\{\frac{2}{5}\}$

201–17 $\{\frac{9}{20}\}$

201–18 $\{\frac{-3}{8}\}$

201–19 $\{\frac{2}{7}\}$

201–20 $\{\frac{-7}{19}\}$

Lesson 138

201–A Yes * 0 * $\frac{-4}{5}$

201–B Yes * $\frac{8}{3}$ * $\frac{5}{6}$ * -7

202–C (Column 1) (0,0) * R_a

202–D (Column 1) Yes * Because of the definition of the sum of two rational numbers

202–E (Column 1) The rational number of arithmetic. 0 is the identity element for addition of rational numbers of arithmetic. * Yes. $0 + c = c$ and $0 + d = d$.

202–F Yes * Using the transitive property, you can obtain condition G from conditions E and F. Since, for each c and d, true statements are obtained from conditions E and F, a true statement is also obtained from condition G.

202–G $x + 0 = x$. * Yes * Commutative property of addition. $0 + x = x + 0$ for each x when the universe is R.

202–H There is no number, other than 0, which when added to a given rational number will give you the same rational number.

202–I True statement

202–J False statement

202–K False statement

202–L True statement

202–M True statement

202–N False statement

202–A Yes * Yes

202–B Yes * Yes

202–C (Column 2) Yes. (1,0) has a component of zero. * Because of the definition of the product of two rational numbers. $1 \cdot c + 0 \cdot d$ is the same as $c + 0$. $1 \cdot d + 0 \cdot c$ is the same as $d + 0$.

202–D (Column 2) $c + 0 = c$ and $d + 0 = d$ because of the identity-element property of addition.

202–E (Column 2) Yes * The transitive property of equality.

203–F $x \cdot 1 = x$. * Yes * The commutative property of multiplication. $1 \cdot x = x \cdot 1$.

203–G There is no number, other other than 1, by which a given number can be multiplied to obtain the given number.

203–H False statement

203–I True statement

203–J True statement

203–K False statement

203–L False statement

203–M True statement

203–N Yes * 0

203–O Yes * The definition of the product of two rational numbers. For each c and d, $0 \cdot c + 0 \cdot d$ is the same as $0 + 0$ and $0 \cdot d + 0 \cdot c$ is the same as $0 + 0$.

203–P $0 + 0 = 0$ because of the identity-element property of addition.

203–Q Yes * Transitive property of equality

203–R Yes * Commutative property of multiplication. $0 \cdot x = x \cdot 0$.

On your own

203–1 A true statement is obtained.

203–2 A true statement is not obtained.

203–3 A true statement is not obtained.

203–4 A true statement is obtained.

203–5 A true statement is not obtained.

203–6 A true statement is obtained.

203–7 A true statement is

obtained.

203–8 A true statement is not obtained.

203–9 A true statement is not obtained.

204–10 False statement

204–11 True statement

204–12 True statement

204–13 False statement

204–14 True statement.

204–15 True statement

204–16 False statement

204–17 False statement

204–18 True statement

204–19 False statement

204–20 True statement

204–21 True statement

204–22 False statement

204–23 True statement

204–24 True statement

204–25 False statement

Keeping skilful

204–1 {26, 27, 28, ..., 33}

204–2 {68}

204–3 {295, 296, 297, ...}

204–4 {(18,107)}

204–5 {(2,0), (3,1)}

204–6 {(2,4), (3,3), (4,2), (5,1), (6,0)}

204–7 $\{c \mid c < 44.26\}$

204–8 $\{a \mid a \geq 121.9\}$

204–9 $\{y \mid \frac{3}{8} < y < 2\}$

204–10 $\{d \mid d \leq \frac{1}{8}\}$

204–11 { }

204–12 $\{3\frac{1}{10}\}$

204–13 $12\frac{1}{16}$

204–14 $\frac{1}{6}$

204–15 $83\frac{1}{6}$

204–16 $156\frac{1}{4}$

204–17 130.65

204–18 1.33056

204–19 1.84

204–20 53.200

Lesson 139

204–A Sentences A, B, C and D

204–B $-\frac{7}{2} * +\frac{7}{2}$

204–C The sum of -23 and $+23$ is 0. * $+23 * -5 * +5$

204–D The sum of 0 and 0 is 0.

204–E $+7 * +7 + -7 = 0$.

204–F Yes * (7,0) * The second component * The first component

204–G $-\frac{3}{4}$

204–H $(\frac{3}{4},0) * (0,\frac{3}{4}) *$ The first component of the basic pair for $+\frac{3}{4}$ is the same as the second component of the basic pair for $-\frac{3}{4}$. The second component of the basic pair for $+\frac{3}{4}$ is the same as the first component of the basic pair for $-\frac{3}{4}$.

205–I $(0,\frac{4}{3})$

205–J (0,5)

205–K $(\frac{2}{5},0)$

205–L (16,0)

205–M (0,0) * (0,0)

205–N $(a,0) + (0,a) = (a,a)$. * R_a * Yes * The sum of (2,0) and (0,2) is (2,2), or 0.

205–O To obtain statement F, a was replaced by $1\frac{3}{2}$. To obtain statement G, a was replaced by $\frac{4}{5}$. * Yes

205–P $(1\frac{3}{2},0)$, or $+1\frac{3}{2}$

205–Q Yes. From the definition of the sum of two rational numbers, you know that for each a, $(a,0) + (0,a) = (a + 0, 0 + a)$ and $(a + 0, 0 + a) = (a,a)$. * Yes. For each a, $(a,a) = 0$.

205–R $(0,a) * (a,0)$

205–S Yes. For each rational number $(a,0)$, there is a rational number $(0,a)$.

205–T Each of the numerals $-(+3)$ and -3 names the additive inverse of $+3$, and a given number has only one additive inverse. * Each of the numerals $-(-\frac{2}{3})$ and $+\frac{2}{3}$ names the additive inverse of $-\frac{2}{3}$.

205–U $-x$ is the additive inverse of x. The sum of a number and its additive inverse is 0.

205–A Yes

206–B $-\frac{2}{3} - +\frac{1}{2} = x$ also means $-\frac{2}{3} = +\frac{1}{2} + x$.

206–C $-(+\frac{1}{2})$

206–D $-(+\frac{1}{2})$ can be replaced by $-\frac{1}{2}$ since $-(+\frac{1}{2}) = -\frac{1}{2}$.

206–E Because of the associative property of addition of rational numbers

206–F $-\frac{1}{2} + +\frac{1}{2} = 0$. The definition of additive inverse

206–G $0 + x = x$. The identity-element property of addition

206–H $-\frac{1}{2} + -\frac{2}{3} = -\frac{2}{3} + -\frac{1}{2}$. The commutative property of addition

206–I Yes * $-\frac{7}{6}$

207–J $+3 - -\frac{7}{6} = x$.
$+3 = -\frac{7}{6} + x$.
$-(-\frac{7}{6}) + +3 = -(-\frac{7}{6}) + (-\frac{7}{6} + x)$.
$+\frac{7}{6} + +3 = +\frac{7}{6} + (-\frac{7}{6} + x)$.
$+\frac{7}{6} + +3 = (+\frac{7}{6} + -\frac{7}{6}) + x$.
$+\frac{7}{6} + +3 = 0 + x$.
$+\frac{7}{6} + +3 = x$.
$+3 + +\frac{7}{6} = x$.
$+3 - -\frac{7}{6} = +3 + +\frac{7}{6}$.

207–K $x - y = z$.
$x = y + z$. Subtraction related to addition.
$-y + x = -y + (y + z)$. Well-defined property of

addition.
$^-y + x = (^-y + y) + z$. Associative property of addition.
$^-y + x = 0 + z$. Additive inverse.
$^-y + x = z$. Identity-element property of addition.
$x + ^-y = z$. Commutative property of addition.
$x - y = x + ^-y$. Transitive property.

207–L $^-2 * ^-3 * ^+3$

207–M $^-2$ and $^+3 * ^+1$

207–N $^+\frac{1}{3}$ and $^-\frac{2}{5}$ * Yes * Yes * $^-\frac{1}{15}$

207–O Add $^+6$ and the additive inverse of $^-\frac{5}{2}$. * Add $^-\frac{1}{4}$ and the additive inverse of $^-5$. * Add x and the additive inverse of y.

207–P No. For example, 3 − 5, 100 − 200, and 4 − 40 are not members of N. * No. For example, $\frac{3}{5} - \frac{4}{5}$, 7.6 − 18, and $2\frac{1}{2} - 21\frac{3}{4}$ are not members of R_a.

207–Q The difference of two rational numbers is also the sum of two rational numbers. Since R is closed under addition, R must also be closed under subtraction. For each x and y, $x - y = x + ^-y$. Since $x + ^-y \in R$, $x - y \in R$.

207–R No * $^+3 - ^+5 = ^-2$ and $^+5 - ^+3 = ^+2$. $^+\frac{1}{2} - ^-\frac{1}{3} = ^+\frac{5}{6}$ and $^-\frac{1}{3} - ^+\frac{1}{2} = ^-\frac{5}{6}$. $^-\frac{13}{2} - ^+\frac{3}{4} = ^-\frac{29}{4}$ and $^+\frac{3}{4} - ^-\frac{13}{2} = ^+\frac{29}{4}$.

207–S $(^+3 - ^-\frac{7}{8}) - ^-5 = ^+\frac{31}{8} - ^-5$. $^+\frac{31}{8} - ^-5 = ^+\frac{71}{8}$. $(^+3 - ^-\frac{7}{8}) - ^-5 = ^+\frac{71}{8}$. $^+3 - (^-\frac{7}{8} - ^-5) = ^+3 - ^+\frac{33}{8}$. $^+3 - ^+\frac{33}{8} = ^-\frac{9}{8}$. $^+3 - (^-\frac{7}{8} - ^-5) = ^-\frac{9}{8}$. Since $(^+3 - ^-\frac{7}{8}) - ^-5$ does not equal $^+3 - (^-\frac{7}{8} - ^-5)$, subtraction of rational numbers does not have the associative property.

On your own

207–1 No. $^-\frac{3}{7} + ^-\frac{7}{3}$ does not equal 0.

207–2 Yes. $^+4 + ^-4 = 0$.

207–3 Yes. $0 + 0 = 0$.

207–4 No. $^+\frac{6}{5} + ^-\frac{5}{6}$ does not equal 0.

207–5 Yes. $^+64 + ^-64 = 0$.

207–6 No. $^+1 + 0$ does not equal 0.

207–7 No. $^+\frac{2}{3} + ^+\frac{1}{3}$ does not equal 0.

207–8 No. $^+3 + ^-\frac{1}{3}$ does not equal 0.

207–9 $^-\frac{7}{4}$

207–10 $^+\frac{4}{1750}$

207–11 0

207–12 $^-\frac{3}{8}$

207–13 $^+27$

207–14 $^-429$

207–15 $^+\frac{11}{12}$

207–16 $^-39$

207–17 $^-\frac{5}{6}$

207–18 $^+14$

207–19 $^+\frac{25}{7}$

207–20 $^+17$

207–21 $^+\frac{1}{8}$

207–22 $^-\frac{90}{7}$

207–23 $^+\frac{67}{8}$

207–24 $^-29$

207–25 $^-\frac{1}{4}$

207–26 $^+\frac{23}{20}$

207–27 $^-55$

207–28 $^+\frac{3}{2}$

207–29 $^-\frac{17}{9}$

207–30 $^+396$

207–31 $^-\frac{5}{8}$

207–32 $^-\frac{94}{57}$

207–33 $^+\frac{8}{21}$

207–34 $^+74$

Lesson 140

208–A $^+1$. $^+1$. $^+1$. $^+1$

208–B $^+\frac{3}{2} * ^+\frac{2}{3}$

208–C Because the product of $^-\frac{4}{5}$ and $^-\frac{5}{4}$ is $^+1 * ^-\frac{4}{5} * ^+\frac{1}{6} * ^+6$

208–D Because the product of $^+1$ and $^+1$ is $^+1$

208–E $^+\frac{3}{2} * ^-\frac{4}{5} * ^+6$

208–F Because $^+\frac{11}{4} \times ^-\frac{4}{11}$ does not equal $^+1$ * Because $^-\frac{15}{7} \times ^+\frac{7}{15}$ does not equal $^+1$

208–G The product of two positive numbers is positive. * The product of two negative numbers is positive.

208–H $^+1$ * Because $^+1 \times ^-1$ does not equal $^+1$

208–I There is no rational number by which you can multiply 0 to obtain $^+1$. * No. Since 0 does not have a reciprocal, 0 cannot be the reciprocal of any rational number.

208–J $(\frac{2}{3},0) * (\frac{3}{2},0)$

208–K Since $\frac{2}{3} \in Z$, $1/\frac{2}{3}$ is the reciprocal of $\frac{2}{3}$. * Yes

208–L $\frac{2}{3} * \frac{3}{2} *$ Yes * $\frac{2}{3} \times \frac{3}{2} = 1$.

209–M $^-\frac{8}{7} * \frac{7}{8} * \frac{8}{7}$

209–N Yes

209–O (3,0) and $(\frac{1}{3},0)$

209–P Yes * They are members of R_a and are reciprocals of each other.

209–Q $(\frac{3}{4},0)$

209–R $(0,\frac{1}{5})$

209–S $(0,\frac{15}{8})$

209–T $(\frac{6}{7},0)$

209–A Z * Yes. By the definition of the product of two

rational numbers, $(a,0) \times (\frac{1}{a},0) = (a \cdot \frac{1}{a} + 0,\ 0 + 0)$, or $(a \cdot \frac{1}{a},0)$. * Yes. $a \cdot \frac{1}{a} = 1$. * Yes. By the transitive property of equality, if $(a,0) \times (\frac{1}{a},0) = (a \cdot \frac{1}{a},0)$ and $(a \cdot \frac{1}{a},0) = (1,0)$, then $(a,0) \times (\frac{1}{a},0) = (1,0)$.

209–B $^{+}1 * (\frac{1}{a},0) * (a,0)$

209–C $(\frac{18}{3},0) * (\frac{3}{18},0) * {}^{+}\frac{3}{18}$

209–D ${}^{+}\frac{5}{16} * {}^{+}\frac{16}{5} * {}^{+}9 * {}^{+}\frac{1}{9}$

209–E $Z * (0,\frac{1}{b})$. $(0,b) \times (0,\frac{1}{b}) = (b \cdot \frac{1}{b},0)$, or $(1,0)$. * $(0,b)$. $(0,\frac{1}{b}) \times (0,b) = (\frac{1}{b} \cdot b,0)$, or $(1,0)$.

209–F $(0,\frac{4}{3}) * (0,\frac{3}{4}) * {}^{-}\frac{3}{4}$

209–G Yes * The universe for a and b is Z. For each $(a,0)$, there is a $(\frac{1}{a},0)$. For each $(0,b)$, there is a $(0,\frac{1}{b})$.

209–H Yes

209–I ${}^{-}\frac{2}{5}$

209–J ${}^{+}\frac{8}{5} \div {}^{-}\frac{2}{5} = y$ also means ${}^{+}\frac{8}{5} = y \cdot {}^{-}\frac{2}{5}$.

210–K ${}^{-}\frac{5}{2} * {}^{-}\frac{5}{2}$

210–L From the associative property of multiplication, $(y \cdot {}^{-}\frac{2}{5})({}^{-}\frac{5}{2})$ is the same as $y({}^{-}\frac{2}{5} \cdot {}^{-}\frac{5}{2})$.

210–M Since ${}^{-}\frac{5}{2}$ is the reciprocal of ${}^{-}\frac{2}{5}$, ${}^{-}\frac{2}{5} \times {}^{-}\frac{5}{2} = 1$.

210–N From the identity-element property of multiplication, $y(1)$ is the same as y.

210–O ${}^{-}4$

210–P ${}^{-}\frac{10}{7} \div {}^{-}\frac{15}{14} = x$.
${}^{-}\frac{10}{7} = x \cdot {}^{-}\frac{15}{14}$.
$({}^{-}\frac{10}{7})({}^{-}\frac{14}{15}) = (x \cdot {}^{-}\frac{15}{14})({}^{-}\frac{14}{15})$.
$({}^{-}\frac{10}{7})({}^{-}\frac{14}{15}) = x(1)$.
$({}^{-}\frac{10}{7})({}^{-}\frac{14}{15}) = x$.
${}^{-}\frac{10}{7} \div {}^{-}\frac{15}{14} = ({}^{-}\frac{10}{7})({}^{-}\frac{14}{15})$.

$* \frac{4}{3}$

210–Q ${}^{+}\frac{3}{4}$

210–R ${}^{-}\frac{2}{11}$

210–S ${}^{+}\frac{55}{23}$

210–T ${}^{-}1$

210–U ${}^{+}\frac{3}{8}$

210–V ${}^{+}363$

210–W ${}^{+}275$

210–X ${}^{+}\frac{2}{3}$

210–Y ${}^{-}20$

211–A $\frac{{}^{+}1}{{}^{-}\frac{2}{7}}$ * The product of ${}^{-}\frac{2}{7}$ and $\frac{{}^{+}1}{{}^{-}\frac{2}{7}}$ is ${}^{+}1$. * Yes

211–B ${}^{+}1/{}^{+}\frac{8}{9}$ * Yes * ${}^{+}\frac{9}{8}$

211–C Yes * ${}^{+}150$ * ${}^{+}\frac{7}{9}$

211–D Yes * Yes * No

211–E Yes

211–F From the quotient property, $x \div y = x \cdot (1/y)$. Since each non-zero rational number has a reciprocal, and since $y \in Z_r$, you know that $1/y \in Z_r$. From the closure property of multiplication, for each x and y, $x \cdot (1/y) \in R$. Therefore, $x \div y \in R$.

211–G The quotient of a given number and 0 would be the same as the product of the given number and the reciprocal of 0. Since 0 does not have a reciprocal, there is no product of a given number and the reciprocal of 0. Hence, there is no quotient of a given number and 0.

211–H 0

211–I No * Examples are: ${}^{+}4 \div {}^{+}2 = {}^{+}2$ and ${}^{+}2 \div {}^{+}4 = {}^{+}\frac{1}{2}$. ${}^{+}\frac{4}{3} \div {}^{-}\frac{2}{9} = {}^{-}6$ and ${}^{-}\frac{2}{9} \div {}^{+}\frac{4}{3} = {}^{-}\frac{1}{6}$. ${}^{-}\frac{3}{5} \div {}^{-}\frac{2}{15} = {}^{+}\frac{9}{2}$ and ${}^{-}\frac{2}{15} \div {}^{-}\frac{3}{5} = {}^{+}\frac{2}{9}$.

211–J No * $({}^{-}\frac{5}{8} \div {}^{+}\frac{15}{4}) \div {}^{-}\frac{7}{12} = {}^{-}\frac{1}{6} \div {}^{-}\frac{7}{12}$.
${}^{-}\frac{1}{6} \div {}^{-}\frac{7}{12} = {}^{+}\frac{2}{7}$.
$({}^{-}\frac{5}{8} \div {}^{+}\frac{15}{4}) \div {}^{-}\frac{7}{12} = {}^{+}\frac{2}{7}$.
${}^{-}\frac{5}{8} \div ({}^{+}\frac{15}{4} \div {}^{-}\frac{7}{12}) = {}^{-}\frac{5}{8} \div {}^{-}\frac{45}{7}$.
${}^{-}\frac{5}{8} \div {}^{-}\frac{45}{7} = {}^{+}\frac{7}{72}$.
$({}^{-}\frac{5}{8} \div {}^{+}\frac{15}{4}) \div {}^{-}\frac{7}{12} = {}^{+}\frac{7}{72}$.
Since $({}^{-}\frac{5}{8} \div {}^{+}\frac{15}{4}) \div {}^{-}\frac{7}{12}$ does not equal ${}^{-}\frac{5}{8} \div ({}^{+}\frac{15}{4} \div {}^{-}\frac{7}{12})$, division of rational numbers does not have the associative property.

On your own

211–1 Reciprocals. ${}^{+}\frac{3}{7} \times {}^{+}\frac{7}{3} = {}^{+}1$.

211–2 Reciprocals. ${}^{-}1 \times {}^{-}1 = {}^{+}1$.

211–3 Not reciprocals. ${}^{-}\frac{7}{10} \times {}^{+}\frac{7}{10}$ does not equal ${}^{+}1$.

211–4 Not reciprocals. ${}^{+}\frac{15}{8} \times {}^{-}\frac{8}{15}$ does not equal ${}^{+}1$.

211–5 Reciprocals. ${}^{-}14 \times {}^{-}\frac{1}{14} = {}^{+}1$.

211–6 Not reciprocals. ${}^{-}\frac{16}{21} \times {}^{+}\frac{21}{16}$ does not equal ${}^{+}1$.

211–7 Not reciprocals. ${}^{+}1 \times {}^{-}1$ does not equal ${}^{+}1$.

211–8 Reciprocals. ${}^{+}1 \times {}^{+}1 = {}^{+}1$.

211–9 ${}^{-}\frac{1}{10}$

211–10 ${}^{+}\frac{1}{17}$

211–11 ${}^{+}\frac{8}{5}$

211–12 ${}^{-}\frac{5}{6}$

211–13 ${}^{-}\frac{11}{2}$

211–14 ${}^{+}\frac{13}{4}$

211–15 ${}^{+}\frac{70}{67}$

211–16 ${}^{-}\frac{24}{23}$

211–17 ${}^{-}\frac{12}{19}$

211–18 ${}^{+}\frac{1}{200}$

211–19 ${}^{-}\frac{1}{159}$

211–20 ${}^{-}\frac{100}{99}$

211–21 ${}^{-}7$

211–22 ${}^{+}\frac{2}{3}$

211–23 ${}^{+}\frac{1}{8}$

211–24 ${}^{+}5$

211–25 ${}^{-}\frac{144}{5}$

211–26 ${}^{+}8$

211–27 ${}^{-}\frac{1}{54}$

211–28 ${}^{+}\frac{4}{325}$

211–29 $^{-}\frac{1}{40}$

211–30 True statement

211–31 False statement

211–32 False statement

211–33 True statement

211–34 True statement

211–35 False statement

Lesson 141

212–A Yes * $(0,\frac{9}{8})$

212–B Yes * Yes

212–C $^{-}\frac{1}{4} + ^{-}\frac{7}{8} = ^{-}\frac{9}{8}$ and $^{-}\frac{9}{8} = ^{-}1\frac{1}{8}$. Therefore, using the transitive property, $^{-}\frac{1}{4} + ^{-}\frac{7}{8} = ^{-}1\frac{1}{8}$.

212–D Yes * $(\frac{11}{10},0)$

212–E $1\frac{1}{10}$ * Yes * Since $(\frac{11}{10},0) = (1\frac{1}{10},0)$, $^{+}\frac{11}{10} = ^{+}1\frac{1}{10}$.

212–F $^{+}\frac{3}{5} - ^{-}\frac{1}{2} = ^{+}\frac{11}{10}$ and $^{+}\frac{11}{10} = ^{+}1\frac{1}{10}$. Therefore, using the transitive property, $^{+}\frac{3}{5} - ^{-}\frac{1}{2} = ^{+}1\frac{1}{10}$.

212–G Yes * $(\frac{45}{20},0)$ * Yes * Yes

212–H $^{+}\frac{5}{2} \times ^{+}\frac{9}{10} = ^{+}\frac{45}{20}$. Since $(\frac{45}{20},0) = (2\frac{1}{4},0)$, the numeral $^{+}2\frac{1}{4}$ names $^{+}\frac{45}{20}$. Therefore, $^{+}\frac{45}{20} = ^{+}2\frac{1}{4}$ and using the transitive property, $^{+}\frac{5}{2} \times ^{+}\frac{9}{10} = ^{+}2\frac{1}{4}$.

212–I Yes * Yes

212–J Since $(0,\frac{20}{18}) = (0,1\frac{1}{9})$, $^{-}\frac{20}{18} = ^{-}1\frac{1}{9}$. * Yes

212–K Yes * Yes * $^{+}3$ * $^{+}\frac{1}{4}$

212–L Yes * $5\frac{1}{2}$ is greater than $2\frac{1}{4}$.

212–M Yes * Difference property

212–N $^{-}6\frac{3}{10} = ^{-}\frac{63}{10}$ and $^{-}\frac{63}{10} = ^{-}5\frac{13}{10}$. * Yes

212–O $^{-}2$ * $^{-}\frac{1}{2}$ * Yes

213–P $^{-}2\frac{1}{2}$ * $^{+}3\frac{4}{5} - ^{+}6\frac{3}{10} = ^{+}3\frac{4}{5} + ^{-}6\frac{3}{10}$. $^{+}3\frac{4}{5} + ^{-}6\frac{3}{10}$ is a negative number since $6\frac{3}{10}$ is greater than $3\frac{4}{5}$. Therefore, $^{+}3\frac{4}{5} - ^{+}6\frac{3}{10}$ is a negative number.

213–Q $^{+}1\frac{2}{5} = ^{+}\frac{7}{5}$ and $^{-}5\frac{1}{4} = ^{-}\frac{21}{4}$. Therefore, using the well-defined property of multiplication, you can replace $^{+}1\frac{2}{5}$ by $^{+}\frac{7}{5}$ and $^{-}5\frac{1}{4}$ by $^{-}\frac{21}{4}$ in the product $^{+}1\frac{2}{5} \times ^{-}5\frac{1}{4}$ without affecting the product.

213–R $^{-}\frac{147}{20}$ * Yes * Yes

213–S $^{-}7\frac{7}{20}$ * The product of a positive number and a negative number is negative.

213–T $^{-}5\frac{2}{3} = ^{-}\frac{17}{3}$ and $^{-}3\frac{1}{2} = ^{-}\frac{7}{2}$.

213–U Yes * Quotient property

213–V $^{+}\frac{34}{21}$ * Yes * Yes

213–W $^{+}1\frac{13}{21}$ * The product of two negative numbers is positive. Since the quotient of two negative numbers is the same as the product of two negative numbers, the quotient of two negative numbers is also positive.

213–A Yes * .5

213–B Yes * Yes * The basic pair for $^{+}\frac{5}{10}$ is $(\frac{5}{10},0)$. $(\frac{5}{10},0) = (.5,0)$. Therefore, the decimal $^{+}.5$ names $^{+}\frac{5}{10}$.

213–C $(1\frac{75}{100},0)$ * 1.75

213–D Yes * Yes * The basic pair for $^{+}1\frac{75}{100}$ is $(1\frac{75}{100},0)$. $(1\frac{75}{100},0) = (1.75,0)$. Therefore, the decimal $^{+}1.75$ names $^{+}1\frac{75}{100}$.

213–E $(0,\frac{3}{1000})$ * .003

213–F The basic pair for $^{-}\frac{3}{1000}$ is $(0,\frac{3}{1000})$. $(0,\frac{3}{1000}) = (0,.003)$. Therefore, the decimal $^{-}.003$ names $^{-}\frac{3}{1000}$.

213–G The basic pair for $^{-}2\frac{1}{5}$ is $(0,2\frac{1}{5})$. $(0,2\frac{1}{5}) = (0,2.2)$. Therefore, the decimal $^{-}2.2$ names $^{-}2\frac{1}{5}$ and statement D is true. The basic pair for $^{-}7\frac{4}{25}$ is $(0,7\frac{4}{25})$. $(0,7\frac{4}{25}) = (0,7.16)$. Therefore, the decimal $^{-}7.16$ names $^{-}7\frac{4}{25}$ and statement E is true. The basic pair for $^{+}\frac{6}{10000}$ is $(\frac{6}{10000},0)$. $(\frac{6}{10000},0) = (.0006,0)$. Therefore, the decimal $^{+}.0006$ names $^{+}\frac{6}{10000}$ and statement F is true.

213–H Yes * Yes

213–I In tenths * 58.4

213–J Positive * $^{+}58.4$

213–K Find the difference of 8.025 and 8.420.

213–L Yes * 4.605

213–M 8.025 is greater than 8.420. * $^{+}4.605$

214–N $^{-}7.35$ and $^{-}16.6$

214–O When you use decimals to find a sum, the decimals should have the same number of digits to the right of the decimal point.

214–P 23.95 * The sum of two negative numbers is negative. * $^{-}23.95$ * $^{-}23.95$

214–Q $^{-}23.043$ and $^{+}6.323$

214–R Subtract 6.323 from 23.043.

214–S 16.720 * 23.043 is greater than 6.323. * $^{-}16.720$

214–T $^{+}2.973$ * $^{+}1.029$

214–U $^{-}8.45$ * $^{-}42.15$

214–V $^{-}14.59$ * $^{+}30.17$

214–W $^{-}11.0566$ * $^{-}1.6086$

214–A $^{+}6.84$ and $^{-}1.05$ * $^{+}6.84 = ^{+}\frac{684}{100}$ and $^{-}1.05 = ^{-}\frac{105}{100}$.

214–B Yes * The product of a positive number and a negative number is

negative.

214–C Yes * Yes

214–D One hundredth times one hundredth equals one ten-thousandth. * 7.1820

214–E Negative * $^{-}7.1820$

214–F $^{-}.16 = {}^{-}\frac{16}{100}$ and $^{-}.5 = {}^{-}\frac{5}{10}$. * Yes

215–G The decimal .16 expresses a number in hundredths. The decimal .5 expresses a number in tenths. One hundredth times one tenth equals one thousandth. * .080

215–H Positive * $^{+}.080$

215–I $^{-}.165$ and $^{+}1.1$

215–J Multiply $^{+}1.1$ by $^{+}10$ * Multiply $^{-}.165$ by $^{+}10$. * Yes

215–K Yes * $^{+}\frac{1}{11}$ * The quotient property tells you that, for each x and y, $x \div y = x(1/y)$. The universe for x is R. The universe for y is Z_r.

215–L $^{-}\frac{15}{100}$ * $^{-}.15$ * $^{-}.15$ * They are the same.

215–M Yes * In hundredths * .15 * .15

215–N A negative number * $^{-}.15$

215–O Multiply both $^{+}.23658$ and $^{+}6.22$ by 100. * Yes

215–P Yes. $\frac{1}{2} \times 622 = 311$. $22 < 311$. * .038. Since the last remainder is less than $\frac{1}{2}$ times the divisor, the quotient should be rounded to the next lesser thousandth.

215–Q Positive * $^{+}.038$

216–R Multiply both $^{+}54.3$ and $^{-}.32$ by 100. * Yes

216–S One

216–T No matter what numbers are being divided, you can multiply both the dividend and the divisor by the same non-zero number without affecting the quotient.

216–U Yes * Since the last remainder is greater than $\frac{1}{2}$ times the divisor, you should round the quotient to the next greater tenth. * $^{-}169.7$

216–V Ten * 8.72

216–W Yes

216–X $^{+}.16$

On your own

216–1 $^{+}3\frac{1}{5}$

216–2 $^{+}3\frac{3}{8}$

216–3 $^{-}10\frac{2}{3}$

216–4 $^{-}1\frac{5}{7}$

216–5 $^{-}1\frac{1}{2}$

216–6 $^{+}2\frac{1}{2}$

216–7 $^{+}1\frac{1}{2}$

216–8 $^{-}1\frac{1}{2}$

216–9 $^{-}.6$

216–10 $^{+}.55$

216–11 $^{-}.75$

216–12 $^{-}1.6$

216–13 $^{+}.04$

216–14 $^{-}1.75$

216–15 $^{-}.002$

216–16 $^{+}.06$

216–17 $^{+}25\frac{5}{12}$ * $^{-}7\frac{11}{12}$

216–18 $^{-}2.14$ * $^{+}4.26$

216–19 $^{-}33$ * $^{-}17\frac{2}{5}$

216–20 $^{-}.22$ * $^{-}5.78$

216–21 $^{-}13.3374$ * $^{+}1.3334$

216–22 $^{+}7\frac{5}{8}$ * $^{+}17\frac{3}{8}$

216–23 $^{+}4.05$ * $^{+}3.25$

216–24 $^{-}.399$ * $^{-}16.455$

216–25 $^{+}.03052$ * $^{+}622.86$

216–26 $^{+}129\frac{1}{6}$ * $^{+}1\frac{43}{50}$

216–27 $^{+}.00195$ * $^{+}.12$

216–28 $^{+}96\frac{1}{10}$, or $^{+}96.1$ * $^{+}1\frac{3}{5}$, or $^{+}1.6$

216–29 $^{+}16.8$ * $^{+}1.64$

216–30 $^{-}76.72$ * $^{-}1.64$

216–31 $^{-}32\frac{11}{48}$ * $^{-}4\frac{1}{68}$

216–32 $^{-}15.151$ * $^{-}12.75$

Lesson 142

217–A $(0, \frac{5}{16})$ * R_a * Second component

217–B 5.0 * 3 * 2

217–C The dividend is expressed in hundredths by the decimal 5.00. * 1 * 4

217–D The dividend is expressed in thousandths by the decimal 5.000. * 2 * 8

217–E 5 * 0

217–F .3125 * Yes * Yes

217–G Since $(0, \frac{5}{16}) = (0, .3125)$, $^{-}\frac{5}{16} = {}^{-}.3125$.

218–H Yes

218–I 2.00 * 6 * 2

218–J 20

218–K Both the first and second digits are 6. * The remainder obtained in the second step is not 0.

218–L 6 * 2

218–M No * No * .667

218–N Since $(\frac{2}{3}, 0) = (.\overline{6}, 0)$. $^{+}\frac{2}{3} = {}^{+}.\overline{6}$. * 6

218–O $^{+}\frac{3}{8} = {}^{+}.375$.

218–P $^{-}\frac{1}{6} = {}^{-}.1\overline{6}$.

218–Q $^{-}\frac{5}{2} = {}^{-}2.5$.

218–R $^{+}\frac{1}{3} = {}^{+}.\overline{3}$.

218–S $^{+}\frac{11}{40} = {}^{+}.275$.

219–A 10 * Yes

219–B 5

219–C The only factors of 2 are 2 and 1. The only factors of 5 are 5 and 1. * $2' = 2$ and $5' = 5$. * Yes. The numeral $2' \cdot 5'$ expresses 10 as a product of primes.

219–D $2^2 \cdot 5^2$

219–E $2' \cdot 3' * 3^2$

219–F $10 = 10^1$. * .3

219–G $1000 = 10^3$. * .023

219–H Multiply both the numerator and denominator of $\frac{3}{8}$ by 125. * .375

219–I Divide both the numerator and denominator of $\frac{27}{90}$ by 9. * .3

219–J .14 * .45

219–K Yes

219–L 2 and 5. $10^2 = 2^2 \times 5^2$. * 2 and 5. $10^3 = 2^3 \times 5^3$. * 2 and 5. $10^5 = 2^5 \times 5^5$. * Yes * 2 and 5

219–M $10 = 2^1 \cdot 5^1$. $100 = 2^2 \cdot 5^2$. $8 = 2^3$. $25 = 5^2$. $40 = 2^3 \cdot 5^1$.

219–N Yes * Yes

219–O 3 * 11 * 5 * $\frac{1}{5}$

220–P Yes * Yes * 1, or 1.0 * 10, or 10.0 * 562, or 562.0

220–Q Yes * When you divide a given number by 1, you obtain the same number.

220–R $\frac{3}{24}$ is not a basic fraction. $\frac{3}{24} = \frac{1}{8}$. $\frac{1}{8}$ is a basic fraction, and each prime factor of 8 is a factor of 10.

220–S $^+\frac{1}{12}$ cannot be named by a terminating decimal. $\frac{1}{12}$ is a basic fraction. 3 is a prime factor of 12, but 3 is not a factor of 10.

220–T $^-\frac{9}{15}$ can be named by a terminating decimal. $^-\frac{9}{15} = ^-\frac{3}{5}$. $^-\frac{3}{5}$ is a basic fraction, and the only prime factor of 5 is a factor of 10.

220–U $^-\frac{5}{9}$ cannot be named by a terminating decimal. $\frac{5}{9}$ is a basic fraction. The prime factor of 9 is 3. 3 is not a prime factor of 10.

220–V $^+\frac{4}{16}$ can be expressed by a terminating decimal. The basic fraction for $\frac{4}{16}$ is $\frac{1}{4}$. The only prime factor of 4 is 2. 2 is a factor of 10.

220–W $^-\frac{6}{30}$ can be expressed by a terminating decimal. The basic fraction for $\frac{6}{30}$ is $\frac{1}{5}$. The only prime factor of 5 is 5. 5 is a factor of 10.

220–X $^-.5$ * $^-\frac{1}{2}$ * Yes * No matter how many digits for zero you write after the digit 5 in the decimal $^-.5$, you will still have a decimal for $^-\frac{1}{2}$.

220–Y $^+.80000000000$ * Yes

220–Z 0

220–A 1 * They are the same.

220–B They are the same.

220–C They are the same.

220–D They are the same.

220–E They are the same.

220–F 2 * 6

220–G The remainder must be less than the divisor. * The only remainders possible are 0, 1, 2, 3, 4, 5, and 6. * Since 7 is the only prime factor of 7 and 7 is not a factor of 10, 1/7 cannot be expressed by a terminating decimal.

221–H 1, 2, 3, 4, 5, and 6

221–I Six * They are the same.

221–J The possible remainders are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

221–K $.\overline{45}$ * 45 * Two

221–L Each remainder must be equal to or less than 16.

221–M 5/17 cannot be expressed by a terminating decimal. * There are just 16 remainders that are greater than 0 but less than or equal to 16.

221–N 26 * 120 * 6237

221–O Every rational number can be indicated by a fraction. For any given fraction, you can find a repeating decimal that names the same rational number as the fraction indicates. Remember that every terminating decimal is also a repeating decimal.

On your own

221–1 0

221–2 6

221–3 0

221–4 03

221–5 3

221–6 Terminating decimal * $\frac{5}{8} = .625$.

221–7 Repeating decimal * 10 * $^-\frac{6}{11} = ^-.\overline{54}$.

221–8 Repeating decimal * 2 * $\frac{7}{3} = 2.\overline{3}$.

221–9 Terminating decimal * $\frac{4}{25} = .16$.

221–10 Terminating decimal * $\frac{33}{3} = 11$.

221–11 Repeating decimal * 12 * $^-\frac{3}{13} = ^-.\overline{230769}$.

221–12 Terminating decimal * $^-\frac{9}{20} = ^-.45$.

221–13 Terminating decimal * $\frac{15}{4} = 3.75$.

221–14 Terminating decimal * $^-\frac{11}{16} = ^-.6875$.

221–15 Repeating decimal * 8 * $\frac{8}{9} = .\overline{8}$.

221–16 Repeating decimal * 2 *

$-\frac{5}{3} = -1.\overline{6}$.

221–17 Terminating decimal *
$\frac{32}{80} = .4$.

221–18 Terminating decimal *
$\frac{3}{500} = .006$.

221–19 Repeating decimal * 23 *
$\frac{7}{24} = .291\overline{6}$.

221–20 Terminating decimal *
$-\frac{76}{125} = -.608$.

Keeping skilful

222–1 $(\frac{2}{5}, 0)$

222–2 $(0, \frac{5}{6})$

222–3 $(\frac{-7}{12}, 0)$

222–4 (0,12)

222–5 $(\frac{25}{30}, 0)$

222–6 (0,0)

222–7 $+\frac{1}{4}$

222–8 $-\frac{4}{5}$

222–9 $+\frac{1}{4}$

222–10 $+\frac{35}{4}$

222–11 $+\frac{28}{3}$

222–12 $-\frac{21}{10}$

222–13 0

222–14 $-\frac{13}{6}$

222–15 $+\frac{13}{60}$

222–16 $+\frac{35}{36}$

222–17 -89

222–18 $-\frac{3}{7}$

222–19 $+\frac{51}{10}$

222–20 $+72$

222–21 $-\frac{100}{80}$, or $-\frac{5}{4}$

222–22 $+868$

222–23 $-\frac{132}{85}$

222–24 $+\frac{125}{3}$

222–25 $-\frac{51}{2}$

222–26 $+7$

222–27 $-\frac{29}{9}$

222–28 $+\frac{2}{15}$

222–29 $+\frac{19}{20}$

222–30 $+\frac{11}{14}$

222–31 $+\frac{42}{5}$

222–32 $-\frac{153}{16}$

222–33 $-\frac{3}{5}$

222–34 0

222–35 $+12$

222–36 $-\frac{42}{11}$

222–37 $-\frac{39}{6}$, or $-\frac{13}{2}$

222–38 $+156$

222–39 $+\frac{40}{11}$

222–40 $+\frac{27}{5}$

222–41 $+\frac{14}{5}$

222–42 $+\frac{101}{6}$

222–43 $+80$

222–44 $-\frac{68}{3}$

222–45 $+155$

222–46 -387

222–47 -65

222–48 $-\frac{17}{88}$

222–49 $+10$

222–50 $-\frac{4}{15}$

222–51 $+\frac{35}{18}$

222–52 $-\frac{5}{12}$

222–53 $-\frac{7}{26}$

222–54 $+\frac{5}{3}$

222–55 $+\frac{27}{2}$

222–56 $-\frac{1}{21}$

222–57 $-\frac{4}{13}$

Checking up

Test 128

222–1 Identity-element property of addition

222–2 Zero property of multiplication

222–3 Identity-element property of multiplication

222–4 Quotient property

222–5 $-9 * +\frac{1}{9}$

222–6 $+\frac{3}{8} * -\frac{8}{3}$

222–7 $-2\frac{4}{5} * +\frac{5}{14}$

222–8 $+\frac{18}{11}$, or $+1\frac{7}{11} * -\frac{11}{18}$

222–9 $-\frac{41}{3}$, or $-13\frac{2}{3} * +\frac{3}{41}$

Test 129

222–10 $+5\frac{2}{3}$

222–11 $-6\frac{4}{5}$

222–12 $-1\frac{5}{6}$

222–13 $+26\frac{1}{2}$

222–14 $12\frac{1}{7}$

222–15 $+.7$

222–16 $-.24$

222–17 $+1.825$

222–18 $-2.\overline{63}$

222–19 $-6.\overline{857142}$

222–20 $+6\frac{5}{6} * +1\frac{5}{6}$

222–21 $+8\frac{1}{15} * -18\frac{4}{15}$

222–22 $-2.3832 * +2.3968$

222–23 $-317.704 * +292.304$

222–24 $-55 * -8\frac{41}{45}$

222–25 $+148\frac{7}{16} * +\frac{45}{76}$

222–26 $+94\frac{1}{6} * +\frac{135}{226}$

222–27 $-109\frac{5}{7} * -\frac{26}{7}$

222–28 $+16,089.216 * +5.77$

222–29 $-.177304 * -1.30$

222–30 $-1.56364 * -5.78$

222–31 $+2.5875 * +30.17$

Lesson 143

223–A If a decimal expresses a rational number and the decimal does not have a symbol for either "positive" or "negative," then it is agreed that the decimal expresses a positive rational number.

223–B 865

223–C Sixty billion * Five billion

223–D 971 * Nine hundred million * Seventy million * One million

223–E 408 * The digit 4 expresses four hundred thousand; the digit 0 expresses zero ten thousand; the digit 8 expresses eight thousand.

223–F Three times

223–G 8 * 10 * The third * Yes. $10^3 = 10 \times 10 \times 10$ and $10 \times 10 \times 10 = 1000$.

223–H Yes * Yes

224–I $8 * \frac{3}{5}$ * A power that has a positive integer for an exponent is a positive integral power.

224–J Three times * Yes

224–K The associative property of multiplication tells

you that the way in which you group the factors in a product does not affect the product. * $\frac{9}{25}$

224–L Yes

224–M $\frac{27}{125}$ * $\frac{27}{125}$

224–N Yes * The third

224–O Yes * 4 * $-\frac{2}{3}$ * The exponent of $(-\frac{2}{3})^4$ is a positive integer.

224–P Yes * The associative property of multiplication tells you that the way in which you group the factors in a product does not affect the product. * $\frac{4}{9}$

224–Q Yes * $-\frac{8}{27}$

224–R Yes * $\frac{16}{81}$ * $\frac{16}{81}$

224–S $\frac{1}{16}$

224–T 16

224–U $-\frac{8}{125}$

224–V 1

224–W $\frac{9}{64}$

224–X $\frac{81}{256}$

224–Y -343

224–Z $\frac{1}{32}$

224–A 3 * 4 * Yes * 10

224–B $10^3 = 10 \times 10 \times 10$ and $10^4 = 10 \times 10 \times 10 \times 10$.

224–C The associative property of multiplication

224–D Seven times * Seven times * Yes * Yes

224–E Yes

224–F 10,000,000 * 10,000,000

224–G $(\frac{3}{4})^2 \cdot (\frac{3}{4})^3$

$(\frac{3}{4} \times \frac{3}{4}) \times (\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4})$

$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

$(\frac{3}{4})^5$. Therefore,

$(\frac{3}{4})^2 \cdot (\frac{3}{4})^3 = (\frac{3}{4})^5$.

$(-4)^3 \cdot (-4)^3$

$(-4 \times -4 \times -4) \times (-4 \times -4 \times -4)$

$-4 \times -4 \times -4 \times -4 \times -4 \times -4$

$(-4)^6$. Therefore,

$(-4)^3 \cdot (-4)^3 = (-4)^6$.

225–H (Column 1, top) False statement

225–I (Column 1) True statement

225–J (Column 1) False statement

225–K (Column 1) True statement

225–A 10^5 and 10^3

225–B 5 * 8 * Yes * 10

225–C Yes * $10^5 = 10 \times 10 \times 10 \times 10 \times 10$ and $10^3 = 10 \times 10 \times 10$.

225–D $10 \times 10 \times 10$, or 1000 * Yes

225–E Three numeral 10's have been crossed out and replaced by numeral 1's in both the name of the dividend and the name of the divisor.

225–F Yes * Yes

225–G Yes

225–H (Column 1, bottom) 100 * 100

225–I (Column 2) $(\frac{5}{8})^6 * (\frac{5}{8})^2$ * Yes * The exponent of the dividend

225–J (Column 2) Yes * Yes

225–K (Column 2)

$\frac{(\frac{5}{8})^6}{(\frac{5}{8})^2}$

$\frac{\frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}}{\frac{5}{8} \times \frac{5}{8}}$

$\frac{\cancel{\frac{5}{8}} \times \cancel{\frac{5}{8}} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8}}{\cancel{\frac{5}{8}} \times \cancel{\frac{5}{8}}}$

$(\frac{5}{8})^4$. Therefore,

$\frac{(\frac{5}{8})^6}{(\frac{5}{8})^2} = (\frac{5}{8})^4$.

$\frac{14^5}{14^2}$

$\frac{14 \times 14 \times 14 \times 14 \times 14}{14 \times 14}$

$\frac{\cancel{14} \times \cancel{14} \times 14 \times 14 \times 14}{\cancel{14} \times \cancel{14}}$

14^3. Therefore,

$\frac{14^5}{14^2} = 14^3$.

$\frac{(-8)^8}{(-8)^5}$

$\frac{-8 \times -8 \times -8 \times -8 \times -8 \times -8 \times -8 \times -8}{-8 \times -8 \times -8 \times -8 \times -8}$

$\frac{\cancel{-8} \times \cancel{-8} \times \cancel{-8} \times \cancel{-8} \times \cancel{-8} \times \cancel{-8} \times -8 \times -8 \times -8}{\cancel{-8} \times \cancel{-8} \times \cancel{-8} \times \cancel{-8} \times \cancel{-8}}$

$(-8)^3$. Therefore

$\frac{(-8)^8}{(-8)^5} = (-8)^3$.

226–L (Column 1) True statement

226–M (Column 1) False statement

226–N (Column 1) True statement

226–O (Column 1) False statement

226–P (Column 1) False statement

226–A The second power of 10 * 6 * 10^2 is the product of a number from 1 to 10 and a power of 10.

226–B The third power of 10 * 1.256 * Yes

226–C Yes * 1.256×10^3 is the product of a number from 1 to 10 and a power of 10.

226–D The fourth power of 10 * Yes * The fourth power of 10

226–E Yes

226–F 8×10^7

226–G 1.4×10^4

226–H 1.87×10^4

226–I 2.4369×10^7

226–J 1.7×10^1

226–K 3.22×10^2

226–L (Column 2) 45,000 and 5100

226–M (Column 2) 45,000 = 4.5×10^4 and 5100 = 5.1×10^3.

226–N (Column 2) Yes * The commutative and associative properties of multiplication

226–O (Column 2) $4.5 \times 5.1 = 22.95$ and $10^4 \times 10^3 = 10^7$.

226–P (Column 2) 229,500,000 * 229,500,000

226–Q 30×6725
$(3 \times 10^1) \times (6.725 \times 10^3)$
$(3 \times 6.725) \times (10^1 \times 10^3)$
20.175×10^4. Therefore, $30 \times 6725 = 20.175 \times 10^4$.

226–R $492 \times 65{,}000{,}000$
$(4.92 \times 10^2) \times (6.5 \times 10^7)$
$(4.92 \times 6.5) \times (10^2 \times 10^7)$
31.980×10^9. Therefore, $492 \times 65{,}000{,}000 = 31.980 \times 10^9$.

226–S $6200 \times 328{,}000$
$(6.2 \times 10^3) \times (3.28 \times 10^5)$
$(6.2 \times 3.28) \times (10^3 \times 10^5)$
20.336×10^8. Therefore, $6200 \times 328{,}000 = 20.336 \times 10^8$.

226–T 201,750 * 31,980,000,000 * 2,033,600,000

227–U 780,000 and 3900

227–V $780{,}000 = 7.8 \times 10^5$ and $3900 = 3.9 \times 10^3$.

227–W The symbol $\frac{7.8 \times 10^5}{3.9 \times 10^3}$ expresses $(7.8 \times 10^5) \div (3.9 \times 10^3)$.

227–X Yes * Yes * Yes

227–Y Yes

227–Z 200 * 200

On your own

227–1 The exponent is 6 and the base is 10. * 1,000,000

227–2 The exponent is 3 and the base is $\frac{3}{4}$. * $\frac{27}{64}$

227–3 The exponent is 5 and the base is $^-4$. * $^-1024$

227–4 The exponent is 4 and the base is $-\frac{2}{11}$. * $\frac{16}{14641}$

227–5 The exponent is 7 and the base is $\frac{1}{10}$. * $\frac{1}{10000000}$

227–6 $(-\frac{3}{8})^9$ * $-\frac{19683}{134217728}$

227–7 $(\frac{2}{3})^3$ * $\frac{8}{27}$

227–8 12^6 * 2,985,984

227–9 5^7 * 78,125

227–10 $(^-2)^9$ * $^-512$

227–11 7^5 * 16,807

227–12 2.2×10^4

227–13 5×10^2

227–14 6.924×10^6

227–15 3.459×10^5

227–16 8.3×10^1

227–17 2.964×10^3

227–18 $8000 = 8 \times 10^3$.
$60{,}000 = 6 \times 10^4$. *
$(8 \times 10^3) \times (6 \times 10^4) = 48 \times 10^7$.
$48 \times 10^7 = 480{,}000{,}000$.

227–19 $51{,}000 = 5.1 \times 10^4$.
$700{,}000 = 7 \times 10^5$. *
$(5.1 \times 10^4) \times (7 \times 10^5) = 35.7 \times 10^9$.
$35.7 \times 10^9 = 35{,}700{,}000{,}000$.

227–20 $256 = 2.56 \times 10^2$.
$23 = 2.3 \times 10^1$. *
$(2.56 \times 10^2) \times (2.3 \times 10^1) = 5.88 \times 10^3$.
$5.888 \times 10^3 = 5888$.

227–21 $45 = 4.5 \times 10^1$.
$1230 = 1.23 \times 10^3$. *
$(4.5 \times 10^1) \times (1.23 \times 10^3) = 5.535 \times 10^4$.
$5.535 \times 10^4 = 55{,}350$.

227–22 $2{,}900{,}000{,}000{,}000 = 2.9 \times 10^{12}$.
$1{,}450{,}000 = 1.45 \times 10^6$. *
$(2.9 \times 10^{12}) \div (1.45 \times 10^6) = 2 \times 10^6$.
$2 \times 10^6 = 2{,}000{,}000$.

227–23 $78{,}000{,}000 = 7.8 \times 10^7$.
$13{,}000 = 1.3 \times 10^4$. *
$(7.8 \times 10^7) \div (1.3 \times 10^4) = 6 \times 10^3$.
$6 \times 10^3 = 6000$.

227–24 $582{,}000{,}000 = 5.82 \times 10^8$.
$1{,}940{,}000 = 1.94 \times 10^6$. *
$(5.82 \times 10^8) \div (1.94 \times 10^6) = 3 \times 10^2$.
$3 \times 10^2 = 300$.

227–25 $67{,}500{,}000 = 6.75 \times 10^7$.
$1{,}350{,}000 = 1.35 \times 10^6$. *
$(6.75 \times 10^7) \div (1.35 \times 10^6) = 5 \times 10^1$.
$5 \times 10^1 = 50$.

227–26 $34{,}000 = 3.4 \times 10^4$.
$170 = 1.7 \times 10^2$. *
$(3.4 \times 10^4) \div (1.7 \times 10^2) = 2 \times 10^2$.
$2 \times 10^2 = 200$.

227–27 $800{,}000 = 8 \times 10^5$.
$1600 = 1.6 \times 10^3$. *
$(8 \times 10^5) \times (1.6 \times 10^3) = 5 \times 10^2$.
$5 \times 10^2 = 500$.

227–28 $12{,}000 = 1.2 \times 10^4$.
$3600 = 3.6 \times 10^3$. *
$(1.2 \times 10^4) \times (3.6 \times 10^3) = .\overline{3} \times 10^1$.
$.\overline{3} \times 10^1 = 3.\overline{3}$.

Keeping skilful

227–1 .8

227–2 $^-.88$

227–3 $^-6.375$

227–4 $5.\overline{3}$

227–5 .0375

227–6 $^-.002$

227–7 16.25

227–8 $^-4.\overline{1}$

227–9 2.125

227–10 $.\overline{18}$

227–11 $^-.1875$

227–12 $^-.011$

227–13 1000

227–14 $\frac{1}{8}$

227–15 1296

227–16 $\frac{1}{256}$

227–17 $^{-}100{,}000$

227–18 4096

227–19 $-\frac{1}{4096}$

227–20 25

227–21 1

227–22 $\frac{1}{100000}$

227–23 4225

227–24 $\frac{1}{4096}$

227–25 19.101

227–26 10.2

227–27 $^{-}18.26$

227–28 6.683

227–29 .65

227–30 59.74

227–31 4.4521

227–32 $^{-}9.471$

227–33 $^{-}6.511$

227–34 $^{-}65.2$

227–35 $^{-}.0374$

227–36 3.12

esson 144

228–A Because of the product property of positive integral powers. The universe for x is R. The universe for m and n is I_p. For each x, m, and n, $x^m \times x^n = x^{m+n}$. * Yes * $3^2 = 9$. $3^3 = 27$. $9 \times 27 = 243$. $243 = 3^5$. Therefore, $3^2 \times 3^3 = 3^5$.

228–B Because of the quotient property of positive integral powers. The universe for x is R. The universe for m and n is I_p. For each x, m, and n, if m is greater than n, $x^m \div x^n = x^{m-n}$. * Yes * $9^4 = 6561$. $9^1 = 9$. $6561 \div 9 = 729$. $729 = 9^3$. Therefore, $9^4 \div 9^1 = 9^3$.

228–C You must know the meaning of the symbol 10^0.

228–D No * Yes * 3 * Yes

228–E 10 * 10 * Yes

228–F If a number is used as a factor, it must be used as a factor at least once. For this reason, we will define 10^0 without using the idea of factor. However, we can use positive integral powers to help define 10^0.

228–G Since $0 + 3 = 3$, 10^3 is the same as 10^{0+3}, and sentence B expresses the same true statement as sentence A.

228–H 1 * Yes

228–I The number 1 is the only identity element for multiplication of rational numbers. Therefore, the only number by which $(^{-}\frac{1}{2})^4$ can be multiplied by to give $(^{-}\frac{1}{2})^4$ is 1. * Since $(^{-}\frac{1}{2})^4$ is the same as $(^{-}\frac{1}{2})^{0+4}$, sentence D expresses the same true statement as sentence C.

228–J Yes * x^n is the same as x^{0+n} because $n = 0 + n$.

228–K We must define x^0 to be equal to 1.

229–L Yes * A power whose exponent is zero is a zero power. The zero power of any non-zero rational number is 1. Therefore, replacing x^0 by 1 in $x^0 \cdot x^0 = x^0$, we obtain $1 \cdot 1 = 1$, which is a true statement.

229–A No * Yes * 2 * Yes

229–B 10 * 10 * Yes

229–C Because $^{-}2 + 2 = 0$. * 0

229–D Yes * By definition, the zero power of any non-zero rational number is equal to 1.

229–E Yes * Since $10^{-2+2} = 10^0$, or 1, statement G is the same as statement E.

229–F 1

229–G If the product of two numbers is 1, each of the numbers is the reciprocal of the other.

229–H Since $(^{-}\frac{3}{4})^{-5+5} = 1$, statement H tells you that the product of $(^{-}\frac{3}{4})^{-5}$ and $(^{-}\frac{3}{4})^5$ is 1. Therefore, $(^{-}\frac{3}{4})^{-5}$ is the reciprocal of $(^{-}\frac{3}{4})^5$. The symbol $1/(^{-}\frac{3}{4})^5$ also names the reciprocal of $(^{-}\frac{3}{4})^5$.

229–I x^{-n}, or $1/x^n$ * The inverse of a positive integer is a negative integer.

229–J $x^{-n+n} = x^0$. If $x^{-n} \cdot x^n = x^{-n+n}$, then $x^{-n} \cdot x^n = x^0$. You know that $x^0 = 1$. If the product of two numbers is 1, the numbers are reciprocals. Therefore, x^{-n} is the reciprocal of x^n. The reciprocal of x^n is $1/x^n$.

230–K (Column 1) $1/x^n$ and x^{-n} are the same. * x^{-n} and x^n are reciprocals of each other.

230–L (Column 1) If we defined 0^{-0} as $1/0^0$, then we would have to define 0^0. If we defined 0^0 to be 0, then $1/0^0$ would not be a number since division by 0 is impossible. Therefore, neither 0^{-0} or 0^0

are defined.

230–M (Column 1) $1/(^{-}6)^{7}$

230–N (Column 1) $1/(\frac{2}{3})^{4}$

230–O (Column 1) $1/5^{2}$

230–P (Column 1) $1/(^{-}\frac{7}{2})^{3}$

230–Q (Column 1) $1/(^{-}100)^{2}$

230–R $1/(\frac{5}{6})^{1}$

230–S $1/43^{3}$

230–T $1/(^{-}\frac{9}{10})^{2}$

230–A From the definition of a negative integral power, 10^{-7} is the same as $1/10^{7}$. * Yes

230–B 10,000, or 10^{4} * Divide both the dividend and the divisor of $10^{4}/10^{7}$ by 10^{4}. * Yes

230–C From the definition of negative integral power, 10^{-3} is the same as $1/10^{3}$. * Yes

230–D Yes * $^{-}8$ * Yes

230–E Yes

230–F $10^{-3} \cdot 10^{-5} = (1/10^{3}) \cdot (1/10^{5})$. $10^{-3} \cdot 10^{-5} = 1/(10^{3} \cdot 10^{5})$. $10^{-3} \cdot 10^{-5} = 1/10^{3+5}$. $10^{-3} \cdot 10^{-5} = 1/10^{8}$. $10^{-3} \cdot 10^{-5} = 10^{-8}$. $10^{9} \cdot 10^{-2} = 10^{9}(1/10^{2})$. $10^{9} \cdot 10^{-2} = 10^{9}/10^{2}$. $10^{9} \cdot 10^{-2} = 10^{7}$.

230–G 10^{0} is the same as 1. * Yes * Add 0 and $^{-}6$. $0 + {}^{-}6 = {}^{-}6$.

230–H $13^{-4} \times 13^{0} = (1/13^{4}) \cdot 1$. $13^{-4} \times 13^{0} = 1/13^{4}$. $13^{-4} \times 13^{0} = 13^{-4}$.

230–I $(\frac{1}{4})^{-5} \times (\frac{1}{4})^{-4} = [1/(\frac{1}{4})^{5}] \times [1/(\frac{1}{4})^{4}]$. $(\frac{1}{4})^{-5} \times (\frac{1}{4})^{-4} = 1/(\frac{1}{4})^{9}$. $(\frac{1}{4})^{-5} \times (\frac{1}{4})^{-4} = (\frac{1}{4})^{-9}$.

230–J $(^{-}17)^{2} \times (^{-}17)^{-6} = (^{-}17)^{2} \times [1/(^{-}17)^{6}]$. $(^{-}17)^{2} \times (^{-}17)^{-6} = (^{-}17)^{2}/(^{-}17)^{6}$. $(^{-}17)^{2} \times (^{-}17)^{-6} = 1/(^{-}17)^{4}$. $(^{-}17)^{2} \times (^{-}17)^{-6} = (^{-}17)^{-4}$.

230–K (Column 2) $(\frac{5}{6})^{-1} \times (\frac{5}{6})^{1} = [1/(\frac{5}{6})^{1}] \times (\frac{5}{6})^{1}$. $(\frac{5}{6})^{-1} \times (\frac{5}{6})^{1} = (\frac{5}{6})^{1} \div (\frac{5}{6})^{1}$. $(\frac{5}{6})^{-1} \times (\frac{5}{6})^{1} = 1 \div 1$. $(\frac{5}{6})^{-1} \times (\frac{5}{6})^{1} = 1$.

230–L (Column 2) $(^{-}12)^{2}$

230–M (Column 2) 19^{-4}

230–N (Column 2) $(\frac{5}{8})^{-10}$

230–O (Column 2) $(^{-}4)^{2}$

230–P (Column 2) $(^{-}\frac{11}{13})^{-2}$

230–Q (Column 2) 141^{-3}

231–A $^{-}6 * {}^{-}2 * 10$

231–B $10^{-6}/10^{-2}$ means $10^{-6} \div 10^{-2}$. From the definition of negative integral powers, $10^{-6} = 1/10^{6}$ and $10^{-2} = 1/10^{2}$. This means that $10^{-6} \div 10^{-2} = (1/10^{6}) \div (1/10^{2})$. Using the transitive property of equality, we can obtain statement P from $10^{-6}/10^{-2} = 10^{-6} \div 10^{-2}$ and $10^{-6} \div 10^{-2} = (1/10^{6}) \div (1/10^{2})$.

231–C Yes * Using the quotient property of rational numbers, $(1/10^{6}) \div (1/10^{2}) = (1/10^{6}) \cdot 10^{2}$. Using the transitive property of equality, statement Q can be obtained from statement P and $(1/10^{6}) \div (1/10^{2}) = (1/10^{6}) \cdot 10^{2}$.

231–D $(1/10^{6}) \cdot 10^{2}$ is the same as $10^{2}/10^{6}$.

231–E 10^{2} * Divide both the dividend and divisor of $10^{2}/10^{6}$ by 10^{2}. * Yes

231–F From the definition of negative integral powers, $1/10^{4}$ is the same as 10^{-4}. * Yes

231–G 10^{-4}

231–H Yes * $^{-}4$ * Yes

231–I $10^{-3}/10^{5} = (1/10^{3}) \div 10^{5}$. $10^{-3}/10^{5} = (1/10^{3}) \times (1/10^{5})$. $10^{-3}/10^{5} = 1/10^{8}$. $10^{-3}/10^{5} = 10^{-8}$. $10^{9}/10^{-6} = 10^{9} \div (1/10^{6})$. $10^{9}/10^{-6} = 10^{9} \times 10^{6}$. $10^{9}/10^{-6} = 10^{15}$.

231–J Yes * $10^{0} = 1$. Therefore, $10^{0}/10^{-4} = 1/10^{-4}$, or 10^{4}.

231–K $12^{3} \div 12^{-4} = 12^{3} \div (1/12^{4})$. $12^{3} \div 12^{-4} = 12^{3} \times 12^{4}$. $12^{3} = 12^{-4} = 12^{7}$.

231–L $(\frac{1}{7})^{0} \div (\frac{1}{7})^{-1} = 1 \div [1/(\frac{1}{7})^{1}]$. $(\frac{1}{7})^{0} \div (\frac{1}{7})^{-1} = 1 \times (\frac{1}{7})^{1}$. $(\frac{1}{7})^{0} \div (\frac{1}{7})^{-1} = (\frac{1}{7})^{1}$.

231–M $(\frac{3}{4})^{-2} \div (\frac{3}{4})^{-5} = [1/(\frac{3}{4})^{2}] \div [1/(\frac{3}{4})^{5}]$. $(\frac{3}{4})^{-2} \div (\frac{3}{4})^{-5} = [1/(\frac{3}{4})^{2}] \times (\frac{3}{4})^{5}$. $(\frac{3}{4})^{-2} \div (\frac{3}{4})^{-5} = (\frac{3}{4})^{5} \div (\frac{3}{4})^{2}$. $(\frac{3}{4})^{-2} \div (\frac{3}{4})^{-5} = (\frac{3}{4})^{3}$.

231–N $(^{-}\frac{3}{8})^{-7} \div (^{-}\frac{3}{8})^{4} = [1/(^{-}\frac{3}{8})^{7}] \div (^{-}\frac{3}{8})^{4}$. $(^{-}\frac{3}{8})^{-7} \div (^{-}\frac{3}{8})^{4} = [1/(^{-}\frac{3}{8})^{7}] \times [1/(^{-}\frac{3}{8})^{4}]$. $(^{-}\frac{3}{8})^{-7} \div (^{-}\frac{3}{8})^{4} = 1/(^{-}\frac{3}{8})^{11}$. $(^{-}\frac{3}{8})^{-7} \div (^{-}\frac{3}{8})^{4} = (^{-}\frac{3}{8})^{-11}$.

231–O 82^{1}

231–P $(^{-}\frac{7}{8})^{-10}$

231–Q 89^{20}

231–R $(^{-}5)^{-2}$

231–S $(^{-}21)^{2}$

231–T $(\frac{6}{11})^{3}$

232–A Seven hundred twenty-four thousand five hundred sixty-nine and thirty-nine thousand eighty-one hundred-thousandths

232–B $10^{0} * 10^{3} * 10^{-2}$

232–C Thirty-two and thirty-eight thousandths *

$3(10^1) + 2(10^0) + 0(10^{-1}) + 3(10^{-2}) + 8(10^{-3})$

232–D Four hundred seventy-six and twenty-five hundredths * $4(10^2) + 7(10^1) + 6(10^0) + 2(10^{-1}) + 5(10^{-2})$

232–E Seven hundred sixty-two hundred-thousandths * $7(10^{-3}) + 6(10^{-4}) + 2(10^{-5})$

232–F One and nine tenths * $1(10^0) + 9(10^{-1})$

232–G Thirty-seven thousand nine hundred twenty-six and three hundred forty-five thousandths * $3(10^4) + 7(10^3) + 9(10^2) + 2(10^1) + 6(10^0) + 3(10^{-1}) + 4(10^{-2}) + 5(10^{-3})$

232–H Eighty-six thousand two hundred fifty-nine hundred-thousandths * $8(10^{-1}) + 6(10^{-2}) + 2(10^{-3}) + 5(10^{-4}) + 9(10^{-5})$

On your own

232–I True statement

232–2 False statement

232–3 True statement

232–4 False statement

232–5 False statement

232–6 True statement

232–7 False statement

232–8 True statement

232–9 False statement

232–10 21^{-2}

232–11 $(^-9)^1$

232–12 $(^-\frac{5}{2})^{-2}$

232–13 $(\frac{3}{3})^{14}$

232–14 12^{-8}

232–15 $(^-62)^{-20}$

232–16 $(^-\frac{5}{12})^0$

232–17 $(\frac{13}{7})^{-16}$

232–18 61^{28}

232–19 $(^-6)^8$

232–20 $(\frac{17}{19})^{-9}$

232–21 $(^-\frac{21}{25})^5$

232–22 41^{-2}

232–23 $(^-57)^4$

232–24 $(^-\frac{1}{33})^6$

232–25 $(\frac{8}{15})^7$

232–26 $6(10^3) + 8(10^2) + 9(10^1) + 4(10^0) + 7(10^{-1}) + 5(10^{-2})$

232–27 $7(10^2) + 0(10^1) + 1(10^0) + 3(10^{-1}) + 9(10^{-2}) + 8(10^{-3})$

232–28 $1(10^{-3}) + 0(10^{-4}) + 4(10^{-5}) + 0(10^{-6}) + 8(10^{-7})$

Lesson 145

233–A The numeral 7.5×10^2 expresses a number as the product of a number from 1 to 10 and a power of 10. * 750

233–B 58.62

233–C 8.6345×10^3 * 1×10^1

233–D 1×10^6 * 2×10^6 * 5.3×10^6

233–E The numeral 6×10^0 expresses a number as the product of a number from 1 to 10 and a power of 10. * Since $10^0 = 1$, $6 \times 10^0 = 6 \times 1$ and $6 \times 1 = 6$.

233–F 3.2×10^0 * 1.5842

233–G 9×10^0, $9.7 \div 10^0$, 9.75×10^0, and 9.752×10^0

233–H Use a decimal to name the rational number. Then express the rational number as the product of itself and 10^0.

233–I .625 is greater than 0 and less than 1. The statement $0 < .625 < 1$ is true.

233–J Yes. $.625 \times 10^1 = .625 \times 10$ and $.625 \times 10 = 6.25$. * Yes. $1 \leq 6.25 < 10$. * $.625 = 6.25 \div 10$ and $6.25 \div 10 = 6.25 \div 10^1$.

233–K Because of the quotient property * Yes * $.625 = 6.25 \div 10^1$ and $6.25 \div 10^1 = 6.25 \times 10^{-1}$.

233–L Yes * The numeral 6.25×10^{-1} expresses a number as the product of a number from 1 to 10 and a power of 10.

233–M 10^2 * Yes * Since you know that $.03 \times 10^2 = 3$, you know that $.03 = 3 \div 10^2$.

233–N Yes * $.03 = 3 \div 10^2$. $3 \div 10^2 = 3 \times 10^{-2}$. $.03 = 3 \times 10^{-2}$, by the transitive property of equality. * 3×10^{-2}

233–O 10^3 * 10^3

233–P $.0048 = 4.8 \div 10^3$. $4.8 \div 10^3 = 4.8 \times 10^{-3}$. $.0048 = 4.8 \times 10^{-3}$. * 4.8×10^{-3}

233–Q $.0007 = 7 \div 10^4$. $7 \div 10^4 = 7 \times 10^{-4}$. $.0007 = 7 \times 10^{-4}$.

234–R (Column 1) 9.93×10^0

234–S 3.6×10^{-5}

234–T 5.3×10^{-2}

234–U 1.68×10^1

234–V 3.2568×10^2

234–W 1.4285×10^{-1}

234–A (Column 1) 3,000,000 and 26,000

234–B (Column 1) 3×10^6 * 2.6×10^4 * Yes

234–C The associative and commutative properties of multiplication *

$(3 \times 10^6) \times (2.6 \times 10^4) = 3 \times (10^6 \times 2.6) \times 10^4$. Associative property of multiplication.
$(3 \times 10^6) \times (2.6 \times 10^4) = 3 \times (2.6 \times 10^6) \times 10^4$. Commutative property of multiplication.
$(3 \times 10^6) \times (2.6 \times 10^4) = (3 \times 2.6) \times (10^6 \times 10^4)$. Associative property of multiplication

234–D $3 \times 2.6 = 7.8$ and $10^6 \times 10^4 = 10^{10}$.

234–E 7.8×10^{10} * 78,000,000,000

234–F $7.5 \times 10^1 * 1.6 \times 10^{-2}$ * Yes

234–G The associative and commutative properties of multiplication

234–H $7.5 \times 1.6 = 12$ and $10^1 \times 10^{-2} = 10^{-1}$. * 12 is not a number from 1 to 10.

234–I Yes * $12 \times 10^{-1} = (1.2 \times 10^1) \times 10^{-1}$. $12 \times 10^{-1} = 1.2 \times (10^1 \times 10^{-1})$. $12 \times 10^{-1} = 1.2 \times 10^0$.

234–J 1.2×10^0 * 1.2

234–K 2.01×10^3 * 2010

234–L 6.8×10^7 * 68,000,000

234–M 3.094×10^3 * 3094

234–N 1.52×10^{-3} * .00152

234–O 8×10^{-5} * .00008

234–P 2.16×10^{-4} * .000216

234–Q 1.395×10^3 * 1395

234–R (Column 2) 6.63×10^{-7} * .000000663

234–A (Column 2) 330,000 and .011

234–B (Column 2) 330,000 = 3.3×10^5 and .011 = 1.1×10^{-2}.

234–C Numerals S and R each express the quotient of 3.3×10^5 and 1.1×10^{-2}.

235–D $3 * 10^7$

235–E 3×10^7

235–F .086 ÷ 4.8.
$(8.6 \times 10^{-2}) \div (4.3 \times 10^0)$.
$(8.6/4.3) \times (10^{-2}/10^0)$.
2×10^{-2} * .02

235–G .0095 ÷ 1.9.
$(9.5 \times 10^{-3}) \div (1.9 \times 10^0)$.
$(9.5/1.9) \times (10^{-3}/10^0)$.
5×10^{-3} * .005

235–H 72,000 ÷ 4,000,000.
$(7.2 \times 10^4) \div (4 \times 10^6)$.
$(7.2/4) \times (10^4/10^6)$.
1.8×10^{-2} * .018

235–I 16,000 ÷ .2.
$(1.6 \times 10^4) \div (2 \times 10^{-1})$.
$(1.6/2) \times (10^4/10^{-1})$
$.8 \times 10^5$.
8×10^4 * 80,000

235–J 15.4 ÷ .07.
$(1.54 \times 10^1) \div (7 \times 10^{-2})$.
$(1.54/7) \times (10^1/10^{-2})$.
$.22 \times 10^3$.
2.2×10^2 * 220

235–K .086 ÷ 4300.
$(8.6 \times 10^{-2}) \div (4.3 \times 10^3)$.
$(8.6/4.3) \times (10^{-2}/10^3)$.
2×10^{-5} * .00002

On your own

235–1 2×10^{-2}

235–2 7.85826×10^5

235–3 4.13×10^0

235–4 3.256×10^{-1}

235–5 6.9×10^{-6}

235–6 1.684×10^1

235–7 3×10^{-3}

235–8 2.5×10^4

235–9 9.9×10^{-3}

235–10 5.168×10^7

235–11 .025

235–12 1.52×10^1

235–13 38

235–14 2.56×10^2

235–15 9.2×10^{-3}

235–16 7×10^{-2}

235–17 6.69×10^8 * 669,600,000

235–18 2.304×10^0 * 2.304

235–19 3.6×10^0 * 3.6

235–20 4.732×10^{-4} * .0004732

235–21 3×10^2 * 300

235–22 5×10^{-5} * .00005

235–23 4×10^0 * 4

235–24 4×10^{-3} * .004

Applying mathematics

235–1 x, number of bushels of tomatoes Mr. Hanson sold on Saturday.
$16\frac{1}{2} + x = 35$.
$\{18\frac{1}{2}\}$. $18\frac{1}{2}$ bu.

235–2 x, Jim's height on Jan. 1
$x < 55 \wedge x + 2\frac{1}{2} > 57$.
$\{x \mid 54\frac{1}{2} < x < 55\}$.
Between $54\frac{1}{2}$ in. and 55 in

235–3 x, number of tons of iron ore mined in Canada in a recent year.
$10627000 + x = 35067380$.
$\{24440380\}$.
24,440,380 tons

235–4 x, amount of Miss Grant's telephone bill for August
$x - 4.35 = 2.20$.
$\{6.55\}$. $6.55

235–5 x, amount of money Alice earned last week.
$x > 10.50 \wedge x - 2.60 < 9.50$.
$\{x \mid 10.50 < x < 12.10\}$.
Between $10.50 and $12.1

236–6 $x < 40 \wedge 102.16 + x > 141.26$.
$\{x \mid 39.10 < x < 40\}$

236–7 $x + y + z = 32 \wedge x - y = 1 \wedge y(2) = 25$.
$\{(13\frac{1}{2}, 12\frac{1}{2}, 6)\}$

236–8 x, amount of interest Mr Sayre owed the bank.
$6/100 \sim x/1950$.
$\{117\}$. $117

236–9 x, rate of commission Mr Lopez received.

192/2560 ~ x/100.
{7.5}. 7.5%

36–10 x, amount of money it will cost the Camera Club to buy 72 ice-cream bars.
12/.95 ~ 72/x v
18/1.25 ~ 72/x.
{5.70,5.00}. It will cost the Camera Club either $5.70 or $5.00 to buy the 72 ice-cream bars.

36–11 x, perimeter of football field in yards.
y, perimeter of football field in feet.
$x = 110 + 63\frac{1}{3} + 110 + 63\frac{1}{3}$ ∧ 3/1 ~ y/x.
{$346\frac{2}{3}$, 1040}. 1040 ft.

36–12 x, area of field in square yards.
32/1 ~ $2x$/20.
{320}. 320 sq. yd.

36–13 x, area in square feet.
y, length of base in feet.
144/1 ~ 360/x ∧
y/1 ~ $x/1\frac{1}{2}$.
{($2\frac{1}{2}$,$1\frac{2}{3}$)}. $1\frac{2}{3}$ ft.

36–14 x/1 ~ 9.61/48.05.
{$\frac{1}{5}$}, or {.2}

36–15 $\frac{4}{5}$/1 ~ $x/60\frac{3}{10}$.
{$48\frac{6}{25}$}, or {48.24}

36–16 .012/1 ~ .3/x.
{25}

ecking up

st 130

6–1 125

6–2 $\frac{1}{16}$

6–3 1

6–4 $^{-}243$

6–5 1

6–6 $\frac{1}{8}$

6–7 $-\frac{243}{32}$

6–8 $\frac{1}{1000}$

6–9 3720

6–10 4.8

6–11 706,000

236–12 .0002175

236–13 1,008,000

236–14 .0000989

236–15 6×10^{8}

236–16 3.1×10^{3}

236–17 4.5×10^{0}

236–18 5.7×10^{-1}

236–19 4.64×10^{-3}

236–20 1.04×10^{-4}

236–21 $6(10^{3}) + 1(10^{2}) + 4(10^{1}) + 5(10^{0}) + 0(10^{-1}) + 3(10^{-2})$

236–22 $1(10^{-1}) + 9(10^{-2}) + 0(10^{-3}) + 7(10^{-4}) + 2(10^{-5}) + 8(10^{-6})$

236–23 $6(10^{4}) + 0(10^{3}) + 8(10^{2}) + 9(10^{1}) + 1(10^{0}) + 2(10^{-1}) + 0(10^{-2}) + 7(10^{-3})$

Test 131

236–24 $(\frac{5}{6})^{9}$

236–25 $(^{-}12)^{1}$

236–26 $(^{-}\frac{7}{8})^{-4}$

236–27 28^{7}

236–28 $(^{-}6)^{4}$

236–29 10^{-7}

236–30 2.48×10^{7}

236–31 9.6×10^{10}

236–32 2.7×10^{2}

236–33 9.6×10^{3}

236–34 5×10^{-6}

236–35 4.81×10^{5}

236–36 3×10^{5}

236–37 3×10^{-4}

Lesson 146

238–A .6/1 ~ 4.2/x. * R_p

238–B The definition of equivalent rate pairs tells you that, since .6/1 is equivalent to 4.2/x, the product of the first component of .6/1 and the second component of 4.2/x is equal to the product of the first component of 4.2/x and the second component of .6/1. The product of .6 and x is .6x. The product of 4.2 and 1 is 4.2.

238–C Yes

239–D (Column 1) 1/.6 * Yes * 1/.6

239–E (Column 1) From condition B, .6x is equal to 4.2. Using the well-defined property of multiplication, .6x may be replaced in the product (1/.6)(.6x) by 4.2 without affecting the product.

239–F (Column 1) Yes * Yes

239–G (Column 1) Associative property * Yes

239–H Definition of multiplicative inverse * Yes

239–I 7

239–J From the identity-element property of multiplication, $1x = x$. Therefore, condition F can be obtained from condition E by using the identity-element property of multiplication.

239–K 7 * Condition F was obtained from condition A by using definitions and properties. * {7}

239–A $m - 3$

239–B Yes. The difference of m and 3 is the same as the sum of m and the additive inverse of 3. $^{-}(3)$ is the additive inverse of 3. * Yes. $^{-}(3)$ is the same as $^{-}3$.

239–C Condition H can be obtained from condition G by using the difference property of rational numbers.

239–D (Column 2) $^+3$ * Yes

239–E (Column 2) The associative property of addition

239–F (Column 2) Yes * Since $^-3 + {}^+3$ is 0, condition K is the same as condition J. * Yes

239–G (Column 2) Yes * Identity-element property of addition * $^+2$ * Yes

240–H $^+2$

240–I $\{^+2\}$

240–J Yes * The difference of $\frac{3}{2}$ and x is the same as the sum of $\frac{3}{2}$ and the additive inverse of x. ^-x is the additive inverse of x.

240–K Condition N can be obtained from condition M by using the difference property of rational numbers.

240–L $0 + {}^-x$ is the same as ^-x.

240–M $-\frac{3}{2}$ * Yes

240–N From condition N, $\frac{3}{2} + {}^-x$ is the same as 5. Therefore, by the well-defined property of addition, $(\frac{3}{2} + {}^-x)$ can be replaced in $(^-\frac{3}{2}) + (\frac{3}{2} + {}^-x)$ by 5 without affecting the sum.

240–O So you can find the sum of $(^-\frac{3}{2})$ and $\frac{3}{2}$. * Associative property of addition

240–P Yes * Condition Q was obtained from condition P by using definitions and properties of rational numbers. * Yes

240–Q Yes * Identity-element property of addition * $\frac{7}{2}$

240–R $-\frac{7}{2}$ * $-\frac{7}{2}$

240–S $\{-\frac{7}{2}\}$

240–A R

241–B $^-2$ * Yes

241–C Condition B can be obtained from condition A by using the well-defined property of addition.

241–D Yes * Associative property of addition

241–E Use the definition of the additive inverse. $2 + {}^-2$ is the same as 0. * Use the identity-element property of addition and the definition of the sum of two rational numbers. $5y + 0$ is the same as $5y$. $^-13 + {}^-2$ is the same as $^-15$.

241–F $\frac{1}{5}$ * Yes

241–G From condition E, $5y$ is the same as $^-15$. Using the well-defined property of multiplication, $5y$ may be replaced in $\frac{1}{5}(5y)$ by $^-15$ without affecting the product.

241–H Associative property * Use the definition of the reciprocal. $\frac{1}{5} \cdot 5$ is the same as 1. * From the identity-element property of multiplication, $1y$ is the same as y. From the definition of the product of two rational numbers, $\frac{1}{5}(^-15)$ is the same as $^-3$.

241–I $^-3$ * Yes

241–J $\{^-3\}$

241–K $\{^-81\}$ *

$\frac{1}{3}a + 7 = {}^-20$.

$(\frac{1}{3}a + 7) + (^-7) = {}^-20 + (^-7)$.

$\frac{1}{3}a + (7 + {}^-7) = {}^-27$.

$\frac{1}{3}a + 0 = {}^-27$.

$\frac{1}{3}a = {}^-27$.

$3(\frac{1}{3}a) = 3(^-27)$.

$(3 \cdot \frac{1}{3})a = {}^-81$.

$1a = {}^-81$.

$a = {}^-81$.

On your own

242–1 True statement

242–2 False statement

242–3 False statement

242–4 True statement

242–5 False statement

242–6 False statement

242–7 True statement

242–8 True statement

242–9 True statement

242–10 False statement

242–11 $x + 4.6 = {}^-3.9$.

$(x + 4.6) + {}^-4.6 = {}^-3.9 + {}^-4.6$.

Well-defined property of addition.

$x + (4.6 + {}^-4.6) = {}^-8.5$.

Associative property of addition.

$x + 0 = {}^-8.5$.

Definition of additive inverse.

$x = {}^-8.5$.

Identity-element property of addition * $\{^-8.5\}$

242–12 $\frac{1}{2}x = \frac{3}{5}$.

$2(\frac{1}{2}x) = 2 \cdot \frac{3}{5}$.

Well-defined property of multiplication.

$(2 \cdot \frac{1}{2})x = \frac{6}{5}$.

Associative property of multiplication.

$1x = \frac{6}{5}$.

Reciprocal property.

$x = \frac{6}{5}$.

Identity-element property of multiplication * $\{\frac{6}{5}\}$

242–13 $x - 17 = 32$.

$x + {}^-17 = 32$.

Difference property.

$(x + {}^-17) + 17 = 32 + 17$

Well-defined property of addition.
$x + (^{-}17 + 17) = 49.$
Associative property of addition.
$x + 0 = 49.$
Definition of additive inverse.
$x = 49.$
Identity-element property of addition * {49}

242–14 $^{-}\frac{3}{4} + x = ^{-}\frac{5}{2}.$
$\frac{3}{4} + (^{-}\frac{3}{4} + x) = \frac{3}{4} + ^{-}\frac{5}{2}.$
Well-defined property of addition.
$(\frac{3}{4} + ^{-}\frac{3}{4}) + x = ^{-}\frac{7}{4}.$
Associative property of addition.
$0 + x = ^{-}\frac{7}{4}.$
Definition of additive inverse.
$x = ^{-}\frac{7}{4}.$
Identity-element property of addition * $\{^{-}\frac{7}{4}\}$

242–15 $6 - x = \frac{2}{3}.$
$6 + ^{-}x = \frac{2}{3}.$
Difference property.
$^{-}6 + (6 + ^{-}x) = ^{-}6 + \frac{2}{3}.$
Well-defined property of addition.
$(^{-}6 + 6) + ^{-}x = ^{-}\frac{16}{3}.$
Associative property of addition.
$0 + ^{-}x = ^{-}\frac{16}{3}.$
Definition of additive inverse.
$^{-}x = ^{-}\frac{16}{3}.$
Identity-element property of addition.
$x = ^{+}\frac{16}{3}.$
Definition of additive inverse * $\{\frac{16}{3}\}$

242–16 $4x + 3 = ^{-}17.$
$(4x + 3) + ^{-}3 = ^{-}17 + ^{-}3.$
Well-defined property of addition.
$4x + (3 + ^{-}3) = ^{-}20.$
Associative property of addition.
$4x + 0 = ^{-}20.$
Additive-inverse property.
$4x = ^{-}20.$
Identity-element property of addition.
$\frac{1}{4}(4x) = \frac{1}{4} \cdot ^{-}20.$
Well-defined property of multiplication.
$(\frac{1}{4} \cdot 4)x = ^{-}5.$
Associative property of multiplication.
$1x = ^{-}5.$
Reciprocal property.
$x = ^{-}5.$
Identity-element property of multiplication * $\{^{-}5\}$

242–17 $\{^{-}42\}$
242–18 {1.57}
242–19 {8}
242–20 $\{4\frac{1}{3}\}$
242–21 $\{^{-}\frac{19}{3}\}$, or $\{^{-}6\frac{1}{3}\}$
242–22 $\{1\frac{1}{6}\}$
242–23 {15.3}
242–24 $\{^{-}\frac{5}{4}\}$, or $\{^{-}1\frac{1}{4}\}$
242–25 $\{^{-}\frac{19}{20}\}$
242–26 {11}
242–27 $\{^{-}10\frac{1}{12}\}$
242–28 {26}
242–29 {19}
242–30 {76.3}
242–31 {4}
242–32 $\{^{-}48\}$
242–33 {.9}
242–34 {12}
242–35 {.92}
242–36 $\{^{-}2\}$

Lesson 147

242–A R * $\{^{-}3\}$
242–B No
242–C Yes * No * Yes * No
242–D No
242–E Yes * Yes * Yes
242–F Yes
242–G Yes * Yes
242–H Yes * Yes
242–I Yes
243–J $^{-}\frac{11}{4} > ^{-}3$ is a true statement. * Yes * $^{-}3 < ^{-}\frac{11}{4}$ and $^{-}\frac{11}{4} < ^{-}\frac{5}{2}$.
243–K Suppose that a is the least member of $\{x \mid x > ^{-}3\}$. From the density property of the set of rational numbers, there is a rational number r such that $^{-}3 < r$ and $r < a$. Since $^{-}3 < r$, r is a member of $\{x \mid x > ^{-}3\}$ and r is also less than a. This means that a, or any rational number you choose, cannot be the least member of $\{x \mid x > ^{-}3\}$.
243–L $3x - 4 = 2.$
243–M {2}
243–N No * No
243–O Yes
243–P $\{x \mid x < 2\}$
243–Q Yes * Because of the density property of the set of rational numbers
243–R $4 - x = 5.$
243–S $\{^{-}1\}$
243–T No * No * No
243–U No
243–V Yes * Yes * Yes
243–W Yes
243–X $\{x \mid x < ^{-}1\}$
243–Y Yes
243–A R
243–B By the horizontal bar above the picture of the number line
243–C No * Yes
243–D No
243–E No

244–F $^{-}\frac{3}{4} - y < 4$. $y + \frac{1}{4} < {}^{-}2$. * The connective "and" * R

244–G $^{-}\frac{3}{4} - y = 4$. * $\{^{-}4\frac{3}{4}\}$, or $\{^{-}\frac{19}{4}\}$

244–H Yes

244–I $\{y \mid y < {}^{-}2\frac{1}{4}\}$

244–J The solution set of the compound condition is $\{y \mid y > {}^{-}\frac{19}{4} \wedge y < {}^{-}\frac{9}{4}\}$. $^{-}\frac{19}{4} < y < {}^{-}\frac{9}{4}$ means the same as $y > {}^{-}\frac{19}{4} \wedge y < {}^{-}\frac{9}{4}$.

244–K By the horizontal bar above the picture of the number line

244–L $4.5 + x > 3.6$. $7.2 - x = 8.1$. * The connective "or"

244–M The solution set of a compound condition that consists of two simple conditions and the connective "or," contains each solution of either, or both, simple conditions.

244–N $\{x \mid x > {}^{-}.9\}$ * $\{x \mid x = {}^{-}.9\}$

244–O $x \geqq {}^{-}.9$ is the same condition as $x > {}^{-}.9 \vee x = {}^{-}.9$. The symbol "$\geqq$" means "greater than or equal to."

244–P Yes * By the vertical stroke through the dot for point $^{-}.9$

244–Q Yes * By the horizontal bar above the picture of the number line

244–R The condition that it is not the case that $4m - 7 > 5$. * $\overline{R}_n$. This is the set of non-negative rational numbers.

244–S $\{m \mid m > 3\}$

244–T Yes. Only those members of $\overline{R}_n$ that are greater than 3, satisfy $4m - 7 > 5$. Therefore, each non-negative rational number that is less than or equal to 3 does not satisfy $4m - 7 > 5$. * $m > 3$ is equivalent to $4m - 7 > 5$. Therefore, $\sim (m > 3)$ is equivalent to $\sim (4m - 7 > 5)$.

245–U By the horizontal bar above the picture of the number line * These dots are not in the graph of the universe, $\overline{R}_n$.

245–V The dot for point 3 * Yes

245–W All the dots for points associated with members of $\overline{R}_n$ that are to the left of the dot for point 3 * Yes

245–X Yes * Yes * Yes

On your own

245–1 True statement

245–2 False statement

245–3 True statement

245–4 False statement

245–5 True statement

245–6 True statement

245–7 False statement

245–8 $\{a \mid a < 29\}$

245–9 $\{b \mid b < 6.7\}$

245–10 $\{m \mid m < 7\}$

245–11 $\{x \mid x > 8\}$

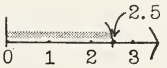
245–12 $\{x \mid \sim(x < 3\frac{7}{8})\}$, or $\{x \mid x \geqq 3\frac{7}{8}\}$

245–13 $\{x \mid \sim(x < 4)\}$, or $\{x \mid x \geqq 4\}$

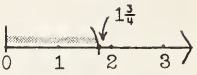
245–14 $\{w \mid {}^{-}2\frac{3}{4} < w < 4\frac{2}{5}\}$

245–15 $\{y \mid y \geqq \frac{3}{4}\}$

245–16



245–17



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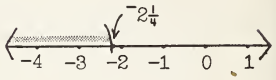

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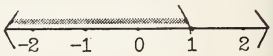

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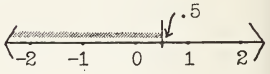
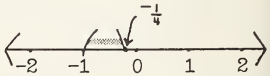
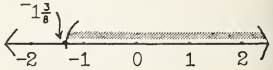


245–21



245–22



Lesson 148

245–A 10 – 15 refers to the temperature at 7 P.M.

245–B Yes

246–C Use the difference property. * Yes

246–D $^{-}5$ * Yes

246–E $\{^{-}5\}$

246–F The average number of miles Mr. Carson travell per hour * A number of hours

246–G How many hours it took Mr. Carson to travel 153.6 miles * The first component of 57.6/1 refers to a number of miles. Therefore, the

first component of the rate pair whose second component you are to find must also refer to a number of miles. 153.6 refers to a number of miles. * The number of hours it took Mr. Carson to travel 153.6 miles

246–H Yes

246–I Use the definition of equivalent rate pairs.

246–J You should find the quotient of 153.6 and 57.6.

246–K When you divide 153.6 by 57.6, the second remainder is the same as the first remainder. Therefore, the second digit obtained in the decimal for the quotient will repeat infinitely. This is indicated by the decimal $2.\overline{6}$.

246–L $\{2.\overline{6}\}$ * $\{2.\overline{6}\}$

246–M It took Mr. Carson 2.7 hours to travel the 153.6 mi.

246–N Yes

246–O The distance from the earth to the sun in miles * The distance from the earth to the moon in miles

246–P The rate pair $(9.3 \times 10^7)/(2.4 \times 10^5)$ represents the comparison of the distance from the earth to the sun with the distance from the earth to the moon. The rate pair $m/1$, and not $1/m$, represents this same comparison. The rate pair $1/m$ represents the comparison of the distance from the earth to the moon with the distance from the earth to the sun.

247–Q $m/1$ is equivalent to $(9.3 \times 10^7)/(2.4 \times 10^5)$.

247–R $(9.3 \times 10^7) = (2.4 \times 10^5)m$ can be obtained from $(9.3 \times 10^7)/(2.4 \times 10^5) \sim m/1$ by using the definition of equivalent ordered pairs.

247–S Yes

247–T 3.875×10^2

247–U Yes * $\{3.875 \times 10^2\}$

247–V 387.5

247–A When rational numbers refer to elevations, it is generally agreed that a negative rational number indicates a number of feet below sea level. Since Swamp City is 5 feet below sea level, $^-5$ represents the elevation of Swamp City.

247–B No

247–C Yes

247–D $3440 * 3440 - n$ represents the difference of the elevation of West City (3440) and the elevation of Silver City (n). The problem states that this difference is greater than 500.

247–E Yes

247–F The connective "and" * Each member of the solution set must satisfy both simple conditions.

247–G Each member of $\{n \mid n < 2940\}$ satisfies $3440 - n > 500$. * No

247–H No * There are rational numbers that are less than 2940 but are not greater than $^-5$.

247–I Yes. The condition $^-5 < n < 2940$ is the same as $^-5 < n \wedge n < 2940$. Since $^-5 < n$ also means $n > {}^-5$ and $n < 2940$ is equivalent to $3440 - n > 500$, the condition $^-5 < n < 2940$ is the same as the condition $n > {}^-5 \wedge 3440 - n > 500$.

248–J Yes

248–K Since the first component of .45/1 refers to an amount of money the oil can cost, the first component of the rate pair you are to find must also refer to an amount of money that the oil can cost. * 5

248–L Yes

248–M Yes

248–N Both .60/1 and $n/5$ represent the same rate and are, therefore, equivalent. * n is the variable for the amount that 5 quarts of oil can cost.

248–O Yes

248–P The union * The solution set of a compound condition that includes two simple conditions and the connective "or," contains all those numbers that satisfy either, or both, of the simple conditions.

248–Q 2.25 * 3.00

248–R {2.25, 3.00}

248–S Either $2.25 or $3.00

248–A $^-3.2 + x < 6.4$ requires that the sum of $^-3.2$ and each x be less than 6.4. Therefore, $\sim(^-3.2 + x < 6.4)$ requires that it is not the case that the sum

of $^-3.2$ and each x be less than 6.4.

248–B No * No

248–C Yes

248–D Yes. Each member that is either greater than or equal to 9.6 is also not less than 9.6. * Each member of $\{x \mid x \geq 9.6\}$ satisfies $\sim(^-3.2 + x < 6.4)$.

248–E Yes

On your own

249–1 x, the temperature in degrees at noon.
$x - {}^-3 = 10$.
$\{7\}$. $^+7$ degrees, or 7 degrees above zero

249–2 x, number of yards gained on the two downs.
$8 + {}^-5 = x$.
$\{3\}$. 3 yd.

249–3 x, lowest temperature in degrees recorded in Quebec that year.
$x - {}^-81.4 = 22.5$.
$\{^-58.9\}$. $^-58.9$ degrees, or 59 degrees below zero

249–4 $^-14\frac{2}{5} + x > 15\frac{7}{10}$.
$\{x \mid x > 30\frac{3}{10}\}$

249–5 $3(6.4) + x = 19.7$.
$\{.5\}$

249–6 $\sim(3\frac{1}{3}x < {}^-16\frac{1}{2})$
$\{x \mid x \geq {}^-4\frac{19}{20}\}$

249–7 $x < 20.5 \wedge 2 - x < {}^-10.7$.
$\{x \mid 12.7 < x < 20.5\}$

249–8 $\sim(9\frac{1}{2} - x > 4)$.
$\{x \mid x \geq 5\frac{1}{2}\}$

249–9 x, number of revolutions the crankshaft makes in 60 minutes.
$(2.6 \times 10^3)/1 \sim x/60$, or $(2.6 \times 10^3)/1 \sim x/(6 \times 10^1)$.
$\{1.56 \times 10^5\}$.
1.56×10^5 revolutions

249–10 x, fraction the height the rocket reached was of the expected height.
$(2.52 \times 10^5)/(2.8 \times 10^5) \sim x/1$.
$\{.9 \times 10^0\}$, or $\{.9\}$. The height the rocket reached was $\frac{9}{10}$ of the expected height.

249–11 x, number of tons of steel produced in Canada in a recent year.
$27/100 \sim x/(5.50 \times 10^6)$, or $(2.7 \times 10^1)/10^2 \sim x/(5.50 \times 10^6)$.
$\{1.485 \times 10^6\}$.
1.485×10^6 tons

249–12 x, amount of interest Mr. Frank will pay.
$6/100 \sim x/600 \vee 5\frac{1}{2}/100 \sim x/600$.
$\{36, 33\}$. Either \$36 or \$33

249–13 x, number of square yards of carpeting Mrs. Connor used.
$0 < x < 17\frac{2}{3} \wedge 17\frac{2}{3} - x < 4$.
$\{x \mid 13\frac{2}{3} < x < 17\frac{2}{3}\}$.
Between $13\frac{2}{3}$ and $17\frac{2}{3}$ square yards of carpeting

249–14 x, amount of money Mrs. Sands had in her checking account before she deposited the \$60.35.
$x < 170 \wedge x + 60.35 > 210$.
$\{x \mid 149.65 < x < 170\}$.
Between \$149.65 and \$170

249–15 x, number of gallons of paint Mr. White had before he painted the kitchen.
$x \leq 4\frac{1}{2} \wedge x - 3\frac{3}{4} > \frac{1}{8}$.
$\{x \mid 3\frac{7}{8} < x \leq 4\frac{1}{2}\}$.
More than $3\frac{7}{8}$ gal. but not more than $4\frac{1}{2}$ gal.

249–16 x, number of days Mr. Allen can spend in France.
$14\frac{1}{2} + x = 22 \vee 10 + x = 22$.
$\{7\frac{1}{2}, 12\}$.
Either $7\frac{1}{2}$ days or 12 days

Checking up

Test 132

250–1 $(\frac{23}{3}, 0)$

250–2 $8\frac{1}{4}$

250–3 $\{(x,y) \mid (x,y) \sim (0, \frac{3}{8})\}$

250–4 Well-defined property of multiplication

250–5 $^-1.\overline{6}$

250–6 $\frac{625}{256}$

250–7 9×10^{-4}

250–8 $^-5\frac{3}{4}$

250–9 $^+8.82$

250–10 $^-7.58$

250–11 24.96

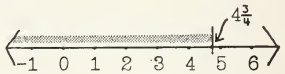
250–12 $^-\frac{5}{4}$, or $^-1\frac{1}{4}$

250–13 $^-\frac{7}{12}$

250–14 .86

250–15 $^-\frac{104}{35}$, or $^-2\frac{34}{35}$

250–16



Test 133

250–17 first

250–18 right

250–19 $\frac{3}{10}$

250–20 $\frac{225}{7}$, or $32\frac{1}{7}$

250–21 $\frac{7}{11}$

250–22 $^-\frac{7}{2}$

250–23 difference

250–24 $^-\frac{12}{11}$

250–25 $2.1\overline{6}$

250–26 .007818

250–27 associative

Test 134

250–28 F

250–29 F

250–30 T

250–31 F

250–32 T

250–33 T

250–34 T

250–35 F

250–36 T

250–37 T

250–38 F

250–39 F

Test 135

251–40 (c) $R_p \cup R_n \cup \{0\}$

251–41 (a) $^-9$

251–42 (b) R_p and $\{0\}$

251–43 (d) identity-element

251–44 (a) R

251–45 (d) $^-\frac{8}{7}$

251–46 (a) $\frac{64}{125}$

251–47 (b) $\{x \mid x < ^-1\}$

251–48 (c) 9.380×10^3

251–49 (d) 13^{12}

251–50 (a) $\frac{1}{7}(7m) = \frac{1}{7}(\frac{13}{3})$

Test 136

251–1 Closure property of addition

251–2 Associative property of multiplication

251–3 Quotient property

251–4 Identity-element property of multiplication

251–5 Commutative property of multiplication

251–6 Associative property of addition

251–7 Difference property

Test 137

251–8 F

251–9 T

251–10 T

251–11 F

251–12 T

251–13 T

252–14 F

252–15 T

252–16 T

252–17 F

252–18 F

Test 138

252–19 (b) $^+2\frac{1}{2}$

252–20 (d) 1 point

252–21 (b) 270

252–22 (d) ||

252–23 (c) square

252–24 (b) $^-2$

252–25 (a) R_n

252–26 (a) 120

252–27 (b) $\{0\}$

252–28 (d) $\{q \mid 5 < q < 9\}$

Test 139

252–29 x, number of hours it would take Mr. Challand to paint 168 ft. of gutters.
$35/1 \sim 168/x$.
$\{4.8\}$. 4.8 hrs.

252–30 x, fraction of baskets made in 76 attempts.
$x/1 \sim 28/76$.
$\{\frac{7}{19}\}$. .368

252–31 x, number of revolutions wheel makes in 9.9×10^3 mi.
$(8.6 \times 10^2)/1 \sim x/(9.9 \times 10^3)$.
$\{8.514 \times 10^6\}$.
8.514×10^6 revolutions

252–32 x, amount of money the driveway will cost per running foot.
$520/80 \sim x/1 \vee 290/80 \sim x/1$.
$\{6.5, 8.625\}$. Either \$6.50 or \$8.625 (\$8.63)

252–33 $2.4\ x > ^-7.5 \div ^-5$.
$\{x \mid x > .625\}$

252–34 $\sim(4\frac{1}{5} + x < ^-2\frac{7}{10})$.
$\{x \mid x \geq ^-6\frac{9}{10}\}$

252–35 $x < 8\frac{1}{2} \wedge ^-\frac{2}{3} - x \geq ^-4\frac{3}{4}$.
$\{x \mid x \leq 4\frac{1}{12}\}$

Lesson 149

253–A Twice * 36 * Yes

253–B $\frac{4}{9}$ * The product of two negative numbers is positive. * Yes

253–C $\frac{16}{25}$ * 49 * Yes

253–D $\frac{16}{25}$ * 49

253–E $^-9 \times ^-9 = 81$, or $(^-9)^2 = 81$.

253–F 144

253–G 100

253–H $\frac{9}{16}$

253–I $\frac{121}{169}$

253–J $\frac{25}{64}$

254–K (Column 1) Yes * Yes

254–L The square of a number is equal to the product of a number and itself. The product of two positive numbers is positive. The product of two negative numbers is positive. Any positive number is greater than zero.

254–M 0

254–N Yes * Yes * Yes * None.

254–A R

254–B a was replaced by $\frac{2}{3}$ and b by $\frac{4}{6}$. * a was replaced by $\frac{2}{3}$ and b by $\frac{4}{6}$. * Yes

254–C Yes. $\frac{2}{3}$ and $\frac{4}{6}$ indicate the same rational number. * $(\frac{2}{3})^2 = \frac{4}{9}$. $(\frac{4}{6})^2 = \frac{16}{36}$. $\frac{4}{9} = \frac{16}{36}$. Therefore, $(\frac{2}{3})^2 = (\frac{4}{6})^2$.

254–D a was replaced by $\frac{7}{8}$ and b by $\frac{21}{24}$. * Yes

254–E Well-defined property of multiplication

254–F Using the well-defined property of multiplication, since $a = b$, you may replace both a's in $a \cdot a$ by b's without affecting the product. Therefore, $a \cdot a = b \cdot b$. Since $a \cdot a = a^2$ and $b \cdot b = b^2$, $a \cdot a = b \cdot b$ is the same as $a^2 = b^2$.

254–G a was replaced by $\frac{8}{9}$ and b by $^-\frac{8}{9}$. * a was replaced

by $\frac{8}{9}$ and b by $\bar{}\frac{8}{9}$. * Yes

254–H Yes * Yes

254–I $\frac{8}{9}$ and $\bar{}\frac{8}{9}$ are additive inverses of each other.

254–J $\bar{}20 * a$ was replaced by 20 and b by $\bar{}20$. * Yes

254–K (Column 2) They are equal. * They are equal.

255–L The squares of the two numbers are equal.

255–M The squares of the two numbers are equal.

255–N No * The two numbers may be additive inverses of each other.

255–O The set whose members are the non-negative rational numbers. This set contains 0 and each positive rational number.

255–P a was replaced by 9 and b by 11. * a was replaced by 9 and b by 11. * Yes

255–Q Yes * $9^2 = 81$.
$11^2 = 121$.
$81 < 121$.
Therefore, $9^2 < 11^2$.

255–R a was replaced by $\frac{3}{4}$ and b by $\frac{7}{8}$. * Yes

255–S The square of the first number is less than the square of the second number.

255–T The first non-negative number is less than the second non-negative number.

255–U Examples are: $\bar{}7 < 5$, but $(\bar{}7)^2 > (5)^2$. $\bar{}20 < \bar{}10$, but $(\bar{}20)^2 > (\bar{}10)^2$.

255–A $\frac{9}{25} * \frac{-7}{4}$ and $\frac{-7}{4} * (\frac{3}{5})^2 = \frac{3}{5} \times \frac{3}{5}$ and $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$. $(\frac{-7}{4})^2 = \frac{-7}{4} \times \frac{-7}{4}$ and $\frac{-7}{4} \times \frac{-7}{4} = \frac{49}{16}$.

255–B $\bar{}12 * \frac{14}{13}$

255–C $\frac{9}{25}$ is also the product of $\frac{-3}{5}$ and $\bar{}\frac{3}{5}$, or $(\frac{-3}{5})^2$.

255–D $\frac{7}{4} * 12 * \frac{14}{13}$ and $\bar{}\frac{14}{13}$

255–E You cannot obtain a negative rational number by multiplying a rational number by itself.

256–F $\frac{5}{3} * (\frac{-5}{3})^2 = \frac{25}{9}$. $(\frac{5}{3})^2 = \frac{25}{9}$.

256–G $\bar{}14 * 14^2 = 196$. $(\bar{}14)^2 = 196$.

256–H $\frac{-17}{9} * (\frac{17}{9})^2 = \frac{289}{81}$. $(\frac{-17}{9})^2 = \frac{289}{81}$.

256–I $23 * (\bar{}23)^2 = 529$.
$23^2 = 529$.

256–J $\frac{25}{32} * (\frac{-25}{32})^2 = \frac{625}{1024}$. $(\frac{25}{32})^2 = \frac{625}{1024}$.

256–K $\bar{}48 * 48^2 = 2304$.
$(\bar{}48)^2 = 2304$.

256–L Yes * Yes

256–M 0 * Yes

256–N Yes * No * 0

256–O The symbol $\sqrt{\ }$ is used to express the principal square root of a number. The principal square root of 64 is 8, not $\bar{}8$.

256–P True statement

256–Q False statement

256–R False statement

256–S False statement

256–T True statement

256–U True statement

256–V 9 * 4 * 10

256–W From the definition of square root $\sqrt{a}$ when used twice as a factor gives a as the product. Therefore, $(\sqrt{a}) \times (\sqrt{a}) = a$, or $(\sqrt{a})^2 = a$.

256–A 9 * Yes * Yes

256–B 1.782 * Yes

256–C 225 * 2401 * 9801 * 17,424 * The principal square root of 15 is 3.873 to the nearer thousandth. The principal square root of 49 is 7. The principal square root of 99 is 9.950 to the nearer thousandth. The principal square root of 132 is 11.489 to the nearer thousandth.

256–D 625 * Find the numeral 625 under the column headed by the symbol n^2. Then to the left of the numeral 625, the blue numeral in the column headed by the letter n expresses the principal square root of 625.

256–E 31 * 54 * 141 * 170

On your own

257–1 169 * 3.606

257–2 361 * 4.359

257–3 30,976 * 13.266

257–4 3844 * 7.874

257–5 5476 * 8.602

257–6 1444 * 6.164

257–7 25 * 2.236

257–8 40,000 * 14.142

257–9 3025 * 7.416

257–10 19,600 * 11.832

257–11 13,924 * 10.863

257–12 961 * 5.568

257–13 16,641 * 11.358

257–14 4900 * 8.367

257–15 36,481 * 13.820

257–16 8464 * 9.592

257–17 11,236 * 10.296

257–18 4489 * 8.185

257–19 9025 * 9.747

257–20 44,944 * 14.560

257–21 17

257–22 24

257–23 33

257–24 182

257–25 66

257–26 156

257–27 215

257–28 120

257–29 True statement

257–30 False statement

257–31 True statement

257–32 False statement

257–33 False statement

257–34 True statement

257–35 The universe for a and b is R. For each a and b, if $a = {}^{-}b$, then $a^2 = b^2$. Since $810 = {}^{-}({}^{-}810)$, $(810)^2 = ({}^{-}810)^2$.

257–36 The universe for a and b is $\overline{R}_n$. For each a and b, if $a < b$, then $a^2 < b^2$. Since $\frac{16}{15} < \frac{17}{15}$, $(\frac{16}{15})^2 < (\frac{17}{15})^2$.

257–37 From the definition of square root, $\sqrt{7}$ when used twice as a factor gives 7 as the product. Therefore, $(\sqrt{7}) \times (\sqrt{7}) = 7$, or $(\sqrt{7})^2 = 7$.

257–38 The universe for each a and b is R. For each a and b, if $a = b$, or if $a = {}^{-}b$, then $a^2 = b^2$. Since $\frac{16}{5} = \frac{32}{10}$, $(\frac{16}{5})^2 = (\frac{32}{10})^2$

257–39 The universe for a and b is $\overline{R}_n$. For each a and b, if $a^2 < b^2$, then $a < b$. Since $289 < 441$, $\sqrt{289} < \sqrt{441}$.

257–40 The universe for a and b is R. For each a and b, if $a = {}^{-}b$, then $a^2 = b^2$. Since $\frac{-2}{5} = {}^{-}(\frac{16}{40})$, $(\frac{-2}{5})^2 = (\frac{16}{40})^2$.

Keeping skilful

257–1 $\{2.53\}$

257–2 $\{41\frac{3}{4}\}$

257–3 $\{r \mid r > 2.3\}$

257–4 $\{4.1\}$

257–5 $\{1\frac{1}{2}\}$

257–6 $\{h \mid h \leq 5\frac{2}{5}\}$

257–7 $\{t \mid 21\frac{7}{10} < t < 24\frac{3}{10}\}$

257–8 $\{c \mid c > 8.8\}$

257–9 $\{\ \}$

257–10 $\{m \mid m \leq {}^{-}1\}$

Lesson 150

257–A 6 * 6

257–B 13 * 13

257–C 22 * 22

257–D Yes * Yes

257–E 25 * 25

257–F 78 * 78

257–G Yes. The universe for x and y is I_p. For each x and y, $xy \in I_p$. * The square of an element of I_p is the product of two elements of I_p where the two elements are the same. Since I_p is closed under multiplication, the square of any element of I_p is also a member of I_p.

258–H (Column 1) $49 = 7^2$ and $7 \in I_p$. * $400 = 20^2$ and $20 \in I_p$.

258–I (Column 1) Yes

258–J No * No

258–K Perfect square. $64 = 8^2$ and $8 \in I_p$.

258–L Perfect square. $121 = 11^2$ and $11 \in I_p$.

258–M Not a perfect square. $121 < 136 < 144$. $121 = 11^2$ and $144 = 12^2$.

258–N Perfect square. $225 = 15^2$ and $15 \in I_p$.

258–O Not a perfect square. $49 < 54 < 64$. $49 = 7^2$ and $64 = 8^2$.

258–A $1 * 4 * 1 < 2 < 4$. $1 = 1^2$ and $4 = 2^2$.

258–B Yes

258–C Yes * Yes

258–D The universe for a and b is $\overline{R}_n$. For each a and b, if $a^2 < b^2$, then $a < b$. Since $2 < (1.5)^2$, $\sqrt{2} < 1.5$. * 1.96 * Yes

258–E Yes * You know that $1.4 < \sqrt{2}$ and $\sqrt{2} < 1.5$. This is expressed by the sentence $1.4 < \sqrt{2} < 1.5$.

258–F Because, so far, you know that $\sqrt{2}$ is between 1.4 and 1.5

258–G Yes * Yes * Since $1.9881 < 2$, $\sqrt{1.9881} < \sqrt{2}$.

258–H (Column 2) 1.41 is less than $\sqrt{2}$ so you should choose a number that is greater than 1.41 but less than 1.5 for your next estimate of $\sqrt{2}$. Since $(1.41)^2$ is close to 2, you should choose a number close to 1.41. * 2.0164

258–I (Column 2) Yes. $(1.42)^2 = 2.0164$. Since $2 < 2.0164$, $\sqrt{2} < \sqrt{2.0164}$, or $\sqrt{2} < 1.42$. * Yes. $(1.41)^2 < 2 < (1.42)^2$. Therefore, $1.41 < \sqrt{2} < 1.42$.

259–J Yes * Since $2 < 2.002225$, $\sqrt{2} < \sqrt{2.002225}$, or $\sqrt{2} < 1.415$.

259–K 1.415 is a little greater than $\sqrt{2}$, so your next estimate of $\sqrt{2}$ should be a little less than 1.415. * 1.999396 * You already know that $\sqrt{2} < 1.415$. Since $1.999396 < 2$, $\sqrt{1.999396} < \sqrt{2}$, or $1.414 < \sqrt{2}$. Therefore, $1.414 < \sqrt{2} < 1.415$.

259–L Because $\sqrt{2}$ is between 1.414 and 1.415

259–M 1.414 * 1.4142

259–N Yes * Yes * Yes.

259–O Each rational number can be expressed by a repeating decimal.

259–A Yes * Yes

259–B Yes * Since $12^2 < 152 < 13^2$, $12 < \sqrt{152} < 13$.

259–C If you multiply both the dividend and the divisor by the same number, the quotient remains the same.

259–D The dividend should be expressed by the numeral 1520.000.

259–E Yes * 12.160

259–F The quotient of 152 and 12.5 is not 12.5.

259–G Yes * Yes * When you divide 152 by 12.5, you obtain the quotient 12.160, which is less than 12.5. This means that 12.5 is greater than $\sqrt{152}$ and 12.160 is less than $\sqrt{152}$.

260–H Yes * Yes

260–I Yes * 12.328

260–J Yes * When you divide 152 by 12.330, you obtain the quotient 12.328, which is less than 12.330. This means that 12.330 is greater than $\sqrt{152}$ and 12.328 is less than $\sqrt{152}$.

260–K $(12.328 + 12.330) \div 2 = 24.658 \div 2$. $24.658 \div 2 = 12.329$. * Yes

260–L Yes

260–M 44 is between the perfect squares of 36 and 49. Therefore, $\sqrt{36} < \sqrt{44} < \sqrt{49}$, or $6 < \sqrt{44} < 7$. There is no integer between 6 and 7. * 6 and 7

260–N Yes * $\sqrt{44}$ will be between 6.5 and the quotient of 44 and 6.5. This will enable you to get a better approximation of $\sqrt{44}$ than 6.5.

260–O When you divide 44 by 6.5, you obtain the quotient 6.769, which is greater than 6.5. This means that 6.5 is less than $\sqrt{44}$ and 6.769 is greater than $\sqrt{44}$. * 13.269 * 6.634

260–P Yes * 6.632

260–Q When you divide 44 by 6.634, you obtain the quotient 6.632, which is less than 6.634. This means that 6.634 is greater than $\sqrt{44}$ and 6.632 is less than $\sqrt{44}$. * Yes

260–R $(6.633)^2 = 43.996689$. * For most work, 6.633 is a reasonably good approximation of $\sqrt{44}$.

260–S 3.742

260–T 9.220

260–U 13.266

260–V 2.449

On your own

260–1 Not a perfect square * 13.379

260–2 Perfect square. $196 = 14^2$.

260–3 Not a perfect square * 14.142

260–4 Perfect square. $484 = 22^2$.

260–5 Perfect square. $900 = 30^2$.

260–6 Not a perfect square * 16.583

260–7 Perfect square. $289 = 17^2$.

260–8 Not a perfect square * 8.367

260–9 Perfect square. $1225 = 35^2$.

260–10 Not a perfect square * 19.105

260–11 Perfect square. $6400 = 80^2$.

260–12 Perfect square. $256 = 16^2$.

260–13 Not a perfect square * 24.698

260–14 Not a perfect square * 14.526

260–15 Perfect square. $2500 = 50^2$.

260–16 Perfect square. $10{,}000 = 100^2$

Lesson 151

261–A (Column 1) Yes * Yes * The repetend of the decimal $1.5\overline{0}$ is 0. The repetend of $^-.125\overline{0}$ is 0. The repetend of $2.\overline{3}$ is 3. The repetend of $.\overline{09}$ is 09. The repetend of $^-14.\overline{0}$ is 0. The repetend of $^-3.\overline{857142}$ is 857142.

261–B (Column 1) $1\frac{1}{2}$, $^-\frac{1}{8}$, and $^-14$

261–C (Column 1) $1.75\overline{0}$

261–D (Column 1) $^-.\overline{6}$

261–E $^-6.\overline{0}$

261–F $.\overline{63}$

261–G $^-15.1\overline{6}$

261–H $.6\overline{0}$

261–I $5.\overline{142857}$

261–J $^-.00\overline{10}$

261–K $^-.\overline{6428571}$

261–L $.4\overline{6}$

261–M Yes * Yes

261–A (Column 2) The three dots in numeral G indicate that the digits in the decimal for the number named go on and on but that the decimal is not a repeating decimal. * No. The decimal for $\sqrt{2}$ is an infinite decimal. $\sqrt{2}$ cannot be named by a terminating decimal.

261–B (Column 2) By the three dots in the numeral * No. $\sqrt{3}$ cannot be named by a terminating decimal.

261–C (Column 2) By the three dots in the numeral * 1 * 2 * 3 * 4

261–D (Column 2) 5 * .10100100010000100000100000000100...

262–E No

262–F The first digit is 3 and the second digit is 2. * The third and fourth digits are the digit 3, the fifth and sixth digits are the digit 2. * The next three digits are the digit 3 and the next three are the digit 2.

262–G 33332222 * The three dots in the numeral tell you that numeral J is an infinite decimal but is not a repeating decimal. * No

262–H Repeating infinite decimal.

262–I Repeating infinite decimal

262–J Repeating infinite decimal

262–K Repeating infinite decimal

262–L Nonrepeating infinite decimal

262–M Nonrepeating infinite decimal

262–N An irrational number can be expressed by a nonrepeating infinite decimal. Any number that can be named by an infinite decimal is a real number. * Yes * Yes

262–O Elements of R are $^{-}2.1\overline{6}$, $.\overline{142857}$, $15.650\overline{0}$ and $^{-}3.0\overline{9}$. * Elements of $\overline{R}$ are 2.2361... and $^{-}.2468101214....$

263–P $\overline{R}$ is an infinite set. * There are infinitely many digits in a nonrepeating infinite decimal. Each of these digits can be replaced to form a nonrepeating infinite decimal for another member of $\overline{R}$.

263–Q Yes * Each of the sets R and $\overline{R}$ is an infinite set.

263–R The portion of the diagram that represents the rectangle and its interior

263–S The portion of the diagram that is not shaded

263–T The portion of the diagram that represents R_p does not overlap the portion of the diagram that represents R_n. * The portion of the diagram that represents R_p and the portion of the diagram that represents R_n are completely within the portion of the diagram that represents R. * No * Yes

263–U R_p, R, and D

263–V R_n, R, and D

263–A Yes * Yes * Yes

263–B Since $1^2 < 2 < 2^2$, $1 < \sqrt{2} < 2$. * The symbol $\sqrt{2}$ is written between the symbols 1 and 2.

263–C Yes * There is an arrow pointing to the approximate location of point $\sqrt{2}$. This indicates that point $\sqrt{2}$ is between point 1.4 and point 1.5.

263–D Yes * Since $1.41 < \sqrt{2} < 1.42$, the point for $\sqrt{2}$ is between the points for 1.41 and 1.42.

264–E There is an arrow that points to the approximate location of point $\sqrt{2}$ between point 1.41 and 1.42.

264–F The subset of ℓ_1 that contains 1.41 and 1.42 and all points between 1.41 and 1.42 * Yes * By an arrow that points to the approximate location of point $\sqrt{2}$

264–G Yes * The dot for $\sqrt{2}$ is between the dot for 1.4142 and the dot for point 1.4143.

264–H $1.7 < \sqrt{3} < 1.8$. * $1.73 < \sqrt{3} < 1.74$. * $1.732 < \sqrt{3} < 1.733$.

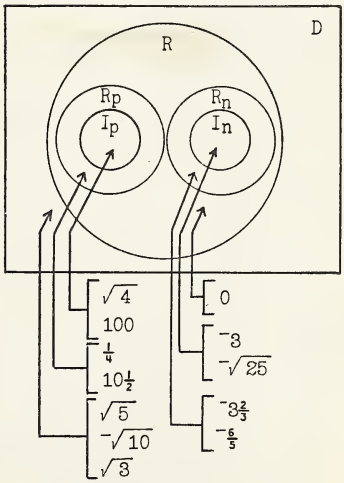
265–I The negative square root of three

265–J Yes * Yes

265–K Points $^{-}2$, $^{-}1.5$, $^{-}1$, $^{-}.5$, 0, .5, 1, 1.5, and 2 * Points $^{-}\sqrt{3}$, $^{-}\sqrt{2}$, $\sqrt{2}$, and $\sqrt{3}$

On your own

265–I See the diagram below.



265–2 See diagram for 207–1.

265–3 See diagram for 207–1.

265–4 See diagram for 207–1.

265–5 See diagram for 207–1.

265–6 See diagram for 207–1.

265–7 See diagram for 207–1.

265–8 See diagram for 207–1.

265–9 See diagram for 207–1.

265–10 See diagram for 207–1.

265–11 See diagram for 207–1.

265–12 See diagram for 207–1.

265–13 3 and 4

265–14 $^-6$ and $^-5$

265–15 10 and 11

265–16 20 and 21

265–17 $^-5$ and $^-4$

265–18 $^-18$ and $^-17$

265–19 64 and 65

265–20 $^-26$ and $^-25$

265–21 110 and 111

Lesson 152

266–A The closure property of addition * Yes * Yes

266–B The closure property of multiplication * Yes * Yes

266–C 2.236 is an approximation, to the nearer thousandth, of $\sqrt{5}$ * Yes

266–D 3.090 * 1.174824

266–E The commutative property of addition

266–F The commutative property of multiplication

266–G Addition * Multiplication

266–H 0 * 1

266–I Inverse property of addition, or additive-inverse property

267–J $^-5$ * .32

267–K $^-\sqrt{48}$ * $\sqrt{6}$ * $\sqrt{112}$

267–L Each real number has an additive inverse. The difference of two real numbers is the same as the sum of the minuend and the additive inverse of the subtrahend. The sum of any two real numbers is a real number.

267–M Inverse property of multiplication, or reciprocal property

267–N $1/\sqrt{16}$ * $\sqrt{16}$ * 1/4.5 * 4.5

267–O $1/\sqrt{145}$ * $1/^-\sqrt{32}$

267–P Each non-zero real number has a reciprocal. The quotient of any two non-zero real numbers is the same as the product of the dividend and the reciprocal of the divisor. The product of any two real numbers is a real number.

267–Q The distributive property of multiplication over addition * Yes

267–R Yes * The set of real numbers is closed, commutative, and associative under both addition and multiplication and multiplication distributes over addition.

267–A No. 0 is not a perfect square. 16, 49, and 144 are perfect squares. * $\sqrt{0} = 0$. $\sqrt{16} = 4$. $\sqrt{49} = 7$. $\sqrt{144} = 12$. * Yes

267–B Yes * No * No * No

267–C It is not true that the square root of each non-negative rational number is a non-negative rational number.

267–D Yes * Yes * Yes

268–E $^-65 = ^-65$. $107 = 107$. $789 = 789$. (Statements B, C, and F) * No. Inequalities are true statements. $32 < 32$, $^-354 > ^-354$, and $96 \neq 96$ are false statements. * Yes

268–F The universe for a is D. For each a, it is not true that $a < a$. For each a, it is not true that $a > a$. For each a, it is not true that $a \neq a$.

268–G Yes * Yes * Yes * Yes * Yes

268–H The universe for a and b is D. For each a and b, if $a < b$, then it is not true that $b < a$. For each a and b, if $a > b$, then it is not true that $b > a$.

268–I Yes

268–J 5.75 * $\frac{23}{4}$ * Yes. 5.75 is the same real number as $5\frac{3}{4}$, or $\frac{23}{4}$. * Yes

268–K 8.125 * $\frac{65}{8}$ * Yes. 8.125 is the same real number as $8\frac{1}{8}$, or $\frac{65}{8}$. * Yes

268–L The universe for a, b, c, and d is D. For each a, b, c, and d, if $a = b$ and $c = d$, then $a + c = b + d$. Since $7.9 = \frac{79}{10}$ and $8.5 = \frac{17}{2}$, $7.9 + 8.5 = \frac{79}{10} + \frac{17}{2}$. * The universe for a, b, c, and d is D. For each a, b, c, and d, if $a = b$ and $c = d$, then $ac = bd$. Since $7.9 = \frac{79}{10}$ and $8.5 = \frac{17}{2}$, $7.9 \times 8.5 = \frac{79}{10} \times \frac{17}{2}$.

On your own

269–1 False statement

269–2 True statement. Commutative property of addition

269–3 False statement

269–4 True statement. Commutative property of multiplication

269–5 True statement. Identity-element property of multiplication

269–6 False statement

269–7 True statement. Distributive property of multiplication over addition

269–8 False statement

269–9 True statement. Identity-element property of addition

269–10 True statement. Inverse property of addition along with the definition of additive inverse

269–11 Closure property of the operation of finding the square root of a non-negative real number. Closure property of addition

269–12 Closure property of the operation of finding the square root of a non-negative real number. Closure property of multiplication

269–13 Closure property of the operation of finding the square root of a non-negative real number. Quotient property. Definition of reciprocal. Closure property of multiplication

269–14 Reflexive property

269–15 Yes * Symmetric property

269–16 Using the symmetric property; since $2 \times \sqrt{625} = \sqrt{2500}$, $\sqrt{2500} = 2 \times \sqrt{625}$. Using the transitive property; since $50 = \sqrt{2500}$ and $\sqrt{2500} = 2 \times \sqrt{625}$, $50 = 2 \times \sqrt{625}$.

269–17 Yes * Yes * Well-defined property of addition and the closure property of finding the square root of a non-negative real number * Well-defined property of multiplication and the closure property of finding the square root of a non-negative real number

269–18 $\sqrt{141}$ is not a rational number. 11.874 is a rational approximation of $\sqrt{141}$. 111.874 is a rational approximation of $\sqrt{10000} + \sqrt{141}$. * 1187.4 is a rational approximation of $\sqrt{10000} \times \sqrt{141}$.

269–19 $\sqrt{9801} + \sqrt{576} = 123$. * $\sqrt{9801} \times \sqrt{576} = 2876$.

269–20 $\sqrt{6889} + \sqrt{900} = 113$. * $\sqrt{6889} \times \sqrt{900} = 2490$.

269–21 $\sqrt{5625} + \sqrt{1764} = 117$. * $\sqrt{5625} \times \sqrt{1764} = 3150$.

Lesson 153

269–A To the left of point 1 * Less than 1

269–B Yes. Point $^-2$ is to the left of point 0. * No. Point $\sqrt{5}$ is not to the left of point 2. * No. Point $\sqrt{2}$ is not to the left of point $^-1$. * Yes. Point $\sqrt{3}$ is to the left of point 2. * No. Point $\sqrt{2}$ is not to the left of point 1.

269–C Yes

270–D (Column 1) 1.732... * Yes

270–E $\sqrt{5}$ is approximately equal to 2.236. * Yes. $2\frac{1}{4} = 2.25\overline{0}$. The first digit in each of the numerals 2.236 and $2.25\overline{0}$ expresses 2 ones. The second digit in each numeral expresses 2 tenths. But the third digit in the numeral 2.236 expresses 3 hundredths and the third digit in the numeral 2.250 expresses 5 hundredths. Therefore, $2.236 < 2.25\overline{0}$ and $\sqrt{5} < 2\frac{1}{4}$. * No. $2\frac{1}{5} = 2.20\overline{0}$. The first digit in each of the numerals 2.236 and $2.20\overline{0}$ expresses 2 ones. The second digit in each numeral expresses 2 tenths. But the third digit in the numeral 2.236 expresses 3 hundredths and the third digit in the numeral $2.20\overline{0}$ expresses zero hundredths. Therefore, $2.236 < 2.20\overline{0}$ is a false statement and $\sqrt{5} < 2\frac{1}{5}$ is a false statement.

270–F Yes * $^-.\overline{3} < ^-.32$ is the same as $^-.33\overline{3} < ^-.32$. The first digit in the decimal $^-.33\overline{3}$ is the same as the first digit in the decimal $^-.32$. The second digit in the decimal $^-.33\overline{3}$ expresses a greater number than the second digit in the decimal $^-.32$. However, since both numbers are negative, $^-.33\overline{3} < ^-.32$ and $^-.\overline{3} < ^-.32$.

270–G $\sqrt{6}$ is approximately equal to 2.449. * No. $^-2.5 = ^-2.50\overline{0}$. $^-2.449 < ^-2.50\overline{0}$ is a false statement. The second digit in the decimal $^-2.449$ expresses a lesser number than the second digit in the decimal $^-2.50\overline{0}$ but both numbers are negative. Since $^-2.449 < ^-2.50\overline{0}$ is a false statement, $^-\sqrt{6} < ^-2.5$ is a false statement. * Yes. Since $^-2.449 < ^-2.448$ is a true statement, $^-\sqrt{6} < ^-2.448$ is a true statement.

270–H Every negative number is less than every positive number. * Yes * Yes

270–I Yes

270–J $\sqrt{15} > \sqrt{14}$. * $^-\frac{1}{3} > ^-\frac{2}{3}$. * $^-.6 > ^-.\overline{6}$.

270–K Yes. Since R is a subset of D, the idea of "greater than" is related to the idea of "less than" for rational numbers in the same way that these ideas are related for real numbers. * Yes. Since I is a subset of D, the idea of "greater than" is related to the idea of "less than" for integers in the same way that these ideas are related for real numbers.

270–L True statement

270–M False statement

270–N False statement

270–O False statement

270–P True statement

270–Q False statement

270–R True statement

270–S True statement

270–T False statement

270–A Yes * 5 * Yes

270–B Yes * $-\frac{2}{3}$

270–C Yes * From the sum property of "less than" for real numbers, since $\sqrt{2}$ is less than $\sqrt{3}$, the sum of $\sqrt{2}$ and $5.\overline{4}$ is less than the sum of $\sqrt{3}$ and $5.\overline{4}$.

270–D (Column 2) Yes. By the closure property of addition, $a + c$ and $b + c$ are real numbers. Therefore, $a + c$ and $b + c$ can be treated in the same way with respect to the ideas of "less than" and "greater than" as any other real numbers. * The universe for each variable is D. For each a, b, and c, if $a > b$, then $a + c > b + c$.

271–E Yes * $\frac{2}{3}$ * Yes * Yes * Yes

271–F Yes * .3 * Yes * Yes * Yes

271–G $\sqrt{19}$ is approximately 4.359. * Yes * Yes * Yes * Because of the positive-multiplier property of "less than" for real numbers

271–H Yes. By the closure property of multiplication, ac and bc are real numbers. Therefore, ac and bc can be treated in the same way with respect to the ideas of "less than" and "greater than" as any other real numbers. * The universe for each variable is D. For each a, b, and c, if $a > b$ and $c > o$, then $ac > bc$.

271–I Yes * $-\frac{1}{2}$ * Yes * Yes * Yes

271–J Yes * -10 * Yes * Yes

271–K Each property concerns a first, a second, and a third number. For each property, the first number is less than the second number. For the positive-multiplier property, 0 is less than the third number. For the negative-multiplier property, the third number is less than 0. For the positive-multiplier property, the product of the first and third numbers is less than the product of the second and third numbers. For the negative-multiplier property, the product of the second and third numbers is less than the product of the first and third numbers.

271–L Yes * Yes * From the negative-multiplier property of "less than" for real numbers, since 1.3 is less than 4.2 and since $-\sqrt{5}$ is less than 0, the product of 1.3 and $-\sqrt{5}$ is greater than the product of 4.2 and $-\sqrt{5}$.

271–M Yes. By the closure property of multiplication, ac and bc are real numbers. Therefore, ac and bc can be treated in the same way in respect to the ideas of "less than" and "greater than" as any other real numbers. * The universe for each variable is D. For each a, b, and c, if $a > b$ and $c < 0$, then $bc < ac$.

271–N Sentence M expresses the sum property of "less tha Sentence N expresses the positive-multiplier property of "less than." Sentence O expresses the negative-multiplier property of "less than."

272–O The second statement is true. * The sum property of "less than"

272–P The second statement is false.

272–Q The second statement is true. * The positive-multiplier property of "less than"

272–R The second statement is true. * The positive-multiplier property of "less than"

272–S The second statement is false.

272–T The second statement is false.

272–U The second statement is false.

272–V The second statement is false.

272–A Yes * Yes

272–B It is possible to find at least one real number between any two given real numbers. The universe for a and b is D. If a and b are any two real numbers, then $\frac{1}{2}(a + b)$ is a real number between a and b. * The set of real numbers has the density property. If, for any two given members of a set of numbers, you can find another member of the set that is between the two given members, then that set of numbers has the density property.

272–C An example is $^{-}1$. * An example is $\frac{1}{2}(\sqrt{8} + 5)$ which is approximately equal to 3.914. * An example is $\frac{1}{2}(^{-}.\overline{3} + ^{-}.3331)$ which is approximately equal to $^{-}.3332$. Notice that $^{-}.333$ is not between $^{-}.\overline{3}$ and $^{-}.3331$. * An example is $\frac{1}{2}(\sqrt{5} + \sqrt{10})$ which is approximately equal to 2.699.

272–D $^{-}\sqrt{6}$ * Yes * No * Yes. Remember that $\overline{R}$ is the symbol for the set of irrational numbers.

272–E $^{-}2$, $^{-}1$, 0, .5, 1, 1.75, 2, and 3 are rational numbers. $^{-}\sqrt{6}$, $^{-}\sqrt{3}$, $\sqrt{5}$, and $\sqrt{8}$ are irrational numbers.

272–F No. Some points in the line do not correspond to members of R. * No. Some points in the line do not correspond to members of I.

On your own

272–1 $^{-}3\frac{1}{5}$, $^{-}2\frac{1}{2}$, $^{-}2\frac{1}{4}$, $^{-}2$, $^{-}\frac{1}{2}$, 0, $1\frac{3}{4}$, 3

272–2 $^{-}1.8$, $^{-}.3$, $^{-}.03$, .02, 1.6, 2.2

272–3 $^{-}8$, $^{-}\sqrt{15}$, $^{-}3$, 2, $\sqrt{7}$, $\sqrt{11}$

273–4 $^{-}.675$, $^{-}.\overline{6}$, $^{-}.5$, .01, .09, .1

273–5 $^{-}\frac{1}{2}$, $^{-}\frac{1}{3}$, $^{-}\frac{1}{5}$, $^{-}\frac{1}{6}$, $^{-}\frac{1}{8}$, $^{-}\frac{1}{10}$

273–6 3.46, 3.463, $\sqrt{12}$, 3.465, 3.47

273–7 The second statement is true. * The sum property of "less than"

273–8 The second statement is true. * The positive-multiplier property of "less than"

273–9 The second statement is false.

273–10 The second statement is true. * The negative-multiplier property of "less than"

273–11 The second statement is false.

273–12 The second statement is false.

273–13 $1\frac{3}{10}$ and $1\frac{1}{4}$

273–14 $^{-}4.432$ and $^{-}4.499$

273–15 $\sqrt{10}$, 3.2, and 3.3

273–16 None

273–17 All of the elements

Keeping skilful

273–1 $\{40\frac{3}{4}\}$

273–2 $\{^{-}49\frac{1}{12}\}$

273–3 $\{x \mid x < 168.2\}$

273–4 $\{m \mid m \leq 33.3\}$

273–5 $\{w \mid 2.15 < w < 5.6\}$

273–6 $\{z \mid z < 5\frac{1}{4}\}$

273–7 12.247

273–8 29.715

273–9 4.472

273–10 26.870

273–11 22.913

273–12 30.512

273–13 36.932

273–14 38.523

Applying mathematics

273–1 x, amount of money Roberta spent for her lunches.
$.55 + .40 + .60 + .30 + .50 = x$.
$\{2.35\}$. \$2.35

273–2 x, amount of money George earned last month.
$x > 12.50 \wedge x < 5.60 + 9.30$.
$\{x \mid 12.50 < x < 14.90\}$.
Between \$12.50 and \$14.90

273–3 x, temperature in Maple City at 6:00 A.M.
$x < 0 \wedge x + 10 > 5$.
$\{x \mid ^{-}5 < x < 0\}$.
Between $^{-}5$ and 0 degrees

273–4 $10 + x = ^{-}2\frac{3}{4} \vee 10 + x = 5\frac{7}{8}$.
$\{^{-}12\frac{3}{4}, ^{-}4\frac{1}{8}\}$

273–5 $.3\frac{3}{4}(\frac{4}{9}) + x = 2\frac{1}{2}$.
$\{\frac{5}{6}\}$

273–6 $12.25 - x \leq 2.5$, or
$\sim(12.25 - x > 2.5)$.
$\{x \mid 9.5 \leq x\}$

273–7 x, number of miles farther Oakville is from Prospect City than Lakeview is.
$40\frac{1}{2} - 20\frac{7}{8} = x$.
$\{19\frac{5}{8}\}$. $19\frac{5}{8}$ miles

273–8 x, number of bushels of tomatoes Mr. Young had before he sold $20\frac{1}{2}$ bu.
$x < 32 \wedge x - 20\frac{1}{2} > 7\frac{1}{2}$.
$\{x \mid 28 < x < 32\}$.
Between 28 and 32 bushels

274–9 x, amount of linen Hilda will have left.
$4\frac{1}{2} - 3\frac{3}{4} = x \vee 4\frac{1}{2} - 2\frac{7}{8} = x$.
$\{\frac{3}{4}, 1\frac{5}{8}\}$. Hilda will have either $\frac{3}{4}$ yd. or $1\frac{5}{8}$ yd. of

linen left.

274–10 x, amount of money Mr. Holland would pay for 15 gal. of gasoline.
$3.69/10 \sim x/15$.
{5.535}. $5.54

274–11 x, number of shots Bob made.
$66\frac{2}{3}/100 \sim 52/x$, or
$\frac{2}{3}/1 \sim 52/x$.
{78}. 78 shots

274–12 x, number of acres of corn Mr. Myers can plant in $8\frac{1}{2}$ hr. $2\frac{1}{3}/1 \sim x/8\frac{1}{2}$ v $4\frac{3}{4}/1 \sim x/8\frac{1}{2}$. $\{19\frac{5}{6}, 40\frac{3}{8}\}$. Mr. Myers can plant either $19\frac{5}{6}$ acres or $40\frac{3}{8}$ acres in $8\frac{1}{2}$ hr.

274–13 x, number of square miles of the earth's surface that are covered with water.
$71/100 \sim x/2 \times 10^8$.
$\{1.42 \times 10^8\}$. 1.42×10^8 sq. mi., or 142,000,000 sq. mi.

274–14 x, number of times as great the speed of sound in wood is as the speed of sound in air.
$x/1 \sim 1.0 \times 10^4/1.1 \times 10^3$.
$\{9.\overline{09}\}$. 9.1 times as great

274–15 x, area of parking lot in square feet.
$670/1 \sim x/195\frac{3}{4}$.
$\{131152\frac{1}{2}\}$. $131,152\frac{1}{2}$ sq. ft.

274–16 x, area of rhombus ABCD in square inches.
$35.4/1 \sim x/20.5$.
{725.7}. 725.7 sq. in.

274–17 x, area of ΔJKL in square feet.
$6.2/1 \sim 2x/2.7$.
{8.37}. 8.37 sq. ft.

Checking up

Test 140

276–1 4.123...

276–2 529

276–3 43,681

276–4 61

276–5 43

276–6 5,041

276–7 24,964

276–8 6.557...

276–9 175

276–10 15.49

276–11 17.61

276–12 21.79

276–13 27.71

Test 141

276–14 Symmetric property

276–15 Transitive property

276–16 Reflexive property

276–17 R and R_n

276–18 R, R_p, I, and I_p

276–19 R, R_n, I, and I_n

276–20 R and R_p

276–21 Well-defined property of addition

276–22 Closure property of addition

276–23 Inverse property of multiplication, or reciprocal property

276–24 Identity-element property of multiplication

276–25 Sum property of "less than"

276–26 Distributive property

276–27 Positive-multiplier property of "less than"

276–28 Commutative property of addition

Lesson 154

276–A $x - 4\frac{1}{2} = {}^{-}\frac{1}{8}$. * D

276–B Yes * The difference property tells you that the difference of two numbers is the same as the sum of the minuend and the additive inverse of the subtrahend.

276–C Since a first number, $x + {}^{-}4\frac{1}{2}$, is equal to a second number, $x - 4\frac{1}{2}$, and the second number is equal to a third number, ${}^{-}\frac{1}{8}$, then the first number, $x + {}^{-}4\frac{1}{2}$, is equal to the third number, ${}^{-}\frac{1}{8}$.

276–D ${}^{+}4\frac{1}{2}$

276–E The associative property of addition * Yes

277–F Since $4\frac{1}{2}$ is the additive inverse of ${}^{-}4\frac{1}{2}$, ${}^{-}4\frac{1}{2} + {}^{+}4\frac{1}{2}$ is the same as 0. * The transitive property of equality tells you that since $x + 0 = x + ({}^{-}\frac{1}{4} + {}^{+}\frac{1}{4})$ and $x + ({}^{-}4\frac{1}{2} + {}^{+}4\frac{1}{2}) = {}^{-}\frac{1}{8} + {}^{+}4\frac{1}{2}$, then $x + 0 = {}^{-}\frac{1}{8} + {}^{+}4\frac{1}{2}$.

277–G Yes

277–H Yes * The identity-element property of addition

277–I ${}^{+}4\frac{3}{8}$ * Yes

277–J $4\frac{3}{8}$ * $\{4\frac{3}{8}\}$

277–K Commutative property of multiplication * Transitive property of equality Since $a(b + c) = ab + ac$ and $ab + ac = ba + ca$, then $a(b + c) = ba + ca$.

277–L Commutative property of multiplication * Transitive property of equality Since $(b + c)a = a(b +$ and $a(b + c) = ba + ca$, then $(b + c)a = ba + ca$.

277–M $2x + 3x = 10$.

277–N Since $(2 + 3)x = 2x + 3$ and $2x + 3x = 10$, then $(2 + 3)x = 10$.

-O Yes * The well-defined property of multiplication * Yes * Yes

-P Use the well-defined property of multiplication. Since $5x = 10$, then $\frac{1}{5}(5x) = \frac{1}{5}(10)$ and $\frac{1}{5}(5x) = \frac{1}{5}(10)$ is equivalent to $x = 2$. * 2

-Q {2}

-R $2x + 3x = 10$. * {2} * Yes * $2(2) + 3(2) = 4 + 6$, or 10.

-S Use the well-defined property of multiplication. Since $x = 2$, the product $5(x)$ is the same as the product $5(2)$. Therefore, $5x = 10$. * Use the well-defined property of multiplication. Since $2 + 3 = 5$, you can replace 5 in $5x$ by $(2 + 3)$ without affecting the product. * Use the distributive property. * Yes

-A $10a = 7(a + 2)$.

-B Yes * Distributive property * Condition B can be obtained from condition A by using a property.

-C Yes

-D Well-defined property of addition

-E Associative property * Condition D was obtained from condition C by using a property.

-F Additive-inverse property * Since $^{-}(7a) + 10a = [^{-}(7a) + (7a)] + 14$ and $[^{-}(7a) + (7a)] + 14 = 0 + 14$, then $^{-}(7a) + 10a = 0 + 14$.

-G Identity-element property of addition tells you that $0 + 14 = 14$. * Yes

278–H $7a + 3a = (7 + 3)a$ and $(7 + 3)a = 10a$. Therefore, $7a + 3a = 10a$, or $10a = 7a + 3a$. * Yes

279–I (Column 1) $[^{-}(7a) + (7a)] + 3a$ is the same as $^{-}(7a) + (7a + 3a)$ by the associative property of addition.

279–J (Column 1) Use the additive-inverse property. * Use the identity-element property of addition.

279–K (Column 1) $3a = 14$. $\frac{1}{3}(3a) = \frac{1}{3}(14)$. Well-defined property of multiplication. $(\frac{1}{3} \cdot 3)a = \frac{14}{3}$. Associative property of multiplication. $1a = \frac{14}{3}$. Reciprocal property. $a = \frac{14}{3}$. Identity-element property of multiplication * $\frac{14}{3}$

279–L (Column 1) $\{\frac{14}{3}\}$

279–M (Column 1) $10(\frac{14}{3}) = 7(\frac{14}{3} + 2)$. $\frac{140}{3} = 7(\frac{20}{3})$. $\frac{140}{3} = \frac{140}{3}$. * Yes

279–A Yes * The first component of $1/(m - .5)$ is 1 and the second component is $m - .5$ The first component of $.8/7.2$ is .8 and the second component is 7.2.

279–B Since $1/(m - .5)$ and $.8/7.2$ are equivalent, the definition of equivalent ordered pairs tells you that the product of the first component of $1/(m - .5)$ and the second component of $.8/7.2$ is equal to the product of the first component of $.8/7.2$ and the second component of $1/(m - .5)$.

279–C Each difference of two numbers is also a sum of two numbers. According to the difference property, the difference of two numbers is the sum of the minuend and the additive inverse of the subtrahend. * Yes

279–D Yes * Yes * Yes * Transitive property

279–E Yes * Because of the difference property. * Yes

279–F $^{+}.4$ * Well-defined property of addition.

279–G The associative property of addition

279–H Use the additive-inverse property. * Yes

279–I (Column 2) Condition S can be obtained from condition R by using the identity-element property of addition.

279–J (Column 2) $1/.8 * 7.6 = .8m$. $\frac{1}{.8}(7.6) = \frac{1}{.8}(.8m)$. Well-defined property of multiplication. $9.5 = (\frac{1}{.8} \cdot .8)m$. Associative property of multiplication. $9.5 = 1m$. Reciprocal property. $9.5 = m$. Identity-element property of multiplication

279–K (Column 2) 9.5

279–L (Column 2) {9.5}

279–M (Column 2) Yes. $1/(9.5 - .5) \sim .8/7.2$. $1/9 \sim .8/7.2$. $1(7.2) = .8(9)$. $7.2 = 7.2$.

On your own

280–I False statement

280–2 False statement

280–3 False statement

280–4 True statement

280–5 True statement

280–6 True statement

280–7 False statement

280–8 False statement

280–9 False statement

280–10 False statement

280–11 $\frac{1}{5}(x - \frac{2}{3}) = \frac{6}{5}$.
$\frac{1}{5}x - \frac{2}{15} = \frac{6}{5}$.
Distributive property.
$\frac{1}{5}x + {}^{-}\frac{2}{15} = \frac{6}{5}$.
Difference property.
$(\frac{1}{5}x + {}^{-}\frac{2}{15}) + \frac{2}{15} = \frac{6}{5} + \frac{2}{15}$.
Well-defined property of addition.
$\frac{1}{5}x + ({}^{-}\frac{2}{15} + \frac{2}{15}) = \frac{4}{3}$.
Associative property of addition.
$\frac{1}{5}x + 0 = \frac{4}{3}$.
Additive-inverse property.
$\frac{1}{5}x = \frac{4}{3}$.
Identity-element property of addition.
$5(\frac{1}{5}x) = 5(\frac{4}{3})$.
Well-defined property of multiplication.
$(5 \cdot \frac{1}{5})x = \frac{20}{3}$.
Associative property of multiplication.
$1x = 6\frac{2}{3}$.
Reciprocal property.
$x = 6\frac{2}{3}$.
Identity-element property of multiplication * $\{6\frac{2}{3}\}$

280–12 $13x + 7x = {}^{-}180$.
$(13 + 7)x = {}^{-}180$.
Distributive property.
$\frac{1}{20}(20x) = \frac{1}{20}({}^{-}180)$.
Well-defined property of multiplication.
$x = {}^{-}9$.
* $\{{}^{-}9\}$

280–13 $(5 - x)/2 \sim 6/1$.
$5 - x = 6(2)$.
Definition of equivalent rate pairs.
$5 + {}^{-}x = 12$.
Difference property.
${}^{-}5 + (5 + {}^{-}x) = {}^{-}5 + 12$.
Well-defined property of addition.
$({}^{-}5 + 5) + {}^{-}x = 7$.
Associative property of addition.
$0 + {}^{-}x = 7$.
Additive-inverse property.
${}^{-}x = 7$.
Identity-element property of addition.
$x = {}^{-}7$.
Definition of additive inverse
* $\{{}^{-}7\}$

280–14 $5(x - 3) = 2x$.
$5x - 15 = 2x$.
Distributive property.
$5x + {}^{-}15 = 2x$.
Difference property.
$(5x + {}^{-}15) + 15 = 2x + 15$.
Well-defined property of addition.
$5x + ({}^{-}15 + 15) = 2x + 15$.
Associative property of addition.
$5x + 0 = 2x + 15$.
Additive-inverse property.
$5x = 2x + 15$.
Identity-element property of addition.
${}^{-}(2x) + 5x = {}^{-}(2x) + (2x + 15)$.
Well-defined property of addition.
${}^{-}(2x) + 5x = [{}^{-}(2x) + 2x] + 15$.
Associative property of addition.
${}^{-}(2x) + 5x = 0 + 15$.
Additive-inverse property.
${}^{-}(2x) + 5x = 15$.
Identity-element property of addition.
${}^{-}(2x) + (2x + 3x) = 15$.
Distributive property.
$[{}^{-}(2x) + 2x] + 3x = 15$.
Associative property of addition.
$0 + 3x = 15$.
Additive-inverse property.
$3x = 15$.
Identity-element property of addition.
$\frac{1}{3}(3x) = \frac{1}{3}(15)$.
Well-defined property of multiplication.
$(\frac{1}{3} \cdot 3)x = 5$.
Associative property of multiplication.
$1x = 5$.
Reciprocal property.
$x = 5$.
Identity-element property of multiplication
* $\{5\}$

280–15 $\{{}^{-}54\}$

280–16 $\{3.64\}$

280–17 $\{{}^{-}13\}$

280–18 $\{{}^{-}2\}$

280–19 $\{\frac{1}{2}\}$

280–20 $\{{}^{-}\frac{5}{8}\}$

280–21 $\{10\frac{1}{3}\}$

280–22 $\{\frac{2}{3}\}$

280–23 $\{\frac{2}{5}\}$

280–24 $\{1\}$

280–25 $\{2.1\}$

280–26 $\{{}^{-}\frac{22}{9}\}$, or $\{{}^{-}2\frac{4}{9}\}$

Lesson 155

280–A $\frac{1}{4}$ * Because $\frac{1}{4}(4) = 1$.

280–B Yes * Yes * The positive-multiplier property of "less than"

280–C Yes

280–D The associative property * Yes

281–E (Column 1) Use the reciprocal property of multiplication. * Use the identity-element property of multiplication. * Yes * Yes

281–F (Column 1) Yes

281–G (Column 1) $\{y \mid y < \frac{7}{4}\}$

281–H The set of real numbers is dense.

281–I $\{y \mid y < \frac{7}{4}\}$ is an infinite set.

281–J Condition G can be obtained from condition F by using the positive-multiplier property of "less than." Condition H can be obtained from condition G by finding the product of 4 and $\frac{7}{4}$.

281–A $^{-}\frac{1}{3} * {}^{-}\frac{1}{3}$

281–B Yes. Every negative number is less than 0. * Yes * The negative-multiplier property of "less than"

281–C Condition K can be obtained from condition J by using the associative property of multiplication.

281–D Condition L can be obtained from condition K by using the reciprocal property. $^{-}\frac{1}{3} \cdot {}^{-}3 = 1$. * Condition M can be obtained from condition L by using the identity-element property of multiplication. $1x = x$.

281–E (Column 2) $\{x \mid x > {}^{-}\frac{4}{3}\}$

281–F (Column 2) The difference property * Yes

281–G (Column 2) Because $^{-}3 + 3 = 0$.

282–H (Column 1) The sum property of "less than"

282–I (Column 1) Yes

282–J (Column 1) Yes. Condition Q can be obtained from condition P by using the associative property of addition. * Yes. Condition R can be obtained from condition Q by using the additive-inverse property. * Yes. Condition S can be obtained from condition R by using the identity-element property of addition and finding the sum of $^{-}3$ and $^{-}2$.

282–K (Column 1) Use the sum property of "less than." * Use the additive-inverse property.

282–L (Column 1) $0 > m + {}^{-}5$.
$0 + 5 > (m + {}^{-}5) + 5$.
Sum property of "less than."
$5 > m + ({}^{-}5 + 5)$.
Identity-element and associative properties of addition.
$5 > m + 0$.
Additive-inverse property.
$5 > m$.
Identity-element property of addition

282–M (Column 1) $\{m \mid 5 > m\}$

282–A $^{-}6a + 2a < 8 \wedge a - \frac{7}{8} < \frac{1}{8}$. * The connective "and" * D

282–B The solution set of a compound condition that includes "and" is the intersection of the solution sets of the two simple conditions that make up the compound condition.

282–C Use the distributive property.
$^{-}6a + 2a$ is the same as $(^{-}6 + 2)a$, and $(^{-}6 + 2)a$ is the same as $^{-}4a$.

282–D $^{-}4a < 8$.
$-\frac{1}{4}(^{-}4a) > -\frac{1}{4}(8)$.
Negative-multiplier property of "less than."
$(^{-}\frac{1}{4} \cdot {}^{-}4)a > {}^{-}2$.
Associative property of multiplication.
$1a > {}^{-}2$.
Reciprocal property.
$a > {}^{-}2$.
Identity-element property of multiplication * Yes

282–E $\{a \mid a > {}^{-}2\}$

282–F $a - \frac{7}{8} < \frac{1}{8}$.
$a + {}^{-}\frac{7}{8} < \frac{1}{8}$.
Difference property.
$(a + {}^{-}\frac{7}{8}) + \frac{7}{8} < \frac{1}{8} + \frac{7}{8}$.
Sum property of "less than"
$a + ({}^{-}\frac{7}{8} + \frac{7}{8}) < 1$.
Associative property of addition.
$a + 0 < 1$.
Additive-inverse property.
$a < 1$.
Identity-element property of addition

282–G $\{a \mid a < 1\}$

282–H (Column 2) Yes

282–I (Column 2) Yes * No * No

282–J (Column 2) $^{-}2$ and 1 * No

282–K (Column 2) No * Yes * By the horizontal bar above the picture of the number line * Yes

282–L (Column 2) $^{-}\frac{3}{4} + r < 2 \vee r + \frac{10}{4} = \frac{21}{4}$. * D

282–M (Column 2) The connective "or" * The union

283–N $^{-}\frac{3}{4} + r < 2$.
$\frac{3}{4} + (^{-}\frac{3}{4} + r) < \frac{3}{4} + 2$.
Sum property of "less than."

$(\frac{3}{4} + \bar{}\frac{3}{4}) + r < \frac{11}{4}$.
Associative property of addition.
$0 + r < \frac{11}{4}$.
Additive-inverse property.
$r < \frac{11}{4}$.
Identity-element property of addition * Yes

283–O $\{r \mid r < \frac{11}{4}\}$

283–P $\{\frac{11}{4}\}$

283–Q Since $r < \frac{11}{4}$ is equivalent to $\bar{}\frac{3}{4} + r < 2$ and $r = \frac{11}{4}$ is equivalent to $r + \frac{10}{4} = \frac{21}{4}$, the solution set of $\bar{}\frac{3}{4} + r < 2 \vee r + \frac{10}{4} = \frac{21}{4}$ is $\{r \mid r < \frac{11}{4} \vee r = \frac{11}{4}\}$.

283–R $\{r \mid r \leq \frac{11}{4}\}$

283–S Yes. $\frac{11}{4}$ is equal to $\frac{11}{4}$. * Yes. 0 is less than $\frac{11}{4}$. * Yes * Yes. Every negative number is less than every positive number. Therefore, every negative number is less than $\frac{11}{4}$.

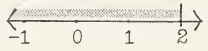
283–T By the vertical stroke through the dot for point $\frac{11}{4}$ * Yes * The dot for point $\frac{11}{4}$ and all the dots for points associated with numbers less than $\frac{11}{4}$ * By the horizontal bar above the picture of the number line

283–U Yes

283–V $.5x + .2 > 1.2$.
$(.5x + .2) + \bar{}.2 > 1.2 + \bar{}.2$.
Sum property of "less than."
$.5x + (.2 + \bar{}.2) > 1$.
Associative property of addition.
$.5x + 0 > 1$.
Additive-inverse property.
$.5x > 1$.
Identity-element property of addition *
$.5x > 1$.
$2(.5x) > 2 \cdot 1$.
Positive-multiplier property of "less than."
$(2 \cdot .5)x > 2$.
Associative property of multiplication.
$1x > 2$.
Reciprocal property.
$x > 2$.
Identity-element property of multiplication

283–W Yes

283–X



On your own

283–1 $x + \bar{}4 < 1\frac{1}{2}$.
$(x + \bar{}4) + 4 < 1\frac{1}{2} + 4$.
Sum property of "less than."
$x + (\bar{}4 + 4) < 5\frac{1}{2}$.
Associative property of addition.
$x + 0 < 5\frac{1}{2}$.
Additive-inverse property.
$x < 5\frac{1}{2}$.
Identity-element property of addition * $\{x \mid x < 5\frac{1}{2}\}$

283–2 $\bar{}12x > 3$.
$(\bar{}\frac{1}{12})\bar{}12x < (\bar{}\frac{1}{12})3$.
Negative-multiplier property of "less than."
$(\bar{}\frac{1}{12} \cdot \bar{}12)x < \bar{}\frac{1}{4}$.
Associative property of multiplication.
$1x < \bar{}\frac{1}{4}$.
Reciprocal property.
$x < \bar{}\frac{1}{4}$.
Identity-element property of multiplication *
$\{x \mid x < \bar{}\frac{1}{4}\}$

283–3 $\frac{1}{10}x < 7$.
$10(\frac{1}{10}x) < 10 \cdot 7$.
Positive-multiplier property of "less than."
$(10 \cdot \frac{1}{10})x < 70$.
Associative property of multiplication.
$1x < 70$.
Reciprocal property.
$x < 70$.
Identity-element property of multiplication *
$\{x \mid x < 70\}$

283–4 $x - .3 < 2.8$.
$x + \bar{}.3 < 2.8$.
Difference property.
$(x + \bar{}.3) + .3 < 2.8 + .3$.
Sum property of "less than."
$x + (\bar{}.3 + .3) < 3.1$.
Associative property of addition.
$x + 0 < 3.1$.
Additive-inverse property.
$x < 3.1$.
Identity-element property of addition *
$\{x \mid x < 3.1\}$

283–5 $\frac{5}{4}x + \frac{1}{2}x > \bar{}14$.
$(\frac{5}{4} + \frac{1}{2})x > \bar{}14$.
Distributive property.
$\frac{7}{4}x > \bar{}14$.
Because $\frac{5}{4} + \frac{1}{2} = \frac{7}{4}$.
$\frac{4}{7}(\frac{7}{4}x) > \frac{4}{7} \cdot \bar{}14$.
Positive-multiplier property of "less than."
$(\frac{4}{7} \cdot \frac{7}{4})x > \bar{}8$.
Associative property of multiplication.
$1x > \bar{}8$.
Reciprocal property.
$x > \bar{}8$.
Identity-element property of multiplication *
$\{x \mid x > \bar{}8\}$

283–6 $15 - x > 12$.
$15 + {}^{-}x > 12$.
Difference property.
$(15 + {}^{-}x) + x > 12 + x$.
Sum property of "less than."
$15 + ({}^{-}x + x) > 12 + x$.
Associative property of addition.
$15 + 0 > 12 + x$.
Additive-inverse property.
$15 > 12 + x$.
Identity-element property of addition.
${}^{-}12 + 15 > {}^{-}12 + (12 + x)$.
Sum property of "less than."
$3 > ({}^{-}12 + 12) + x$.
Associative property of addition.
$3 > 0 + x$.
Additive-inverse property.
$3 > x$.
Identity-element property of addition.
$x < 3$.
"Greater than" related to "less than" *
$\{x \mid x < 3\}$

283–7 True statement

283–8 False statement

283–9 True statement

283–10 False statement

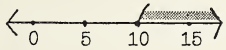
283–11 False statement

283–12 True statement

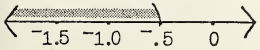
283–13 True statement

283–14 True statement

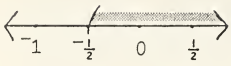
283–15 $\{x \mid x > 10\}$



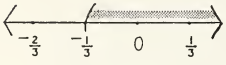
283–16 $\{x \mid x < {}^{-}.5\}$



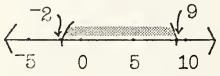
283–17 $\{x \mid x > {}^{-}\frac{1}{2}\}$



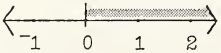
283–18 $\{r \mid r > {}^{-}\frac{1}{3}\}$



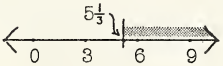
283–19 $\{y \mid {}^{-}2 < y < 9\}$



283–20 $\{x \mid x \geq 0\}$



283–21 $\{k \mid k \geq 5\frac{1}{3}\}$



Lesson 156

284–A N × N

284–B Both components of $(\sqrt{5}, {}^{-}8)$ are real numbers * Yes * Yes * Yes

284–C D is an infinite set.

284–D 0 * 0

284–E The first axis * $(\frac{3}{2}, 0)$

284–F The second axis * 1

284–G The second axis * The first axis

285–H Yes * $\frac{3}{2}$

285–I The components of (2,1) * Point B is located 2 units to the right of the second axis so the first coördinate is 2. Point B is located 1 unit above the first axis so the second coördinate is 1.

285–J The first axis * Yes

285–K The first axis * 2 units to the left of the second axis

285–L The second axis * Below the first axis

285–M Point D is located 1 unit to the left of the second axis and 1 unit above the first axis.

285–N Point E is located $\frac{5}{2}$ units to the left of the second axis and $\frac{3}{2}$ units above the first axis. * $({}^{-}\frac{5}{2}, \frac{3}{2})$

285–O $({}^{-}\frac{3}{2}, {}^{-}\frac{1}{2})$ * Yes

285–P Point G is located 1 unit to the left of the second axis and $\frac{5}{2}$ units below the first axis. * $({}^{-}1, {}^{-}\frac{5}{2})$

285–Q Point H is located in the second axis $\frac{3}{2}$ units below the first axis. This means that the first coördinate of point H is 0 and the second coördinate is ${}^{-}\frac{3}{2}$.

285–R Since point I is located 2 units to the right of the second axis the first coördinate of point I is 2. Since point I is located $\frac{3}{2}$ units below the first axis, the second coördinate of point I is ${}^{-}\frac{3}{2}$.

285–S Point J is located $\frac{3}{2}$ units to the right of the second axis and $\frac{5}{2}$ units below the first axis. * $(\frac{3}{2}, {}^{-}\frac{5}{2})$

285–T To the right * To the left

285–U Above the first axis * Below the first axis

285–V Point $(15, {}^{-}6)$ is located 15 units to the right of the second axis and 6 units below the first axis. * Point $({}^{-}5\frac{3}{4}, 18)$

is located $5\frac{3}{4}$ units to the left of the second axis and 18 units above the first axis. * Point $(-\sqrt{17}, -\sqrt{17})$ is located $\sqrt{17}$ units to the left of the second axis and $\sqrt{17}$ units below the first axis.

285–W Points A and B are in the first quadrant. Points D and E are in the second quadrant. Points F and G are in the third quadrant. Points I and J are in the fourth quadrant.

285–X First * Third

285–Y Fourth * Second

285–Z Point $(-8,2)$ is in the second quadrant. Point $(.\overline{8},.6)$ is in the first quadrant. Point $(-\sqrt{2}, -11)$ is in the third quadrant. Point $(4,-4)$ is in the fourth quadrant.

On your own

286–1 Point A is located $\sqrt{2}$ units to the right of the second axis and $\sqrt{2}$ units above the first axis.

286–2 Point B is located $\sqrt{3}$ units to the left of the second axis and $\frac{5}{3}$ units above the first axis.

286–3 Point C is located $2\frac{1}{4}$ units to the right of the second axis and 3 units above the first axis.

286–4 Point D is located $1\frac{1}{2}$ units to the left of the second axis and in the first axis.

286–5 Point E is located in the second axis and 2.7 units below the first axis.

286–6 Point F: $(2,-1)$
Point G: $(-8,-2)$
Point H: $(-8,3)$
Point I: $(-1,-1)$
Point J: $(3,-2)$
Point K: $(-2,-3)$
Point L: $(-2,1)$
Point M: $(3,2)$
Point N: $(1,1)$

286–7 Points B, H, and L

286–8 Points A, C, M, and N

286–9 Points F and J

286–10 Points G, I, and K

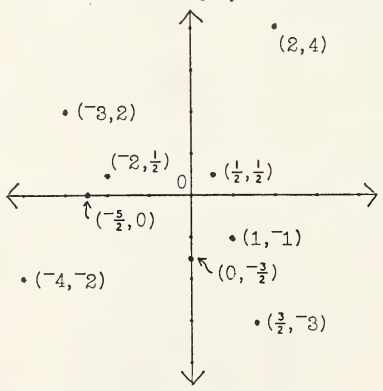
286–11 Point G, point $(0,-2)$, and point J

286–12 Point H, point $(-3,0)$, and point G

286–13 $\sqrt{2}$

286–14 Each point whose second coordinate is $\sqrt{3}$ is contained in a line. This line is $\sqrt{3}$ units above the origin and is parallel to the first axis.

286–15 See the graph below.



286–16 See the graph for 228–15.

286–17 See the graph for 228–15.

286–18 See the graph for 228–15.

286–19 See the graph for 228–15.

286–20 See the graph for 228–15.

286–21 See the graph for 228–15.

286–22 See the graph for 228–15.

286–23 See the graph for 228–15

Keeping skilful

286–1 $3\frac{13}{16}$

286–2 $-\frac{2}{15}$

286–3 $-\frac{1}{12}$

286–4 $\frac{1}{24}$

286–5 325

286–6 $-15\frac{3}{20}$

286–7 $-14\frac{1}{2}$

286–8 $-2\frac{3}{4}$

286–9 $5\frac{1}{2}$

286–10 $-29\frac{3}{4}$

286–11 $\frac{11}{18}$

286–12 $-14\frac{1}{12}$

286–13 $\frac{9}{16}$

286–14 384

286–15 .958

286–16 95.4

286–17 .1824

286–18 −328

286–19 .267

286–20 −1020

286–21 4.45

286–22 $.40\overline{2}$

286–23 −4.999

286–24 −11.11

286–25 −1.354

286–26 $-.8\overline{56}$

286–27 −66.01

286–28 −25

286–29 $-.4024$

286–30 59.4

Lesson 157

287–A $b = 2(-\frac{1}{4}) + 1$. * $\frac{1}{2}$ * $\frac{1}{2}$ * Yes

287–B

a	$-\frac{1}{4}$	$\frac{1}{2}$	3	0	−1	−2	$-\frac{1}{2}$	$\frac{3}{4}$	−3
b	$\frac{1}{2}$	2	7	1	−1	−3	0	$\frac{5}{2}$	−5

287–C $\{(-\frac{1}{4},\frac{1}{2}), (\frac{1}{2},2), (3,7), (0,1), (-1,-1), (-2,-3), (-\frac{1}{2},0), (\frac{3}{4},\frac{5}{2}), (-3,-5)\}$ * Yes * Yes

287–D Examples are (1,3) and

$(\frac{1}{4}, \frac{3}{2})$ * Yes * For each member of D, there is a member in $\{(a,b) \mid b = 2a + 1\}$. D is an infinite set.

287–E First axis * With the numeral $^-2$ * Second axis * The dot for point $(^-3,0)$ and the dot for point $(0,^-3)$

287–F A: $(\frac{3}{4}, \frac{5}{2})$. B: $(\frac{1}{2}, 2)$. C: $(0,1)$. D: $(^-\frac{1}{4}, \frac{1}{2})$. E: $(^-\frac{1}{2}, 0)$. F: $(^-1, ^-1)$. G: $(^-2, ^-3)$. * Yes

287–G Yes * Yes * All the dots for points associated with members of $\{(a,b) \mid b = 2a + 1\}$ are not shown in D2.

287–H 0, 1, $^-\frac{1}{2}$, $^-1$, and $^-3$ * D * Infinite

287–I 1 * 3 * 0 * $^-1$ * $^-5$ * If a is replaced by 0, b must be replaced by a number greater than 1. If a is replaced by 1, b must be replaced by a number greater than 3. If a is replaced by $^-\frac{1}{2}$, b must be replaced by a number greater than 0. If a is replaced by $^-1$, b must be replaced by a number greater than $^-1$. If a is replaced by $^-3$, b must be replaced by a number greater than $^-5$.

287–J $\{(0,1\frac{1}{2}), (0,2), (0,3), (0,15), (1,3\frac{1}{4}), (1,4), (1,4\frac{1}{2}), (1,7), (^-\frac{1}{2}, \frac{1}{2}), (^-\frac{1}{2}, 1), (^-\frac{1}{2}, 3), (^-\frac{1}{2}, 8), (^-1, ^-\frac{1}{2}), (^-1, 0), (^-1, 2), (^-1, 5), (^-3, ^-4), (^-3, ^-2), (^-3, 0), (^-3, 1)\}$ * Yes

287–K For each replacement of a in $2a + 1$, you obtain a real number. For each real number, there are infinitely many other real numbers that are greater. * Yes

288–L A: $(1,3\frac{1}{4})$. B: $(^-\frac{1}{2}, 3)$. C: $(^-1,2)$. D: $(0,1\frac{1}{2})$. E: $(\frac{1}{2}, 1)$. F: $(^-\frac{1}{2}, \frac{1}{2})$. G: $(^-3,1)$. H: $(^-1, ^-\frac{1}{2})$. I: $(^-3, ^-2)$. J: $(0,2)$. K: $(^-3,0)$. * Yes

288–M Yes * A line separates each plane that includes the line into two half-planes. The line is the boundary of both half planes, and the line is not included in either half-plane. All the dots in the graph of $\{(a,b) \mid b > 2a + 1\}$ represent points in one of the half-planes determined by ℓ_2.

288–N The graph of $\{(a,b) \mid b < 2a + 1\}$ would be located to the right of the graph of $\{(a,b) \mid b > 2a + 1\}$.

288–O The graph of $\{(a,b) \mid b \geq 2a + 1\}$ is the union of the graph of $\{(a,b) \mid b = 2a + 1\}$ and the graph of $\{(a,b) \mid b > 2a + 1\}$. The graph of $\{(a,b) \mid b = 2a + 1\}$ is a picture of a line and the graph of $\{(a,b) \mid b > 2a + 1\}$ is a picture of a half-plane.

288–A Each member of the solution set of the compound condition that includes simple conditions and the connective "and" must be a member of both solution sets of the simple conditions.

289–B R: (1,2). O: (0,0). T: $(^-\frac{1}{2}, ^-1)$. U: $(^-1, ^-2)$ * ℓ_3 * Yes

289–C Use a straightedge to draw a picture of the line determined by the two points represented by the two dots. * Two points determine exactly one line.

289–D 3 * (0,3)

289–E (3,0)

289–F (3,0) * (0,3) * ℓ_4 * Yes

289–G 1 * Yes * By the letter W

289–H (1,2) * Just the dot for point R * $\{(1,2)\}$

289–I The well-defined property of addition * Yes

289–J By using the distributive property of addition * Yes

289–K $y = 2x \wedge 3x = 3$.
$y = 2x \wedge \frac{1}{3}(3x) = \frac{1}{3}(3)$.
Well-defined property of multiplication.
$y = 2x \wedge (\frac{1}{3} \cdot 3)x = 1$.
Associative property of multiplication.
$y = 2x \wedge 1x = 1$.
Reciprocal property.
$y = 2x \wedge x = 1$.
Identity-element property of multiplication * Yes

290–L Replace x in $y = 2x$ by 1 and use the identity-element property of multiplication. * $\{(1,2)\}$ * Condition A is equivalent to condition E.

290–M $x = y + 2 \wedge 2y = x + 1$.
$x = y + 2 \wedge 2y = (y + 2) + 1$.
$x = y + 2 \wedge 2y = y + 3$.
$x = y + 2 \wedge {}^-y + 2y = {}^-y + (y + 3)$.
$x = y + 2 \wedge {}^-y + 2y =$

$(^{-}y + y) + 3$.
$x = y + 2 \wedge y = 3$.
$x = 3 + 2 \wedge y = 3$.
Replace y by 3.
$x = 5 \wedge y = 3$. The solution set is $\{(5,3)\}$.

290–A $(x + y + 2z = 12) \wedge (x + y = 6) \wedge (x - 4 = 2)$. * Three * The connective "and"

290–B $\{(x,y,z) \mid x \in D \wedge y \in D \wedge z \in D\}$

290–C The intersection

290–D $x - 4 = 2$.

290–E 6

290–F $x + y = 6$

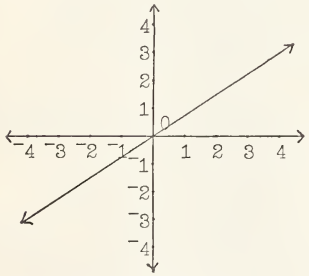
290–G $6 + y = 6$. * 0

290–H $6 + 0 + 2z = 12$. * 3

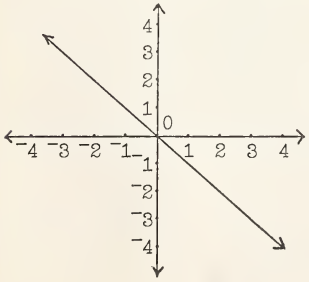
290–I $\{(6,0,3)\}$

On your own

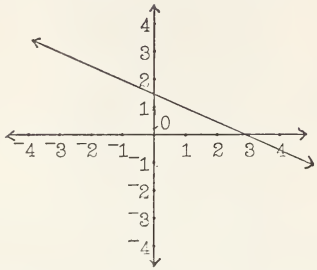
290–1 $\{(x,y) \mid 4y = 3x\}$ *



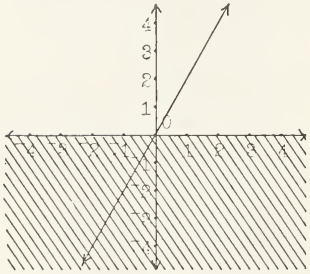
290–2 $\{(x,y) \mid y = ^{-}x\}$ *



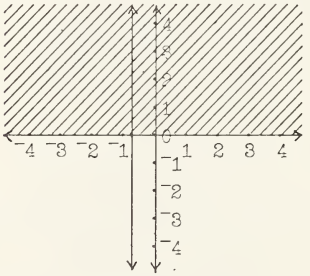
290–3 $\{(x,y) \mid ^{-}2x = 4y - 6\}$ *



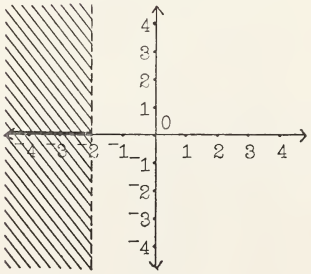
290–4 $\{(x,y) \mid x = \frac{1}{2}y \wedge y \leq 0\}$ *



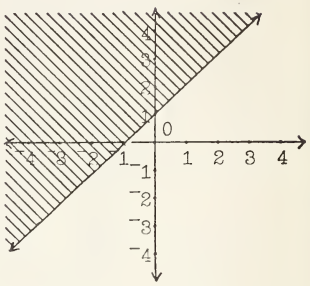
290–5 $\{(x,y) \mid ^{-}x = \frac{3}{4} \wedge y \geq 0\}$ *



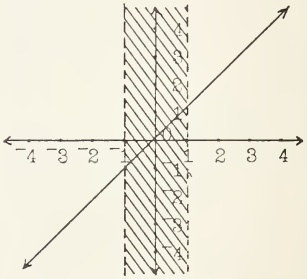
290–6 $\{(x,y) \mid y = 0 \wedge x < ^{-}2\}$ *



290–7 $\{(x,y) \mid 3y \geq 3x + 3\}$ *



290–8 $\{(x,y) \mid y = x \wedge ^{-}1 < x \leq 1\}$ *



290–9 $\{(\frac{1}{2}, 2\frac{1}{2}, 2\frac{1}{2})\}$

290–10 $\{(12, ^{-}6, 0)\}$

291–11 $\{(18, 3, 25\frac{1}{2})\}$

291–12 $\{(16, ^{-}8, ^{-}8)\}$

291–13 Point A: (1,2).
Point B: (0,1).
Point C: $(^{-}1,0)$.
Point D: $(^{-}2, ^{-}1)$.

291–14 1 * Yes

291–15 An example is $x \geq ^{-}1$.

291–16 An example is $x = y \wedge x \geq 0$. Another example is $x = y \wedge y \geq 0$.

291–17 An example is $y = ^{-}2$.

Keeping skilful

291–1 $\{x \mid \frac{4}{5} < x < 1\frac{7}{10}\}$

291–2 $\{y \mid y \geq ^{-}7.3\}$

291–3 $\{(^{-}.3, 9.9)\}$

291–4 $\{(12,3)\}$

291–5 $\{(5\frac{1}{24}, 12\frac{2}{3}, 32\frac{7}{24})\}$

291-6 $\{(12\frac{1}{2}, 10, 25)\}$

Lesson 158

292-A Yes

292-B No

292-C The difference of the elevation of Mt. Robson and the elevation of Ripple Rock.

292-D $12972 - y = 13019$.
$12972 + {}^{-}y = 13019$.
Difference property.
${}^{-}12972 + (12972 + {}^{-}y) = {}^{-}12972 + 13019$.
Well-defined property of addition.
$({}^{-}12972 + 12972) + {}^{-}y = 47$.
Associative property of addition.
$0 + {}^{-}y = 47$.
Additive-inverse property.
${}^{-}y = 47$.
Identity-element property of addition * Yes

292-E ${}^{-}47$

292-F $\{{}^{-}47\}$

292-G $12\frac{3}{4} - c$ refers to the number of feet of wire Mr. Rogers had left. This number is greater than $7\frac{1}{2}$. * The number of feet of copper wire Mr. Rogers used

292-H Yes

292-I Yes * Yes * Yes * Yes

292-J Yes * No * No

292-K No

293-L No * Numbers that are less than or equal to 0 satisfy $c < 5\frac{1}{4}$ but do not satisfy $c > 0$.

293-M Yes * $c > 0$ also means $0 < c$. * Yes * Since $c < 5\frac{1}{4}$ is equivalent to $12\frac{3}{4} - c > 7\frac{1}{2}$ and $0 < c$ is equivalent to $c > 0$, the condition $0 < c < 5\frac{1}{4}$ is equivalent to $12\frac{3}{4} - c > 7\frac{1}{2} \wedge c > 0$. Therefore, $0 < c < 5\frac{1}{4}$ has the same solution set as $12\frac{3}{4} - c > 7\frac{1}{2} \wedge c > 0$.

293-N Yes * Yes

293-O $n + 114.5$ refers to the number of acres of land Mr. Brennan owned after he bought the 114.5 acres. This number is less than 350. * The number of acres of land Mr. Brennan owned before he bought the 114.5 acres

293-P Yes

293-Q Yes. $n < 235.5$ is equivalent to $n + 114.5 < 350$. Therefore, any solution of $n < 235.5$ is a solution of $n + 114.5 < 350$. 95 is a solution of $n < 235.5$. * Yes. 0 is less than 235.5. * Yes. ${}^{-}20$ is less than 235.5. * Yes. $n < 235.5$ is equivalent to $n + 114.5 < 350$.

293-R Yes * Yes * No * No

294-S No * All number less than 0 satisfy $n < 235.5$ but do not satisfy $n \geqq 0$.

294-T Yes. $n \geqq 0$ also means $0 \leqq n$. * Yes. Since $0 \leqq n$ is equivalent to $n \geqq 0$ and $n < 235.5$ is equivalent to $n + 114.5 < 350$, the solution set of $0 \leqq n < 235.5$ is the same as the solution set of $n + 114.5 < 350 \wedge n \geqq 0$.

294-A The number of drapery hooks Mrs. Turner had after she bought the 24

294-B She needed 31 and did not have enough

294-C To the number of drapery hooks Mrs. Turner had before she bought 24 more

294-D $x > 26$ is equivalent to $x + 24 > 50$.

294-E 31

294-F Yes * Yes * Yes * Yes

294-G Yes * No * No

294-H No

295-I Yes

295-J No * Any number that is not an integer and that satisfies $26 < x < 31$ does not satisfy $x \in I$.

295-K $\{27, 28, 29, 30\}$

295-L Yes

295-M Yes *
$5m = m + {}^{-}6\frac{2}{3}$.
${}^{-}m + 5m = {}^{-}m + (m + {}^{-}6\frac{2}{3})$.
Well-defined property of addition.
$5m + {}^{-}m = ({}^{-}m + m) + {}^{-}6\frac{2}{3}$.
Associative and commutative properties of addition.
$5m - m = 0 + {}^{-}6\frac{2}{3}$.
Difference property and additive-inverse property.
$(5-1)m = {}^{-}6\frac{2}{3}$.
Distributive property and identity-element property of addition.
$4m = {}^{-}6\frac{2}{3}$.
Well-defined property of addition

295-N ${}^{-}1\frac{2}{3}$

295-O $\{{}^{-}1\frac{2}{3}\}$

295-P Yes

On your own

295-I x, number of post cards Nancy used.
$x > 14 \wedge 35 - x > 17 \wedge x \in I$.
$\{15, 16, 17\}$. Nancy used

15, 16, or 17 post cards.

295–2 x, number of feet sound travels in 60 seconds.
$1088/1 \sim x/60$.
{65280}. About 65,280 ft.

295–3 x, number of batteries Frank can buy.
$.20/1 \sim 1.25/x$.
{6.25}. 6 batteries

295–4 x, number of feet above water.
$12\frac{1}{2} + 9\frac{1}{4} + x < 35 \wedge x > 0$.
$\{x \mid 0 < x < 13\frac{1}{4}\}$. Between 0 and $13\frac{1}{4}$ ft.

295–5 x, amount Mrs. Allen had in savings account before she deposited $96.75.
$x + 96.75 < 185 \wedge x \geq 0$.
$\{x \mid 0 \leq x < 88.25\}$. Mrs. Allen had from $0 to $88.25 in her savings account before she deposited the $96.75.

295–6 x, amount Barbara spent for art supplies.
$x > 4.75 \wedge 10.00 - x > 2.50$.
$\{x \mid 4.75 < x < 7.50\}$. Barbara spent between $4.75 and $7.50 for art supplies.

295–7 x, amount of paint Mr. Henderson used to paint his garage.
$5\frac{1}{2} - x > 1\frac{5}{8} \wedge x > 0$.
$\{x \mid 0 < x < 3\frac{7}{8}\}$. Mr. Henderson used between 0 and $3\frac{7}{8}$ gal. of paint to paint his garage.

295–8 x, amount it will cost the Emerson Shipping Company to buy the rope.
$.02/1 \sim x/10000$ v
$1.95/100 \sim x/10000$.
{200.00, 195.00}. Either $195.00 or $200.00

295–9 $x > {}^{-}6\frac{3}{4} \wedge 4\frac{1}{8} + x < 2\frac{1}{2}$.
$\{x \mid {}^{-}6\frac{3}{4} < x < {}^{-}1\frac{5}{8}\}$

295–10 $4(3\frac{1}{2} + x) = {}^{-}8\frac{1}{3} \div 2\frac{1}{2}$.
$\{{}^{-}4\frac{1}{3}\}$

Lesson 159

296–A A number of feet in the length of the field * A number of feet in the width of the field

296–B Since the first component of 5/1 refers to a number of feet in the length of the field, a, not b, should be the first component of the rate pair equivalent to 5/1. Since the second component of 5/1 refers to a number of feet of the width of the field, b, not a, should be the second component of the rate pair.

296–C Since 5/1 and a/b represent the same comparison, they are equivalent rate pairs.

296–D Yes

296–E By using the definition of equivalent rate pairs * Each ordered pair that satisfies $a = 5b$ has a first component that is five times the second component.

296–F Yes

296–G Condition C was obtained from condition B by using the distributive property. The distributive property tells you that $5b + b$ is the same as $(5 + 1)b$. Condition D was obtained from condition C by using the well-defined property of multiplication. The well-defined property of multiplication tells you that you can replace $5 + 1$ by 6 in the product $(5 + 1)b$ without affecting the product. Condition E was obtained from condition D by using the distributive property. Condition F was obtained from condition E by using the well-defined property of multiplication.

296–H $12b = 900$.
$\frac{1}{12}(12b) = \frac{1}{12}(900)$.
Well-defined property of multiplication.
$(\frac{1}{12} \cdot 12)b = 75$.
Associative property of multiplication.
$1b = 75$.
Reciprocal property.
$b = 75$.
Identity-element property of multiplication

297–I (Column 1) 375

297–J (Column 1) {(375,75)}

297–K (Column 1) The length is 375 ft. and the width is 75 ft.

297–L Yes * Yes

297–A No * The two trains travelled for the same amount of time, but since the two trains travelled at different rates, they travelled different distances.

297–B No

297–C No

297–D The total distance travelled by both trains

297–E In how many hours the two trains were 475 mi. apart

297–F The first component refers to the number of miles the

passenger train travelled in 1 hour. The second component refers to the number of hours it took the passenger train to travel 60 mi.

297–G The first component of 60/1 refers to a number of miles the passenger train travelled, so x, not z, should be the first component of a rate pair that is equivalent to 60/1. The second component of 60/1 refers to a number of hours the passenger train travelled, so z not x, should be the second component of the rate pair.

297–H Since 60/1 and x/z represent the same rate, they are equivalent rate pairs.

297–I (Column 2) The first component refers to the number of miles the freight train travelled in 1 hour. The second component refers to the number of hours it took the freight train to travel 35 mi.

297–J (Column 2) y refers to the distance the freight train travelled in z hours. Since it is assumed that the rate did not vary, y/z is equivalent to 35/1.

297–K (Column 2) Since 35/1 and y/z represent the same rate, they are equivalent rate pairs.

298–L By using the definition of equivalent rate pairs

298–M By using the definition of equivalent rate pairs

298–N $60z + 35z = 475$.

298–O Yes * Condition G was obtained from $60z + 35z = 475$ by using the distributive property. Condition H was obtained from condition G by using the well-defined property of multiplication.

298–P $95z = 475$.
$\frac{1}{95}(95z) = \frac{1}{95}(475)$.
Well-defined property of multiplication.
$(\frac{1}{95} \cdot 95)z = 5$.
Associative property of multiplication.
$1z = 5$.
Reciprocal property.
$z = 5$.
Identity-element property of multiplication * 5

298–Q 300 * 175

298–R If the passenger train travelled 5 hours at the rate of 60 miles per hour, then the passenger train travelled 300 miles. If the freight train travelled 5 hours at the rate of 35 miles per hour, then the freight train travelled 175 miles. If the passenger train travelled 300 miles and the freight train travelled 175 miles, then the trains were 475 miles apart.

On your own

298–I x, distance from Winnipeg to Regina.
y, distance from Winnipeg to Edmonton.
$\frac{3}{7}/1 \sim x/y \wedge y - x = 420$.
{(315,735)}. The distance from Winnipeg to Regina by commercial air line is 315 mi. and the distance from Winnipeg to Edmonton is 735 mi.

298–2 x, temperature on Friday.
y, temperature on Saturday.
$x - \bar{}12 = 5 \wedge y - x = 4$.
{($\bar{}7$,$\bar{}8$)}. The temperature in Milwaukee was 7 degrees below zero on Friday and 8 degrees below zero on Saturday.

299–3 x, elevation of Ottawa.
y, elevation of Calgary.
$305 - 90 = x \wedge \frac{1}{16}/1 \sim x/y$.
{(215, 3445)}.
The elevation of Ottawa is 215 ft. and the elevation of Calgary is 3445 ft.

299–4 x, length of piece of pine.
y, length of piece of oak.
z, length of piece of birch.
$x + y + z = 36 \wedge 4/1 \sim y/x \wedge 5/1 \sim z/x$.
{(3.6,14.4,18)}. The length of the piece of pine was 3.6 ft. The length of the piece of oak was 14.4 ft. The length of the piece of birch was 18 ft.

299–5 x, regular price of refrigerator.
y, amount of discount.
$20/100 \sim y/x \wedge x - y = 335$.
{(418.75,83.75)}. The regular price of the refrigerator was $418.75. The discount was $83.75.

299–6 x, amount that Jane spent.
y, amount that Martha spent.
$x + y < 3.50 \wedge x = 1.05 + .25 + .95 \wedge y > 0$.

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$\{(x,y) \mid x = 2.25 \wedge 0 < y < 1.25\}$.
Jane spent \$2.25 and Martha spent between \$0 and \$1.25.

299–7 x, population of Hartville in 1910.
y, population of Hartville in 1920.
$35105 + 431 = x \wedge x - 230 = y$.
$\{(35536,35306)\}$. 35,306

299–8 x, distance from school that Jim was after 3 hours.
y, distance from school that Sam was after 3 hours.
z, distance apart the boys were after 3 hours.
$5/1 \sim x/3 \wedge 7\frac{1}{2}/1 \sim y/3 \wedge x + y = z$.
$\{(15,22\frac{1}{2},37\frac{1}{2})\}$. $37\frac{1}{2}$ mi.

299–9 x, amount Clyde weighs.
y, amount Henry weighs.
$x + y = 265 \wedge x - y = 7\frac{1}{2}$.
$\{(136\frac{1}{4},128\frac{3}{4})\}$. Clyde weighs $136\frac{1}{4}$ lb. Henry weighs $128\frac{3}{4}$ lb.

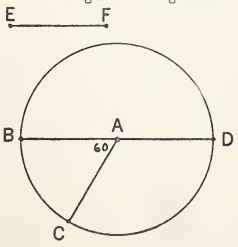
299–10 $x + y > {}^{-}6 \wedge x = 2(3\frac{1}{2})$.
$\{(x,y) \mid x = 7 \wedge y > {}^{-}13\}$

299–11 $x - y < 3 \wedge y + 3\frac{1}{2} = 6$.
$\{(x,y) \mid x < 5\frac{1}{2} \wedge y = 2\frac{1}{2}\}$

299–12 $x + y + z = {}^{-}10\frac{1}{2} \wedge 4/1 \sim y/x \wedge 2/1 \sim z/x$.
$\{({}^{-}1\frac{1}{2},{}^{-}6,{}^{-}3)\}$

Keeping skilful

299–1 See drawing below. $\overline{EF}$ is the given segment.



299–2 See drawing for 241–1.

299–3 See drawing for 241–1.

299–4 $60 * 300$

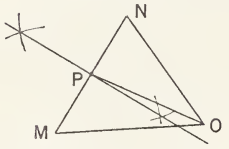
299–5 Semicircle BCD

299–6 180

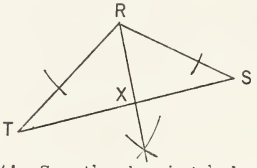
299–7 Minor $\overset{\frown}{CD}$

299–8 120

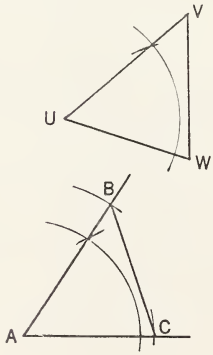
299–9 See the drawing below.



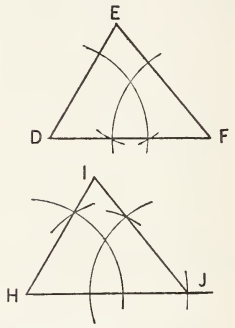
299–10 See the drawing below. $\angle R$ is the obtuse angle of ΔRST. Segment RX is the bisector of $\angle R$ of ΔRST. Ray RX is the bisector of $\angle R$.



299–11 See the drawing below. $\Delta ABC \cong \Delta UVW$. $\overline{AC}$ was constructed congruent to $\overline{UW}$. $\angle A$ was constructed congruent to $\angle U$. $\overline{AB}$ was constructed congruent to $\overline{UV}$.



299–12 See the drawing below. $\Delta HIJ \cong \Delta DEF$. $\overline{HJ}$ was constructed congruent to $\overline{DF}$. $\angle H$ was constructed congruent to $\angle D$. $\angle J$ was constructed congruent to $\angle F$.



Applying mathematics

300–1 x, Don's age now.
$x + 12 > 30 \wedge x - 11 < 15$.
$\{x \mid 18 < x < 26\}$.
Don is between 18 and 26 years old.

300–2 x, distance from Fort William to Sarnia.
$3/1 \sim 1512/x$.
$\{504\}$. 504 mi.

300–3 x, the difference of the amount of money Mr. Berry had on October 1 and the amount he had on October 31.
$500 - {}^{-}200 = x$.
$\{700\}$. \$700

300–4 x, number of acres Mr. Johnson will have left in his farm.
$205 - 40 = x \vee 205 - 25\frac{1}{2} = x$.
$\{165,179\frac{1}{2}\}$. Mr. Johnson will have either 165 acres left or $179\frac{1}{2}$ acres left.

300–5 x, rate of discount.
y, amount of discount.
$y/35.95 \sim x/100 \wedge$

$y = 35.95 - 28.76$.
$\{(20,7.19)\}$. 20%

300–6 x, lowest temperature recorded in Canada that year.
$x - {}^{-}81.4 = 191.6$, or
${}^{-}81.4 + 194.8 = x$.
$\{113.4\}$. 113.4 degrees

300–7 x, number of hours Al worked.
$x + 6\frac{1}{2} < 12 \wedge x > 0$.
$\{x \mid 0 < x < 5\frac{1}{2}\}$. Al worked between 0 and $5\frac{1}{2}$ hr.

300–8 x, amount spent for lights.
y, amount spent for wall switches.
z, amount spent in all.
$1/10.95 \sim 2/x \wedge 2/1.15 \sim 8/y \wedge x + y + 6.95 = z$.
$\{(21.90,4.60,33.45)\}$.
\$33.45

300–9 x, number of degrees the temperature rises in 3 hours.
y, average rise per hour in degrees.
$27.9 - {}^{-}15 = x \wedge x/3 \sim y/1$.
$\{(42.9,14.3)\}$. 14.3 degrees

300–10 x, population of Morton City.
y, population of Huntsville.
z, population of Warner.
$9/1 \sim x/y \wedge 2/1 \sim z/y \wedge x + z = 278762$.
$\{(228078,25342,50684)\}$.
The population of Morton City is 228,078. The population of Huntsville is 25,342. The population of Warner is 50,684.

300–11 x, number of five-cent stamps Ray bought.
y, number of four-cent stamps Ray bought.
$x + y < 12 \wedge x + 6 = 10 \wedge y \in I_p$.
$\{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7)\}$. Ray bought 4 five-cent stamps and from 1 through 7 four-cent stamps.

300–12 x, length of side BA.
$x + 7\frac{1}{2} + 10 \geq 25\frac{1}{2} \wedge x < 7\frac{1}{2} + 10$.
$\{x \mid 8 \leq x < 17\frac{1}{2}\}$.
Side BA is from 8 inches in length to $17\frac{1}{2}$ inches in length.

300–13 x, length of altitude.
y, area of parallelogram.
$2.5/1 \sim 11.25/x \wedge 11.25/1 \sim y/x$.
$\{(4.5,50.625)\}$.
50.625 sq. ft.

300–14 $3\frac{1}{2}/5\frac{1}{2} \sim 12/x \vee 3\frac{1}{2}/5\frac{1}{2} \sim 15/x$.
$\{18\frac{6}{7}, 23\frac{4}{7}\}$. The rate pairs are $12/18\frac{6}{7}$ or $15/23\frac{4}{7}$.

300–15 $x + y \leq 2.5 \wedge x = {}^{-}.68 \div {}^{-}.4$.
$\{(x,y) \mid x = 1.7 \wedge y \leq .8\}$

300–16 $\sim(x > {}^{-}25.5 - {}^{-}16.4)$, or $x \leq {}^{-}25.5 - {}^{-}16.4$.
$\{x \mid x \leq {}^{-}9.1\}$

300–17 $x = 5y \wedge x + y = 13\frac{1}{2} \wedge y - z = 1\frac{1}{2}$.
$\{(11\frac{1}{4}, 2\frac{1}{4}, \frac{3}{4})\}$

Checking up

Test 142

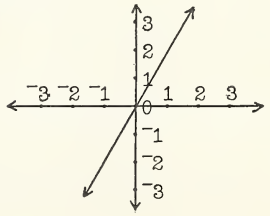
301–1 F
301–2 T
301–3 F
301–4 T
301–5 T
301–6 T
301–7 F
301–8 T
301–9 F
301–10 F

Test 143

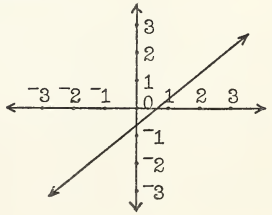
301–11 10
301–12 irrational
301–13 1
301–14 left
301–15 ${}^{-}8$
301–16 well-defined
301–17 density
301–18 8.90
301–19 second
301–20 20

Test 144

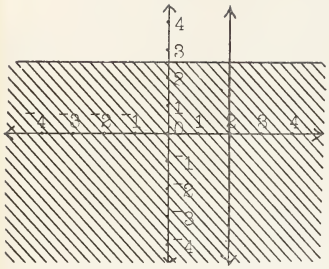
301–21 $\{{}^{-}11\}$
301–22 $\{1.2\}$
301–23 $\{{}^{-}1\frac{1}{4}\}$
301–24 $\{m \mid m < {}^{-}4\frac{4}{7}\}$
301–25 $\{k \mid k < {}^{-}4.7\}$
301–26 $\{x \mid x \geq 1.2\}$
301–27 $\{(1\frac{1}{2}, 7, 3\frac{1}{2})\}$
301–28 Identity-element property of multiplication
301–29 Distributive property
301–30 Identity-element property of addition
301–31 Negative-multiplier property of "less than"
301–32 Definition of equivalent rate pairs
301–33



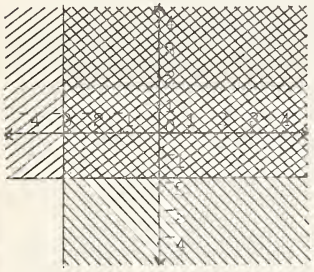
301–34



301–35



301–36



Test 145

302–1 F

302–2 T

302–3 F

302–4 T

302–5 F

302–6 F

302–7 T

302–8 T

302–9 T

302–10 T

Test 146

302–11 6 H

302–12 78

302–13 .0826

302–14 terminal

302–15 s–s–s

302–16 $1/-\frac{1}{7}$, or -7

302–17 well-defined

302–18 closure

302–19 $-\frac{32}{1000}$, or $-\frac{8}{125}$

302–20 solution set

Test 147

302–21 x, amount it costs for the box seat.
$3\frac{1}{3}/1 \sim x/.75$.
{2.50}. $2.50

302–22 x, per cent of hour used for commercials.
$14/60 \sim x/100$.
$\{23\frac{1}{3}\}$. $23\frac{1}{3}\%$

302–23 x, average reading.
y, sum of the five readings.
$x/1 \sim y/5 \wedge y = 48 + 65 + 110 + 216 + 205$.
{(128.8,644)}. 128.8 ft.

302–24 x, amount walk cost.
y, amount Mr. Parker had to pay.
$1.60/1 \sim x/45 \wedge 50/100 \sim y/x$.
{(72,86)}. $86.00

302–25 x, amount of line Don has.
y, amount of line Oscar has.
$x + y < 120 \wedge x = 25 + 40 \wedge y > 0$.
$\{(x,y) \mid x = 65 \wedge 0 < y < 55\}$.
Don has 65 yd. of line and Oscar has between 0 yd. and 55 yd.

302–26 $5.7x \geq -1.9$.
$\{x \mid x \geq -\frac{1}{3}\}$

302–27 $x - y = 2\frac{3}{4} \wedge 6\frac{2}{3}y = -\frac{60}{9}$.
$\{(1\frac{3}{4},-1)\}$

302–28 $-8.2 - x = -4 \wedge x + y = -5 \wedge xy = z$.
{(−4.2,−.8,3.36)}

Lesson 160

303–A 1

303–B 1

303–C Point V was located so that $\overline{UV} \cong \overline{MN}$. Point W was located so that $\overline{VW} \cong \overline{MN}$. Point X was located so that $\overline{WX} \cong \overline{MN}$. Point Y was located so that $\overline{XY} \cong \overline{MN}$. * 1

304–D 4 * 5 * Because point T is to the right of point X, the coördinate of T is greater than the coördinate of X. This means that the absolute difference of the coördinates of the endpoints of $\overline{ST}$ is greater than the absolute difference of the coördinates of the endpoints of $\overline{SX}$. * Because point T is to the left of point Y, the coördinate of T is less than the coördinate of Y.

304–E Yes * .4 * 4.4

304–F No

304–G The segments represented by the different pieces of chalk may not be congruent.

304–H Yes

305–A 1 metre * Yes

305–B 1 decimetre * Yes * 1 decimetre

305–C 1 centimetre * 1 centimetre

305–D 1 millimetre

305–E A decimetre * A centimetre * They are shorter than the metre.

305–F 1 decametre * 1 hectometre * 1 kilometre * 10 metres is the same as 10^1m. 100 metres is the same as 10^2m. 1000 metres is the same as 10^3m.

305–G A decametre, a hectometre, and a kilometre.

305–H A kilometre * A hectometre * A kilometre

305–I The prefixes for the names of the units of measure express powers of ten.

305–J .01/1

305–K 10/1 * 1000/1 * .001/1

305–L For the problem in D8, you want to find how many kilometres is the same length as 50,000 metres. The rate pair 1000/1 represents the comparison of the number of metres in 1 kilometre with 1 kilometre.

305–M The first component of 1000/1 refers to a number of metres and so does the first component of $50{,}000/x$. The second component of 1000/1 refers to a number of kilometres and so does the second component of $50{,}000/x$. * The number of kilometres that is the same length as 50,000 metres

305–N {50} * The length of the walk is 50 kilometres.

306–O .01/1 represents the comparison of the number of metres in 1 centimetre with 1 centimetre. The first component of .01/1 refers to a number of metres and so does the first component of $x/2.1$. The second component of .01/1 refers to a number of centimetres and so does the second component of $x/2.1$. * The number of metres in 2.1 centimetres

306–P Yes

306–Q Yes
2.1 centimetres * The number of millimetres in x metres

306–R The number of metres in

306–S $.001/1 \sim x/y$.

306–T {(.021,21)}

306–U 21 mm.

307–A The inch * The metre * The mile

307–B .394/1 represents the comparison of the number of inches in 1 centimetre with 1 centimetre.

307–C The first component of .394/1 refers to a number of inches and so does the first component of $x/76$. The second component of .394/1 refers to a number of centimetres and so does the second component of $x/76$. * The number of inches in 76 centimetres

307–D {29.944} * 29.944 in.

307–E 1.094/1 represents the comparison of the number of yards in 1 metre with 1 metre. * $k/1.3$ * $1.094/1 \sim k/1.3$.

307–F 36/1 represents the comparison of the number of inches in 1 yard with 1 yard.

307–G h/k * The first component of 36/1 refers to a number of inches so h should be a first component. Since both the second component of 36/1 and k refer to a number of yards, k should be a second component.

307–H {(1.4222,51.1992)} * 51.1992 in.

On your own

307–1 Assign 1 to K, 2 to L, 4 to M, and 5 to N.

307–2 $m(\overline{JK}) = 1$. * $m(\overline{JL}) = 2$. * $m(\overline{LM}) = 2$.

307–3 $m(\overline{JM}) = 4$. * $m(\overline{JN}) = 5$. * P is to the right of M.

Therefore, the coördinate of P is greater than 4 and $m(\overline{JP}) > 4$. * P is to the left of N. Therefore, the coördinate of P is less than 5 and $m(\overline{JP}) < 5$.

307–4 $m(\overline{MP}) = \frac{1}{2}$. * $m(\overline{JP}) = 4\frac{1}{2}$.

308–5 .001 kilometre

308–6 1000 millimetres

308–7 .1 centimetre

308–8 500 metres

308–9 2000 kilometres

308–10 420 decametres

308–11 x, length of car in metres.
$.01/1 \sim x/488$.
{4.88}. 4.88 m.

308–12 x, length of steeplechase in kilometres.
$1000/1 \sim 3000/x$.
{3}. 3 km.

308–13 x, number of centimetres of lace that Carol bought.
$.01/1 \sim \frac{3}{4}/x$.
{75}. 75 cm.

308–14 x, bore of engine in inches.
$.039/1 \sim x/78$.
{3.042}. 3.042 in.

308–15 x, thickness of sheet of metal in inches.
$.394/1 \sim x/(8 \times 10^{-4})$.
$\{3.152 \times 10^{-4}\}$.
3.152×10^{-4} in.

308–16 x, apogee of moon in kilometres.
$.621/1 \sim 252710/x$.
$\{406940\frac{260}{621}\}$. 406,940 km.

308–17 x, height in inches.
y, height in centimetres.
$12/1 \sim x/6 \wedge .394/1 \sim x/y$.
$\{(72, 182\frac{146}{197})\}$. 183 cm.

Applying mathematics

309–1 (Column 1) Radio communications wave

309–2 (Column 1) Infrared wave

309–3 (Column 1) Microwave

309–4 (Column 1) X ray

309–5 (Column 1) Radio communications wave

309–6 (Column 1) Infrared wave

309–7 Ultraviolet wave

309–8 Gamma ray

309–9 x, number of times the microwave is as long as the X ray.
$x/1 \sim 10^{-2}/10^{-9}$.
$\{10^{7}\}$. 10^{7} times

309–10 x, fraction the length of the ultraviolet wave is of the length of the infrared wave.
$x/1 \sim 10^{-7}/10^{-5}$.
$\{10^{-2}\}$. The length of the ultraviolet wave is $\frac{1}{100}$ the length of the infrared wave.

309–11 x, length of radio communications wave in decametres.
$10/1 \sim 10^{2}/x$.
{10}. 10 dkm.

309–12 x, length of X ray in millimetres.
$.001/1 \sim 10^{-10}/x$.
$\{10^{-7}\}$. 10^{-7} mm.

309–13 x, length of ultraviolet wave in metres.
$.01/1 \sim x/10^{-6}$.
$\{10^{-8}\}$. 10^{-8} m.

309–14 x, length of infrared wave in kilometres.
$1000/1 \sim 10^{-5}/x$.
$\{10^{-8}\}$. 10^{-8} km.

309–15 x, length of gamma ray in metres.
$.01/1 \sim x/10^{-12}$.
$\{10^{-14}\}$. 10^{-14} m.

309–16 x, length of radio communications wave in kilometres.
$1000/1 \sim (6.5 \times 10^{2})/x$.
{.65}. .65 km.

309–17 x, length of microwave in hectometres.
$100/1 \sim 2.2 \times 10^{-2}/x$.
$\{2.2 \times 10^{-4}\}$. 2.2×10^{-4} hm.

309–18 x, length of radio communications wave in metres.
y, length of wave in millimetres.

$.01/1 \sim x/85 \wedge .001/1 \sim x/y$.
$\{(.85,850)\}$. 850 mm.

309–19 x, length of ultraviolet wave in metres.
y, length of wave in centimetres.
$.001/1 \sim x/(7.5 \times 10^{-5}) \wedge .01/1 \sim x/y$.
$\{(7.5 \times 10^{-8}, 7.5 \times 10^{-6})\}$.
7.5×10^{-6} cm.

309–1 (Column 2) x, number of ounces the tomato juice weighed.
$.035/1 \sim x/720.8$.
25.228 oz.

309–2 (Column 2) x, number of pounds of cheese.
$2.1/1 \sim x/(2.65 \times 10^5)$.
583,000 lb.

309–3 (Column 2) x, number of kilolitres of oil the tank holds.
$1000/1 \sim 4100/x$. 4.1 kl.

309–4 (Column 2) x, number of milligrams of medicine Mr. Campbell takes each day.
$1000/1 \sim x/.0025$. 2.5 mg.

309–5 (Column 2) x, number of millilitres of glucose solution Mr. Harvey prepared.
$1000/1 \sim x/2.5$. 2500 ml.

309–6 (Column 2) x, weight of sample in grams.
$1000/1 \sim 500/x$. .5 g.

310–7 x, Betty's weight in kilograms.
$2.2/1 \sim 95/x$. 43.2 kg.

310–8 x, number of grams of hydrogen chloride that the 20 litres of solution contained.
$79/1 \sim x/20$. 1580 g.

310–9 x, number of pounds of squash that Mrs. Wilmington bought.
$2.2/1 \sim x/3.5$. 7.7 lb.

310–10 x, weight of candy bar in grams.
$.035/1 \sim 2/x$. 57.1 g.

310–11 x, number of calories in 1 kilogram of butter.
y, number of kilograms that is the same weight as 100 g.
$1000/1 \sim 100/y \wedge 716/y \sim x/1$.
7160 calories

310–12 x, number of pounds that is the same weight as 20 kg.
y, number of pounds of excess baggage Miss Klein has.
$2.2/1 \sim x/20 \wedge 51 - x = y$.
7 lb.

310–13 x, number of quarts of oil Mr. Heath needs to fill the crankcase.
y, amount the oil will cost.
$.88/1 \sim x/5.5 \wedge .45/1 \sim y/x$. $2.68.
This answer assumes that Mr. Heath can buy oil by the hundredth of a quart.

310–14 x, number of quarts of gasoline Mr. Green bought.
y, number of gallons of gasoline Mr. Green bought.
$.88/1 \sim x/45 \wedge 4/1 \sim x/y$. 9.9 gal.

310–15 x, number of grams of silver that the chemist dissolved in acid.
y, number of kilograms of silver that the chemist dissolved in acid.
$1000/1 \sim 750/x \wedge 1000/1 \sim x/y$.
7.5×10^{-4} kg.

310–16 x, number of quarts of gasoline the tank holds.
y, number of liters of gasoline the tank holds.
$4/1 \sim x/15.9 \wedge .88/1 \sim x/y$. 72.3 l.

Lesson 161

310–A 2 inches.

310–B 8 * $\frac{1}{8}$ inch

310–C Yes * Yes * Yes

310–D 3 in. * 3 in.

310–E 3 in.

311–F Two * Six

311–G $\frac{7}{2}$ in. * $3\frac{1}{2}$ in. * $3\frac{1}{2}$ in.

311–H Four * Twelve

311–I $3\frac{2}{4}$ in. * $3\frac{2}{4}$ in.

311–J A physical measure of $3\frac{2}{4}$ in. indicates that $\frac{1}{4}$ inch was used as a unit of length. A physical measure of $3\frac{1}{2}$ in. indicates that $\frac{1}{2}$ inch was used as a unit of length.

311–K $\frac{1}{12}$ ft. * 1 cm. * $\frac{1}{3}$ yd.

311–L A unit of length of 1 yd. was used to obtain a physical measure of 14 yd. and a unit of $\frac{1}{3}$ yd. was used to obtain a physical measure of $14\frac{0}{3}$ yd.

311–M .1 mm. * .001 km. * .01 mi.

311–N A different unit of length was used to obtain each physical measure. 45 m. indicates that 1 m. was used as the unit of length. 45.0 m. indicates that .1 m. was used as the unit of length.

312–O 4 ft.

312–P Yes * Yes * Yes

312–Q Pipes D and E * $4\frac{1}{2}$ ft.

312–R Pipe G * Pipes B and C

312–S Pipe F

312–T $4\frac{0}{2}$ ft. is 8 unit lengths. $4\frac{1}{2}$ ft. is 9 unit lengths. We use $4\frac{0}{2}$ ft., rather than $4\frac{1}{2}$ ft., because $4\frac{0}{2}$ ft. is an even number of unit lengths.

312–U $4\frac{2}{4}$ ft. * $4\frac{2}{4}$ ft. is 18, or an even number of, unit lengths. $4\frac{1}{4}$ ft. is 17, or an odd number of, unit lengths.

312–V No * No

312–A The set of numbers that contains two given numbers and all the numbers between the two given numbers is a closed interval. If the numbers are members of D, then the set is a closed interval in D. The two given numbers are the limits of the interval. * $4\frac{3}{4}$ and $5\frac{1}{4}$

313–B $\frac{1}{4}$

313–C Yes

313–D $\frac{1}{3}$ ft. * $\frac{1}{6}$ ft.

313–E .1 in. * .05 in.

313–F .01 mm. * .005 mm.

313–G .001 m. * .0005 m.

313–H $\frac{1}{8}$ yd. * $\frac{1}{16}$ yd.

313–I 1 km. * .5 km.

313–J Multiply the greatest possible error by 2.

313–K 1 ft. * 1000 ft. * 1 ft.

313–L 2600 ft. * 2600 ft. ±5 * 2600 ft. ±50

313–M 10 yd. * 100 mi. * 10 mi.

313–N Yes * Yes * Add the greatest possible error of the physical measure of $5\frac{0}{2}$ ft. to $5\frac{0}{2}$.

313–O 5.75 and 5.85 * $\{x \mid 5.75 \leq x \leq 5.85\}$

313–P $\{x \mid 7150 \leq x \leq 7250\}$

313–Q $\{x \mid 25.35 \leq x \leq 25.45\}$. U = D.

313–R The tolerance is used to find the limits of the interval. The lower limit is found by subtracting the tolerance from 25.4. The upper limit is found by adding the tolerance to 25.4. * The greatest possible error is used to find the limits of the interval determined by the physical measure of 25.4. To find the lower limit, the greatest possible error is subtracted from 25.4. To find the upper limit, the greatest possible error is added to 25.4.

313–S The unit of length used to obtain 25.4 is .1. One half of .1 is .05.

314–T $\{x \mid 9.345 \leq x \leq 9.355\}$

On your own

314–1 $\frac{1}{32}$ in. * $\frac{1}{64}$ * $8\frac{5}{64}$ and $8\frac{7}{64}$ * $\{x \mid 8\frac{5}{64} \leq x \leq 8\frac{7}{64}\}$

314–2 .01 cm. * .005 * .035 and .045 * $\{x \mid .035 \leq x \leq .045\}$

314–3 10 ft. * 5 * 595 and 605 * $\{x \mid 595 \leq x \leq 605\}$

314–4 .1 mi. * .05 * 13.55 and 13.65 * $\{x \mid 13.55 \leq x \leq 13.65\}$

314–5 $\frac{1}{3}$ yd. * $\frac{1}{6}$ * $18\frac{3}{6}$ and $18\frac{5}{6}$ * $\{x \mid 18\frac{3}{6} \leq x \leq 18\frac{5}{6}\}$

314–6 1 km. * .5 * 48.5 and 49.5 * $\{x \mid 48.5 \leq x \leq 49.5\}$

314–7 $\frac{1}{8}$ in. * $\frac{1}{16}$ * $54\frac{3}{16}$ and $54\frac{5}{16}$ * $\{x \mid 54\frac{3}{16} \leq x \leq 54\frac{5}{16}\}$

314–8 100 yd. * 50 * 1850 and 1950 * $\{x \mid 1850 \leq x \leq 1950\}$

314–9 $\frac{1}{12}$ ft. * $\frac{1}{24}$ * $15\frac{13}{24}$ and $15\frac{15}{24}$ * $\{x \mid 15\frac{13}{24} \leq x \leq 15\frac{15}{24}\}$

314–10 .0001 cm. * .00005 * .00395 and .00405 * $\{x \mid .00395 \leq x \leq .00405\}$

314–11 10 m. * 5 * 315 and 325 * $\{x \mid 315 \leq x \leq 325\}$

314–12 $\frac{1}{2}$ in. * $\frac{1}{4}$ * $15\frac{3}{4}$ and $16\frac{1}{4}$ * $\{x \mid 15\frac{3}{4} \leq x \leq 16\frac{1}{4}\}$

314–13 .1 km. * .05 * 162.85 and 162.95 * $\{x \mid 162.85 \leq x \leq 162.95\}$

314–14 1 ft. * .5 * 3409.5 and 3410.5 * $\{x \mid 3409.5 \leq x \leq 3410.5\}$

314–15 1000 mi. * 500 * 249,500 and 250,500 * $\{x \mid 249500 \leq x \leq 250500\}$

314–16 $\frac{1}{8}$ in. * $\frac{1}{16}$ * $999\frac{15}{16}$ and $1000\frac{1}{16}$ * $\{x \mid 999\frac{15}{16} \leq x \leq 1000\frac{1}{16}\}$

314–17 .1 mm. * .05 * 3.95 and 4.05 * $\{x \mid 3.95 \leq x \leq 4.05\}$

314–18 $\frac{1}{2}$ yd. * $\frac{1}{4}$ * $59\frac{3}{4}$ and $60\frac{1}{4}$ * $\{x \mid 59\frac{3}{4} \leq x \leq 60\frac{1}{4}\}$

314–19 .1 mm. * .05 * .25 and .35 * $\{x \mid .25 \leq x \leq .35\}$

314–20 .01 cm. * .005 * 15.995 and 16.005 * $\{x \mid 15.995 \leq x \leq 16.005\}$

314–21 $\{x \mid 14.75 \leq x \leq 14.85\}$

314–22 $\{w \mid 36\frac{3}{4} \leq w \leq 37\frac{1}{4}\}$

314–23 $\{a \mid 620.5 \leq a \leq 621.5\}$

314–24 $\{x \mid 10\frac{7}{128} \leq x \leq 10\frac{9}{128}\}$

314–25 $\{x \mid 9.045 \leq x \leq 9.055\}$

314–26 $\{z \mid \frac{3}{8} \leq z \leq \frac{5}{8}\}$

314–27 $\{m \mid 5.95 \leq m \leq 6.05\}$

314–28 $\{x \mid 33\frac{1}{6} \leq x \leq 33\frac{3}{6}\}$

314–29 $\{n \mid \frac{13}{2000} \leq n \leq \frac{15}{2000}\}$, or

$\{n \mid .0065 \leq n \leq .0075\}$

Lesson 162

314–A $6\frac{1}{4}$ m. and $5\frac{1}{2}$ m.

314–B $\frac{1}{8}$ m. * $\frac{1}{4}$ m.

314–C $\frac{1}{4}$ m. is less than $\frac{1}{2}$ m. * The greatest possible error of $6\frac{1}{4}$ m. is less than the greatest possible error of $5\frac{1}{2}$ m.

314–D The greatest possible error of 1.005 cm. is .0005 cm. The greatest possible error of 6.04 cm. is .005 cm. .0005 cm. is less than .005 cm.

314–E 135 yd. * The greatest possible error of 135 yd. is $\frac{1}{2}$ yd. The greatest possible error of 6.2 mi. is .05 mi. $\frac{1}{2}$ yd. is less than .05 mi.

314–F Yes. Each has a greatest possible error of $\frac{1}{24}$ ft. * No. They both have the same greatest possible error.

315–G (Column 1) $3\frac{1}{36}$ yd.

315–H (Column 1) 10.80 km.

315–I (Column 1) $6\frac{4}{8}$ rd.

315–J (Column 1) $8\frac{3}{4}$ in.

315–K .01 mm.

315–L 6 ft.

315–A .1 mi. * .05 mi.

315–B .05/2.5 ~ r/1. * D * The first component of .05/2.5 refers to the greatest possible error of 2.5 mi. and the second component refers to the length of 2.5 mi. * Yes

315–C Condition J can be obtained from condition I by using the definition of equivalent rate pairs. * Condition K can be obtained from condition J by using the well-defined property of multiplication and the multiplicative inverse of 2.5.

315–D .02 * Yes

315–E .005 cm. * .05 cm. * 2.25 cm.

315–F Yes * By using the definition of equivalent rate pairs

315–G (Column 2) .005 and 2.25 * Yes * Yes

315–H (Column 2) .05/35.0 ~ x/1.

315–I (Column 2) .001

315–J (Column 2) 35.0 cm.

316–K .5 mi. * .5 mi. * They have the same greatest possible error.

316–L .5/20 ~ x/1. $20x = .5$. $x = .025$. The relative error of 20 mi. is .025. * .5/50 ~ x/1. $50x = .5$. $x = .01$. The relative error of 50 mi. is .01.

316–M 50 mi. * 50 mi.

316–N $4\frac{3}{4}$ in. is more precise. * $\frac{1}{4}/4\frac{1}{2} \sim x/1$. $4\frac{1}{2}x = \frac{1}{4}$. $x = \frac{1}{18}$. The relative error of $4\frac{1}{2}$ in. is $\frac{1}{18}$. $\frac{1}{8}/4\frac{3}{4} \sim x/1$. $4\frac{3}{4}x = \frac{1}{8}$. $x = \frac{1}{38}$. The relative error of $4\frac{3}{4}$ in. is $\frac{1}{38}$. * $4\frac{3}{4}$ in. is more accurate.

316–O Both lengths have the same precision. * .005/2.08 ~ x/1. $2.08x = .005$. $x = \frac{.005}{2.08}$. The relative error of 2.08 km. is about .0024. .005/6.45 ~ x/1. $6.45x = .005$. $x = \frac{.005}{6.45}$. The relative error of 6.45 km. is about .0008. * 6.45 km. is more accurate.

316–P .0002 mm. is more precise. * .00005/.0002 ~ x/1. $.0002x = .00005$. $x = \frac{.00005}{.0002}$. The relative error of .0002 mm. is .25. .5/65 ~ x/1. $65x = .5$. $x = \frac{.5}{65}$. The relative error of 65 mm. is about .008. * 65 mm. is more accurate.

316–Q $6\frac{1}{2}$ yd. is more precise. * $\frac{1}{2}/16 \sim x/1$. $16x = \frac{1}{2}$. $x = \frac{\frac{1}{2}}{16}$. The relative error of 16 yd. is $\frac{1}{32}$. $\frac{1}{4}/6\frac{1}{2} \sim x/1$. $6\frac{1}{2}x = \frac{1}{4}$. $x = \frac{\frac{1}{4}}{6\frac{1}{2}}$. The relative error of $6\frac{1}{2}$ yd. is $\frac{1}{26}$. * 16 yd. is more accurate.

316–A 1 ft.

316–B .01 ft. * 5

316–C .0001 cm. * 62

316–D Only the digits 6 and 2 are needed to express 62 ten-thousandths of a centimetre. * 2

316–E .0001 cm. * The three digits 1, 4, and 5 are necessary to express the 145 unit lengths in .0145 cm.

316–F .001 in. * The digit 2 and the digit 0 that follows the digit 2 are significant.

316–G Two * Both the digits 2 and 0 are necessary to express the number of unit lengths in .20 in.

316–H .01 mi. * 2

316–I The symbol ±5 means that the greatest possible error is 5. Since the greatest possible error

is $\frac{1}{2}$ the unit of length, the unit of length is 10 mi. * 750

316–J Only the three digits 7, 5, and 0 are necessary to express 750 10-mile units of length.

317–K .002 * .100

317–L 20 ft. is more accurate. * Yes

317–M The relative error of .0062 cm. is .008. The relative error of .0145 is is .003.

317–N .0145 cm. * .0145

317–O .0067 * .0007

317–P 7500 mi. ±5 * 7500 ±5

317–A 10,000 mi. * 9287

317–B Only the digits 9, 2, 8, and 7 express the number of 10,000-mile units in 92,870,000 mi.

317–C Yes * Yes

317–D Yes. Because the unit of measure is 1000 miles, you need 5 digits to express the number of units in 92,870,000 mi. Therefore, you should use five digits to express the number from 1 to 10 in the scientific notation for 92,870,000 ±500.

317–E .0000001 cm.

317–F The digits 1 and 4 and the digit 0 to the right of the digit 4.

317–G Yes * The notation 1.40×10^{-5} indicates three significant digits and the notation 1.4×10^{-5} indicates only two significant digits. The numeral .0000140 has 3 significant digits.

On your own

317–1 The relative error of $8\frac{1}{2}$ in. is $\frac{1}{34}$. The relative error of 19 in. is $\frac{1}{38}$. * $8\frac{1}{2}$ in. is more precise. * 19 in. is more accurate.

317–2 The relative error of 6 ft. is $\frac{1}{12}$. The relative error of 5 in. is $\frac{1}{10}$. * 5 in. is more precise. * 6 ft. is more accurate.

317–3 The relative error of $3\frac{2}{3}$ yd. is $\frac{1}{22}$. The relative error of $1\frac{1}{2}$ yd. is $\frac{1}{6}$. * $3\frac{2}{3}$ yd. is more precise. * $3\frac{2}{3}$ yd. is more accurate.

317–4 The relative error of 10.2 m. is .005. The relative error of 5.45 m. is .001. * 5.45 m. is more precise and more accurate.

317–5 The relative error of $10\frac{1}{2}$ ft. is $\frac{1}{42}$. The relative error of $12\frac{0}{4}$ ft. is $\frac{1}{96}$. * $12\frac{0}{4}$ ft. is more precise and more accurate.

317–6 The relative error of .025 cm. is .020. The relative error of .25 cm. is .020. * .025 cm. is more precise. * .025 cm. and .25 cm. have the same accuracy.

317–7 The relative error of .0008 mm. is .062. The relative error of .032 mm. is .016. * .0008 mm. is more precise. * .032 mm. is more accurate.

317–8 The relative error of $4\frac{5}{36}$ yd. is $\frac{1}{298}$. The relative error of 35 in. is $\frac{1}{70}$. * $4\frac{5}{36}$ yd. and 35 in. have the same precision. * $4\frac{5}{36}$ yd. is more accurate.

317–9 The relative error of 1.65 km. is .003. The relative error of .80 km. is .006. * 1.65 km. and .80 km. have the same precision. * 1.65 km. is more accurate.

317–10 The relative error of 500 mi. is .001. The relative error of 5000 mi. ±5 is .001. * 500 mi. is more precise than 5000 mi. ±5. * 500 mi. and 5000 mi. ±5 have the same accuracy.

318–11 .001 cm. * All the digits in 14.030 are significant. * 1.4030×10^{1} cm.

318–12 1 yd. * All the digits in 160 are significant. * 1.60×10^{2} yd.

318–13 .00001 mm. * The last two digits in .00050 are significant. * 5.0×10^{-4} mm.

318–14 .01 km. * The digit 2 is the only significant digit in .02. * 2×10^{-2} km.

318–15 1000 mi. * The first five digits in 38,000,000 are significant. * 3.8000×10^{7} mi.

318–16 .1 ft. * All the digits are significant in 300.0. * 3.000×10^{2} ft.

318–17 .01 cm. * The digit 6 is the only significant digit in .06. * 6×10^{-2} cm.

318–18 10 km. * The first five digits in 350,000 are

significant. *
3.5000×10^5 km.

Keeping skilful

318–1 $\{-20\}$

318–2 $\{a \mid a \leq \frac{13}{20}\}$

318–3 $\{20\}$

318–4 $\{x \mid x \leq 4\}$

318–5 $\{d \mid -7.25 < d < -2.25\}$

318–6 $\{\frac{13}{40}, \frac{1}{10}\}$

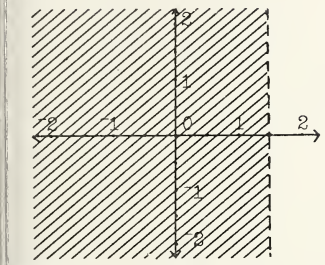
318–7 $\{y \mid y < 1.5\}$

318–8 $\{(-6\frac{2}{3}, -13\frac{1}{3})\}$

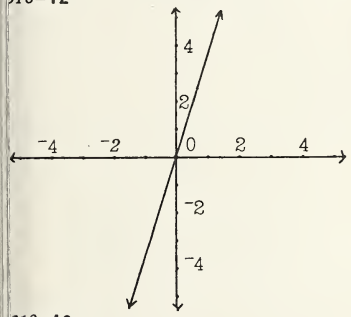
318–9 $\{(7,2)\}$

318–10 $\{(.4,.5,.9)\}$

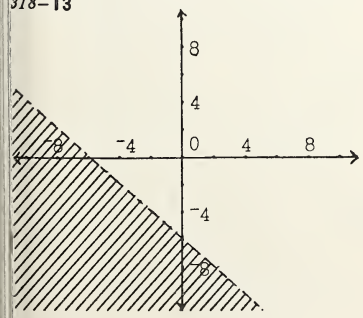
318–11



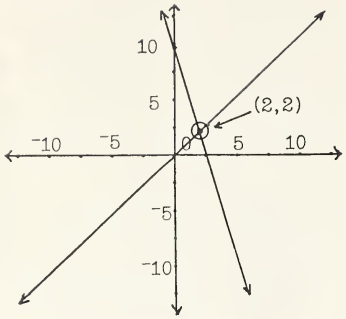
318–12



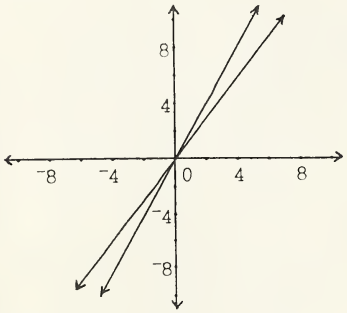
318–13



318–14



318–15



318–16 6.5 litres

318–17 5500 milligrams

318–18 800 litres

318–19 .15 kilograms

318–20 72,000,000 millilitres or 7.2×10^7 millilitres

318–21 .0056 kilograms

Lesson 163

318–A The greatest possible error of 3 kilometres is .5 kilometre. The greatest possible error of 7.1 kilometres is .05 kilometre. The greatest possible error of 2.8 kilometres is .05 kilometres.

318–B Yes * 7.1 kilometres * 2.8 kilometres

318–C The lesser limit of interval A is 2.5. The lesser limit of interval B is 7.05. The lesser limit of interval C is 2.75. * 12.30 * Since 12.30 is the sum of the lesser limits, the sum of the mathematical measures is at least as great as 12.30 km.

318–D The greater limit of interval A is 3.5. The greater limit of interval B is 7.15. The greater limit of interval C is 2.85. * 13.50 * Since 13.50 is the sum of the greater limits, the sum of the mathematical measures is not greater than 13.50 km.

318–E From exercises C and D, you know that w is greater than or equal to 12.30 and that w is less than or equal to 13.50. This condition is expressed by the sentence $12.30 \leq w \leq 13.50$. * Yes * By the bar above the picture of the number line

319–F 3 km., 7.1 km., and 2.8 km. * Yes * The greatest possible error of 12.9 is .05. The lesser limit of the interval determined by 12.9 is 12.9 − .05, or 12.85. The greater limit of the interval is 12.9 + .05, or 12.95. The interval determined by a recorded measure is a closed interval.

319–G 3 km. has a greatest possible error that is

greater than the greatest possible error of either 7.1 km. or 2.8 km. * Yes

319–H 13 km.

319–I $\{w \mid 12.5 \leq w \leq 13.5\}$

319–J The graph indicated by the horizontal bar above the picture of the number line. * The graph indicated by the horizontal bar below the picture of the number line

319–K The difference of 11.32 and 7.8 is 3.52. The least precise of 11.32 yd. and 7.8 yd. is 7.8 yd. Therefore, you round 3.52 to 3.5. A good approximation of the difference of 11.32 yd. and 7.8 yd. is 3.5 yd.

320–L Yes * The perimeter of a polygon is equal to the sum of the lengths of the sides.

320–M Yes

320–N $15\frac{3}{4}$ ft. * The greatest possible error of $15\frac{3}{4}$ is greater than the greatest possible error of $20\frac{1}{12}$.

320–O $15\frac{3}{4}$ ft. is expressed in $\frac{1}{4}$-foot units. Since $15\frac{3}{4}$ ft. is the least precise measure, the sum should be rounded to the nearer $\frac{1}{4}$ foot.

320–P $71\frac{8}{12}$ is greater than $71\frac{2}{4}$ and less than $71\frac{3}{4}$.

320–Q $71\frac{3}{4}$. $71\frac{8}{12}$ is nearer $71\frac{3}{4}$ than $71\frac{2}{4}$.

320–A Yes

320–B 3.25 and 2.5 * Yes

320–C The greatest possible error of 3.25 is .005. The lesser limit of the interval determined by 3.25 is 3.25 − .005, or 3.245. The greater limit is 3.25 + .005 or 3.255. Since a physical measure determines a closed interval, 3.25 determines the interval $\{h \mid 3.245 \leq h \leq 3.255\}$. * Yes

320–D Because the mathematical measure of the length is greater than or equal to 3.245 ft. and the mathematical measure of the width is greater than or equal to 2.45 * Yes

320–E Because the mathematical measure of the length is less than or equal to 3.255 ft. and the mathematical measure of the width is less than or equal to 2.55 ft. * Yes

320–F If m is a variable for the area of the piece of wood, then you know that m is greater than or equal to 7.95025 and less than or equal to 8.30025. This condition is expressed by the sentence $7.95025 \leq m \leq 8.30025$.

320–G Yes * The graph of $\{m \mid 7.95025 \leq m \leq 8.30025\}$ contains the dots for point 7.95025, point 8.30025, and all points between point 7.95025 and point 8.30025.

321–H Yes * Yes * A recorded measure determines a closed interval. The greatest possible error of 8.125 is .0005. Therefore, the limits of the closed interval determined by 8.125 are 8.125 − .0005 and 8.125 + .0005. Standard names for these limits are 8.1245 and 8.1255.

321–I The numeral 2.5 has fewer significant digits than the numeral 3.25.

321–J Yes * 8.1

321–K $\{m \mid 8.05 \leq m \leq 8.15\}$

321–L The area of the piece of wood is more likely to be a member of the interval determined by 8.1 sq. ft. than the interval determined by 8.125 sq. ft.

321–M 5.23×10^3

321–N 9×10^1

On your own

321–1 .08 in.

321–2 124 mi.

321–3 299 cm.

321–4 18600 km. ±50, or 1.86×10^4 km.

321–5 45.8

321–6 2.5×10^8

321–7 .1

321–8 16

322–9 x, thickness of 158 leaves of the book.
$(6.7 \times 10^{-3})/1 \sim x/158$.
{1.0586}. 1.1 cm. Notice that the rules of approximation were not used to obtain the solution set. The rules were used, however, to obtain the answer.

322–10 x, total precipitation in Chatham for the three days.
$x = .07 + 1.3 + .52$.
{1.89}. 1.9 in.

322–11 x, difference of the elevations of Mount Clemenceau and Mount Robson.

$x = 15{,}300 - 12{,}000$.
{3,300}. 3,300 ft.
or 3.3×10^3 ft.

322–12 x, average measure.
$2.98/25 \sim x/1$.
{.1192}. .119 mm.

Keeping skilful

322–1 .1 cm. * .05 cm.

322–2 .1 ft. * .05 ft.

322–3 1 in. * .5 in.

322–4 $\frac{1}{4}$ mi. * $\frac{1}{8}$ mi.

322–5 1 mm. * .5 mm.

322–6 $\frac{1}{12}$ in. * $\frac{1}{24}$ in.

322–7 10 ft. * 5 ft.

322–8 100 yd. * 50 yd.

322–9 $\frac{1}{14}$

322–10 $\frac{1}{100}$

322–11 .02

322–12 $\frac{1}{40}$

322–13 .01

322–14 .02

322–15 .007

322–16 .004

322–17 $\{x \mid 16.05 \leq x \leq 16.15\}$

322–18 $\{y \mid 2\frac{13}{16} \leq y \leq 2\frac{15}{16}\}$

322–19 $\{a \mid \frac{41}{2000} \leq a \leq \frac{43}{2000}\}$

322–20 $\{b \mid .00055 \leq b \leq .00065\}$

322–21 $\{m \mid .0045 \leq m \leq .0055\}$

322–22 $\{y \mid 151.5 \leq y \leq 152.5\}$

Applying mathematics

323–1 x, weight of hydrogen in grams.
$36.95 - 36.83 = x$.
{.12}. .12 g.

323–2 x, number of inches of precipitation Kenton City had during January and February.
$x = 2.16 + 1.9$. {4.06}.
4.1 in. Notice that the rules of approximation were not used to obtain the solution set.

323–3 x, number of pounds Alice gained.
$103\frac{1}{4} + x < 111 \wedge x > 0$.
$\{x \mid 0 < x < 7\frac{3}{4}\}$.
Between 0 and 8 pounds

323–4 x, number of times the area of Ontario is as large as the area of Saskatchewan.
$(5.95 \times 10^5)/(2.52 \times 10^5) \sim x/1$.
$\{2.36\overline{11}\}$. 2.36 times

323–5 x, number of yards of material Mrs. Perry bought.
$x - 10\frac{1}{2} = \frac{7}{6}$. $\{11\frac{3}{6}\}$.
$11\frac{1}{2}$ yd.

323–6 x, number of feet of lumber Dan used to build the shelf.
$9\frac{1}{2} - x = 2\frac{1}{4}$. $\{7\frac{1}{4}\}$. $7\frac{0}{2}$ ft.

323–7 x, number of inches of rain that Saskatoon had on Sunday and Monday.
$.37 + .8 > x \wedge x \geq 0$.
$\{x \mid 0 \leq x < 1.17\}$. From 0 to 1.2 in.

324–8 x, part of his farm that Mr. Hawkins planted in corn. $x/1 \sim 45/225$.
$\{\frac{1}{5}\}$. .20

324–9 x, number of miles Mr. Mills drove on Saturday.
$680.2 - x > 375 \wedge x \geq 0$.
$\{x \mid 0 \leq x < 305.2\}$.
From 0 to 305 mi.

324–10 x, area of piece of slate in square metres.
$1.24/1 \sim x/.62$. {.7688}.
.77 square metres

324–11 x, perimeter of window in feet.
$x = 8\frac{1}{2} + 4\frac{3}{4} + 8\frac{1}{2} + 4\frac{3}{4}$.
$\{26\frac{1}{2}\}$. $26\frac{1}{2}$ ft.

324–12 x, area of piece of aluminum in square centimetres.
$40.2/1 \sim x/30.6$.
{1230.12}.
1230 sq. cm. ±5

324–13 x, number of rods of fencing Mr. Hall needs to enclose the field.
$x = 300.6 + 285.2 + 300.6 + 285.2$.
{1171.6}. 1171.6 rods

324–14 x, area of scarf in square inches.
$32/1 \sim 2x/16.5$.
{264}. 260 sq. in. ±5

324–15 x, length of third side of patio in feet.
$97\frac{1}{2} = 30\frac{1}{6} + 30\frac{1}{6} + x$.
$\{37\frac{1}{6}\}$. $37\frac{0}{2}$ ft.

324–16 x, area of window in square inches.
$96/1 \sim x/96$. {9216}.
9200 sq. in. ±50

324–17 x, area of table top in square inches.
$24/1 \sim x/20.5$. {492}.
490 sq. in. ±5

Checking up

Test 148

324–1 .036 m.

324–2 15,000 m.

324–3 4.17 m.

324–4 2080 m.

324–5 .5012 m.

324–6 .000099 m.

324–7 19.7 in.

324–8 118.1 in.

324–9 1.7 in.

324–10 $\frac{1}{2}$ ft. * $\frac{1}{4}$ ft. *
$\{x \mid 4\frac{1}{4} \leq x \leq 4\frac{3}{4}\}$

324–11 1 mm. * $\frac{1}{2}$ mm. *
$\{x \mid 68\frac{1}{2} \leq x \leq 69\frac{1}{2}\}$

324–12 .1 mi. * .05 mi. *
$\{x \mid 18.85 \leq x \leq 18.95\}$

324–13 .01 km. * .005 km. *
$\{x \mid 32.505 \leq x \leq 32.515\}$

324–14 $\frac{1}{16}$ in. * $\frac{1}{32}$ in. *
$\{x \mid 5\frac{25}{32} \leq x \leq 5\frac{27}{32}\}$

324–15 .001 m. * .0005 m. *
$\{x \mid 2.0475 \leq x \leq 2.0485\}$

Test 149

324–16 .062

324–17 .016

324–18 .020

324–19 .017

324–20 .004

324–21 .083

324–22 2×10^{-3} cm.

324–23 7.09×10^{0} in.

324–24 4.00000×10^{5} m.

324–25 8.0×10^{1} ft.

324–26 5.0000×10^{6} mi.

324–27 1.000063×10^{2} km.

324–28 265 in.

324–29 4, or 4×10^{0}

324–30 1.55×10^{8}

324–31 $8\frac{1}{2}$ ft.

Lesson 164

325–A A triangle with no two of its sides congruent is a scalene triangle. No two sides of $\triangle ABC$ are congruent. No two sides of $\triangle DEF$ are congruent.

325–B The corresponding sides are $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, and $\overline{AC}$ and $\overline{DF}$. The corresponding angles are $\angle A$ and $\angle D$, $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$.

325–C Yes

325–D No. $\overline{AB}$ is not congruent to $\overline{DE}$, $\overline{BC}$ is not congruent to $\overline{EF}$, and $\overline{AC}$ is not congruent to $\overline{DF}$. * No. If $\triangle ABC \cong \triangle DEF$, then $ABC \leftrightarrow DEF$ must be a congruence correspondence. However, you know that $ABC \leftrightarrow DEF$ is not a congruence correspondence because corresponding sides are not congruent.

325–E Yes * 5/10 * 4/8 ~ 5/10 because $4 \cdot 10 = 5 \cdot 8$.

325–F 6/12 * Yes

325–G The comparison of $m(\overline{AB})$ with $m(\overline{DE})$ * The comparison of $m(\overline{AC})$ with $m(\overline{DF})$ * Yes

325–H Yes

326–I Corresponding angles are $\angle M$ and $\angle R$, $\angle P$ and $\angle S$, and $\angle Q$ and $\angle T$. Corresponding sides are $\overline{MP}$ and $\overline{RS}$, $\overline{PQ}$ and $\overline{ST}$, and $\overline{MQ}$ and $\overline{RT}$.

326–J Two angles that have the same measure are congruent. $\angle M^{o} = \angle R^{o}$. $\angle P^{o} = \angle S^{o}$. $\angle Q^{o} = \angle T^{o}$.

326–K 2.8/2.8 represents the comparison of $m(\overline{MP})$ with $m(\overline{RS})$. 1.8/1.8 represents the comparison of $m(\overline{PQ})$ with $m(\overline{ST})$. 2.0/2.0 represents the comparison of $m(\overline{MQ})$ with $m(\overline{RT})$. * Yes. 2.8/2.8, 1.8/1.8, and 2.0/2.0 are members of the same proportional relation. * Yes. Since there is a similarity correspondence between the vertices of $\triangle MPQ$ and $\triangle RST$, $\triangle MPQ \sim \triangle RST$.

326–L The corresponding angles and the corresponding sides of $\triangle MPQ$ and $\triangle RST$ are congruent.

326–M Yes

326–N Corresponding angles of two similar triangles are congruent. * The measures of corresponding sides of similar triangles are proportional.

327–A $\angle G$ and $\angle J$, $\angle H$ and $\angle K$, and $\angle I$ and $\angle L$ * Corresponding angles of similar triangles are congruent.

327–B For any two triangles, if the corresponding angles are congruent, then the two triangles are similar.

327–C The measures of corresponding sides of similar triangles are proportional.

327–D For any two triangles, if the measures of the corresponding sides are proportional, then the two triangles are similar.

327–E The corresponding angles are $\angle U$ and $\angle Q$, $\angle S$ and $\angle R$, and $\angle T$ and $\angle P$. The corresponding sides are $\overline{US}$ and $\overline{QR}$, $\overline{ST}$ and $\overline{RP}$, and $\overline{UT}$ and $\overline{QP}$.

327–F The sum of the measures of the angles of a triangle is 180. Therefore, in $\triangle UST$, $\angle S^{o} + 30 + 80 = 180$, and $\angle S^{o} = 70$. * In $\triangle QRP$, $\angle R^{o} + 30 + 80 = 180$, and $\angle R^{o} = 70$. * Yes. Angles that have the same measure are congruent.

327–G The corresponding angles of $\triangle UST$ and $\triangle QRP$ are congruent. For any two triangles, if the corresponding angles are congruent, then the two triangles are similar.

328–H Yes

328–I 6/7.5 * $\overline{DE}$ corresponds to $\overline{AB}$ and $\overline{EF}$ corresponds to $\overline{BC}$. * 4/5

328–J $6 \times 5 = 4 \times 7.5$.

328–K 7.5/9 * $\overline{EF}$ corresponds to $\overline{BC}$ and $\overline{DF}$ corresponds to $\overline{AC}$. * 5/6

328–L Yes. $7.5 \times 6 = 5 \times 9$.

328–M Yes * $m(\overline{AB})/m(\overline{AC}) = 6/9$, $m(\overline{DE})/m(\overline{DF}) = 4{:}6$, and $6/9 \sim 4/6$.

328–N Because a represents $m(\overline{UV})$ and d represents $m(\overline{XY})$

328–O b/e

328–P The rate pairs a/d and b/e represent comparisons of the measures of corresponding sides of the two triangles. The measures of corresponding sides of two similar triangles are proportional.

328–Q By using the definition of equivalent rate pairs. * By using the commutative property of multiplication

328–R The definition of equivalent rate pairs. * Yes

328–S Yes * $a/d \sim b/e$ and $a/b \sim d/e$ are equivalent conditions.

328–T $\overline{UV}$ and $\overline{VW}$ * $\overline{XY}$ and $\overline{YZ}$ * Yes

329–U $m(\overline{VW})/m(\overline{YZ}) \sim m(\overline{UW})/m(\overline{XZ})$.
$b/e \sim c/f$.
$b \times f = c \times e$.
$b \times f = e \times c$.
$b/c \sim e/f$.
$m(\overline{VW})/m(\overline{UW}) \sim m(\overline{YZ})/m(\overline{XZ})$.

329–V $m(\overline{UV})/m(\overline{XY}) \sim m(\overline{UW})/m(\overline{XZ})$.
$a/d \sim c/f$.
$a \times f = c \times d$.
$a \times f = d \times c$.
$a/c \sim d/f$.
$m(\overline{UV})/m(\overline{UW}) \sim m(\overline{XY})/m(\overline{XZ})$.

On your own

329–1 40 * 70 * 70 * 70

329–2 $5/7 \sim 8/x$. * $11\frac{1}{5}$

329–3 6

329–4 90 * 90

329–5 45 * 45 * 45

329–6 $m(\overline{MN})/m(\overline{NO}) \sim m(\overline{PQ})/m(\overline{QR})$. Using x as a variable for $m(\overline{QR})$, we have $1/\sqrt{2} \sim 3/x$. The solution of $1/\sqrt{2} \sim 3/x$ is $3\sqrt{2}$. * 3

329–7 In an isosceles triangle, the angles opposite the congruent sides are congruent. * 60 * 7

329–8 60 * 3 * 3

330–9 $\Delta UVW \sim \Delta XYZ$. * a-a property of similarity for triangles

330–10 ΔUVW is not similar to ΔXYZ.

330–11 You cannot tell if the two triangles are similar.

330–12 $\Delta UVW \sim \Delta XYZ$. * a-a-a property of similarity for triangles

330–13 You cannot tell if the two triangles are similar.

330–14 $\Delta UVW \sim \Delta XYZ$. * Proportional measures property of similarity for triangles

330–15 $\Delta UVW \sim \Delta XYZ$. * a-a-a property of similarity for triangles

330–16 You cannot tell if the two triangles are similar.

330–17 $\Delta UVW \sim \Delta XYZ$. * Congruent triangles are similar.

Keeping skilful

330–1 5.2

330–2 15.9

330–3 52.2

330–4 32.1

330–5 .49

330–6 .599

330–7 7.90×10^5, or 790,000 ±500

330–8 2.1

330–9 578

330–10 32

330–11 10.26

330–12 775.9

330–13 166.2

330–14 264

330–15 4.7×10^0, or 4.7

330–16 2.3×10^{10} or 23,000,000,000 ±500,000,000

Lesson 165

330–A $\angle C$ * Yes

330–B {A, B} * Side AB

330–C The sum of the measures of the angles of a triangle is 180. If one of the angles has a measure of 90, the other two each have a measure less than 90. An angle that has a measure of less than 90 is an acute angle.

330–D Side BC

330–E Sides AB and AC

330–F Side AC

331–G (Column 1) An acute angle * Side AC * Side BC

331–H (Column 1) 180 * 90 * 90

331–I You know that the sum of the measures of the angles of any triangle is 180 and the measure of a right angle is 90. Therefore, the sum of the measures of the acute angles of a right triangle is 90.

331–A ΔDEH, ΔDFI, and ΔDGJ * Yes.

331–B Since two angles of ΔDEH are congruent to two angles of ΔDFI, ΔDEH is similar to ΔDFI. Since two angles of ΔDFI are congruent to two angles of ΔDGJ, ΔDFI is similar to ΔDGJ. Since two angles of ΔDEH are congruent to two angles of ΔDGJ, ΔDEH is similar to ΔDGJ.

331–C In ΔDEH, $\overline{DE}$ is the hypotenuse, side EH is opposite $\angle D$, and side DH is adjacent to $\angle D$. In ΔDFI, $\overline{DF}$ is the hypotenuse, side FI

is opposite ∠D, and side DI is adjacent to ∠D. In ΔDGJ, $\overline{DG}$ is the hypotenuse, side GJ is opposite ∠D, and side DJ is adjacent to ∠D.

331–D Yes * For ΔDEH, 3/4. For ΔDFI, 6/8.

331–E Yes * For any two similar triangles, the rate pair that represents the comparison of the measures of any two sides of one triangle is equivalent to the rate pair that represents the comparison of the measures of the corresponding sides of the other triangle. * Yes * Yes

331–F There are infinitely many points in $\overrightarrow{DJ}$, other than point D, each of which determines a unique right triangle that has ∠D as an acute angle and a subset of $\overrightarrow{DG}$ as a hypotenuse. * Yes

331–G (Column 2) Yes * Any right triangle that includes ∠D is similar to any other right triangle that includes ∠D. For any two similar triangles, the measures of any two sides of one triangle are proportional to the measures of the corresponding sides of the other triangle.

331–H (Column 2) $\{\frac{3}{4}\}$ * $\frac{3}{4}$, or .75

332–I 9/15 * For ΔDEH, 3/5. For ΔDFI, 6/10.

332–J Each right triangle that has ∠D as an acute angle is similar to any other right triangle that has ∠D as an acute angle. For any two similar triangles, the measures of any two sides of one triangle are proportional to the measures of the corresponding sides of the other triangle.

332–K $\frac{3}{5}$, or .6

332–L 12/15 * For ΔDEH, 4/5. For ΔDFI, 8/10.

332–M Each of the triangles DEH, DFI, and DGJ is similar to each of the other triangles. For any two similar triangles, the measures of any two sides of one triangle are proportional to the measures of the corresponding sides of the other triangle.

332–N Yes * .8/1 ~ 4/5 because .8(5) = 4(1). Each member of {4/5,...} is equivalent to each of the other members of {4/5,...} by the definition of a proportional relation. Since .8/1 is equivalent to 4/5, .8/1 is a member of {4/5,...} and hence, each member of {4/5,...} is equivalent to .8/1.

On your own

333–1 ∠K * ∠J

333–2 $\frac{20}{21}$ * $\frac{21}{20}$ * If the product of two numbers is 1, each number is the reciprocal of the other. The product of $\frac{20}{21}$ and $\frac{21}{20}$ is 1.

333–3 $\frac{40}{9}$

333–4 ∠R

333–5 $\frac{24}{10}$

333–6 ∠T

333–7 $\frac{20}{29}$ * Yes

333–8 $\frac{21}{29}$ * ∠J

333–9 .2195 * ∠Q

333–10 .9756

333–11 The sine of ∠S is .9231 and the cosine of ∠S is .3846. The sine of ∠T is .3846 and the cosine of ∠T is .9231.

Lesson 166

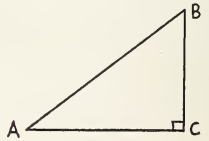
334–A The numeral .5774 appears under the heading "tan x."

334–B .5000

334–C .8660

334–D sin 29 ≈ .4848. * cos 55 ≈ .5736. * tan 89 ≈ 57.2900.

334–E sin 62 ≈ .8829. * cos 28 ≈ .8829. * For the right triangle represented below, ∠A is the complement of ∠B. From the diagram you can see that (sine of ∠A)/1 = $m(\overline{BC})/m(\overline{AB})$ and (cosine of ∠B)/1 = $m(\overline{BC})/m(\overline{AB})$. Therefore, the sine of ∠A is the same as the cosine of ∠B.



334–F 15 * 15

335–G (Column 1, top) 54 * 52

335–H (Column 1) Yes. The angle whose measure is greater also has a greater tangent. * Yes. The angle whose measure is greater also has a greater sine. * No. The angle whose measure is greater has a lesser cosine. These facts may be seen by

examining the table you have used.

335–A Yes

335–B 4

335–C Tangent

335–D Yes

335–E Use the definition of equivalent rate pairs. * Yes

335–F .4245

335–G (Column 1, bottom) Replace tan 23 in a = (tan 23)4 by .4245.

335–H (Column 2) {1.6980} * 1.698

335–I $\overline{LN}$ * Adjacent * $\overline{LM}$ * 25

335–J Cosine of $\angle L$

336–K The principal steps involved in obtaining $\cos x = .4000$ from $(\cos x)/1 \sim 10/25$ are expressed below.
$(\cos x)/1 \sim 10/25$.
$(\cos x)\ 25 = 10$.
Definition of equivalent rate pairs. Cos $x = \frac{10}{25}$, or cos x = .4000. Well-defined property of multiplication. * .4000

336–L No

336–M Yes

336–N .4067 − .4000, or .0067. * .4000 − .3907, or .0093.

336–O 66

336–P 624 * 75

336–Q The measure of the side opposite $\angle D$

336–R Yes

336–S Use the definition of equivalent rate pairs. * From the table you know that sin 75 ≈ .9659. Therefore, you can replace sin 75 in x = (sin 75)624 by .9659 to obtain x = (.9659)624.

336–T {602.7216}

336–U 603

336–V The measure in degrees of $\angle H$

337–W No * Yes * Yes

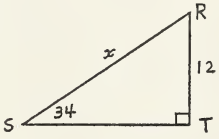
337–X $(\tan r)/1 \sim 228/54$.

337–Y Use the definition of equivalent rate pairs. * Use the well-defined property of multiplication.

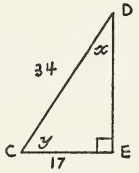
337–Z 77 * The absolute difference of tan 77 and 4.2222 is less than the absolute difference of tan 76 and 4.2222.

On your own

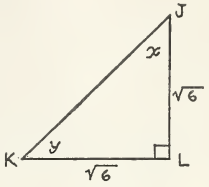
337–1 See the sketch below.
$(\sin 34)/1 \sim 12/x$.
m($\overline{RS}$) is 21.5.



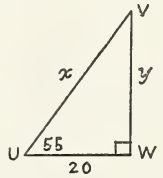
337–2 See the sketch below.
$(\sin x)/1 \sim 17/34$.
$\angle D^o$ = 80. *
$x + y$ = 90. $\angle C^o$ = 60.



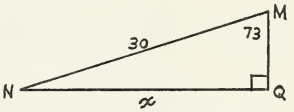
337–3 See the sketch at the top of the next column.
$(\tan x)/1 \sim \sqrt{6}/\sqrt{6}$.
$\angle J^o$ = 45. *
$x = y$. $\angle K^o$ = 45.



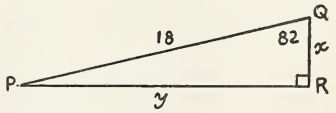
337–4 See the sketch below.
$(\cos 55)/1 \sim 20/x$.
m($\overline{UV}$) is 34.87. *
$(\tan 55)/1 \sim y/20$.
m($\overline{VW}$) is 28.56.



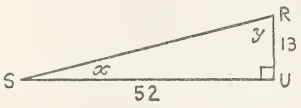
337–5 See the sketch below.
$(\sin 73)/1 \sim x/30$.
m($\overline{NQ}$) is 28.69.



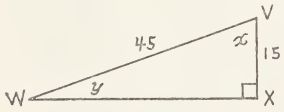
337–6 See the sketch below.
$(\cos 82)/1 \sim x/18$.
m($\overline{QR}$) is 2.506. *
$(\sin 82)/1 \sim y/18$.
m($\overline{RP}$) is 17.825.



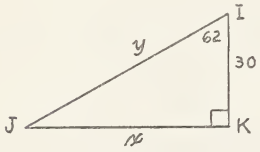
337–7 See the sketch at the top of the next page.
$(\tan x)/1 \sim 13/52$.
$\angle S^o$ is 14. *
$x + y$ = 90. $\angle R^o$ is 76.



337–8 See the sketch below.
$(\cos x)/1 \sim 15/45$.
$\angle V^{\circ}$ is 71. *
$x + y = 90$. $\angle W^{\circ}$ is 19.



337–9 See the sketch below.
$(\tan 62)/1 \sim x/30$.
$m(\overline{JK})$ is 56.42. *
$(\cos 62)/1 \sim 30/y$.
$m(\overline{JI})$ is 68.90.



Keeping skilful

337–1 .05
337–2 $\bar{}2.25$
337–3 $\bar{}.32$
337–4 $.\overline{63}$
337–5 .13
337–6 $\bar{}.6875$
337–7 8.875
337–8 $\bar{}18.\overline{3}$
337–9 7.4
337–10 $\bar{}2.\overline{7}$
337–11 .0875
337–12 $\bar{}.013$
337–13 78.519
337–14 94.7
337–15 8.9
337–16 5032.7
337–17 $\bar{}1.96$
337–18 62.01
337–19 6.958
337–20 $\bar{}42.8$
337–21 $\bar{}19.524$
337–22 207.27
337–23 $\bar{}.24375$
337–24 7590
337–25 $\bar{}106.75$
337–26 14.851

Lesson 167

338–A The other triangle is determined by the points suggested by the top of Don's head, the spot where he is standing, and the end of his shadow.
338–B 280 * The height of the radio tower * No
338–C $\overline{UW}$ * $\overline{VW}$ * $m(\overline{UW}) = 89$ and $m(\overline{VW}) = 69$. * In inches
338–D $\angle Z$ and $\angle W$ are right angles.
339–E $\angle U$
339–F If two angles of one triangle are congruent to two angles of another triangle, the two triangles are similar.
$\angle U \cong \angle X$ and $\angle W \cong \angle Z$.
339–G $\overline{UV}$ and $\overline{XY}$, $\overline{UW}$ and $\overline{XZ}$, and $\overline{VW}$ and $\overline{YZ}$
339–H Yes
339–I Yes
339–J For any two similar triangles, the rate pair that represents the comparison of the measures of any two sides of one triangle is equivalent to the rate pair that represents the comparison of the measures of the corresponding sides of the other triangle. * Yes
339–K $\{495\frac{5}{13}\}$
339–L The least accurate measures given in the problem are expressed by numerals that each have exactly two significant digits.
339–M The height of the radio tower is 5.0×10^2 ft.
339–A $\overline{QR}$
339–B $\angle P$ * 9 * Yes
339–C 90 * A right triangle
340–D $\overline{PR}$
340–E Yes * 235
340–F $(\tan 9)/1$
340–G Yes * Yes
340–H By using the definition of equivalent rate pairs
340–I Yes * $235 = .1584\,a$. * 235 and .1584
340–J 4 * 3 * 3
340–K Yes * $\angle J$
340–L $\overline{JK}$ * 48
340–M $\overline{JL}$ suggests the distance from the bottom of the ladder to the base of the wall. This distance is 10.8 ft.
340–N $\angle J$
341–O $(\cos x)/1$
341–P 10.8/48 represents the comparison of the measure of the side adjacent to $\angle J$ with the measure of the hypotenuse. By definition, this rate pair is equivalent to $(\cos x)/1$ where x is a variable for the measure of $\angle J$.
341–Q Use the definition of equivalent rate pairs. * 10.8 and 48
341–R .2250

341–S 77 * The measure of the angle of elevation of the top of the ladder from the bottom is 77.

341–T 13 * The acute angles of a right triangle are complementary angles and the sum of their measures is 90.

341–U $\overline{KL}$

341–V (sin 77)/1 or (tan 77)/1 * To find $m(\overline{KL})$, you must use a trigonometric rate pair that is equivalent to a rate pair having $m(\overline{KL})$ as one component. (sin 77)/1 is equivalent to $m(\overline{KL})/48$. (tan 77)/1 is equivalent to $m(\overline{KL})/10.8$.

341–W Yes * $d/10.8$. Remember that d is a variable for $m(\overline{KL})$.

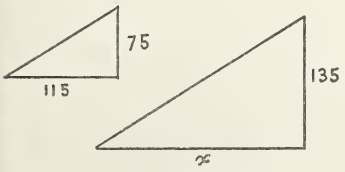
342–X Use the definition of equivalent rate pairs. * Use the table to find sin 77. Then replace (sin 77) in d = (sin 77)48 by (.9744).

342–Y 48 * 47 * 47 ft.

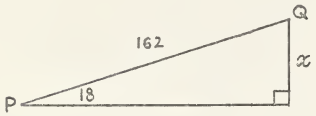
342–Z 47 ft. * They are the same.

On your own

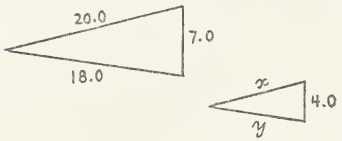
342–1 See the sketches below.
x, length of shadow cast by 135-foot building in feet.
$75/115 \sim 135/x$.
2.1×10^2 ft.



342–2 See the sketch below.
x, vertical height of hill in feet.
$x/162 \sim (\sin 18)/1$.
5.01×10^1 ft.

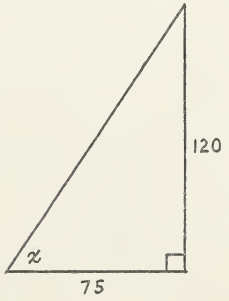


342–3 See the sketches below.
x, length in inches of longest edge of smaller pennant.
$7.0/20.0 \sim 4.0/x$.
1.1×10^1 in.

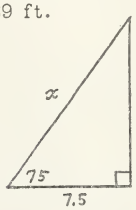


342–4 See the sketch for 284–3.
y, length in inches of third side of smaller pennant.
$7.0/18.0 \sim 4.0/y$.
1.0×10^1 in.

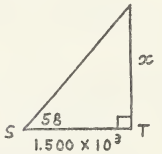
342–5 See the sketch below.
x, measure of angle of elevation.
$120/75 \sim (\tan x)/1$.
58



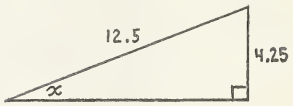
342–6 See the sketch below.
x, length of ladder in feet.
$7.5/x \sim (\cos 75)/1$.
29 ft.



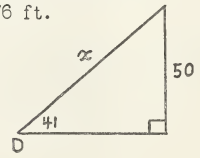
342–7 See the sketch below.
x, height of airplane above point T in feet.
$x/1500 \sim (\tan 58)/1$.
2.400×10^3 ft.



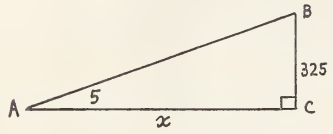
342–8 See the sketch below.
x, measure of angle of elevation.
$4.25/12.5 \sim (\sin x)/1$.
20



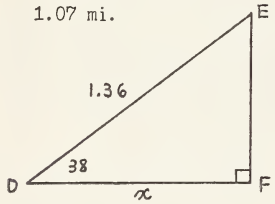
342–9 See the sketch below.
x, length of guy wire in feet.
$50/x \sim (\sin 41)/1$.
76 ft.



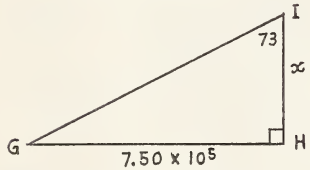
342–10 See the sketch below.
$(\tan 5)/1 \sim 325/x$.
3.71×10^3 ft.



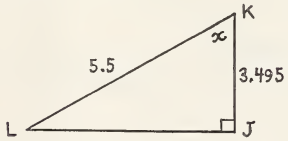
342–11 See the sketch below.
$(\cos 38)/1 \sim x/1.36$.
1.07 mi.



342–12 See the sketch below.
$(\tan 73)/1 \sim (7.50 \times 10^5)/x$.
2.29×10^5 mm.



342–13 See the sketch below.
$(\cos x)/1 \sim 3.495/5.5$.
51

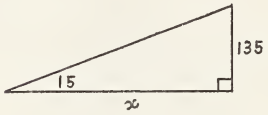


Applying mathematics

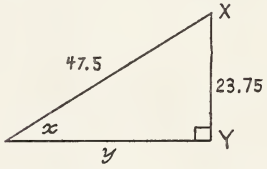
343–1 x, Ruth's height in inches.
$68/42.5 \sim x/40$.
64 in.

343–2 x, length of shortest side of smaller piece of canvas in feet.
$23.5/13.0 \sim 14.0/x$.
7.74 ft.

343–3 See the sketch below.
x, distance in feet from base of lighthouse to boat.
$135/x \sim (\tan 15)/1$.
504 ft.

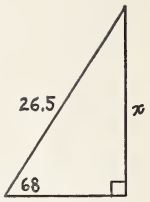


343–4 See the sketch below.
x, measure of angle of elevation of point X from the stake.
$23.75/47.5 \sim (\sin x)/1$.
30

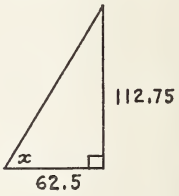


343–5 See sketch for 285–4.
y, distance in feet from stake to point Y.
$y/47.5 \sim (\cos 30)/1$, or
$23.75/y \sim (\tan 30)/1$.
41.1 ft.

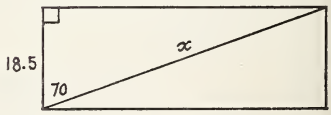
343–6 See the sketch at the top of the next column.
x, distance in feet from base of wall to top of ladder.
$x/26.5 \sim (\sin 68)/1$.
24.6 ft.



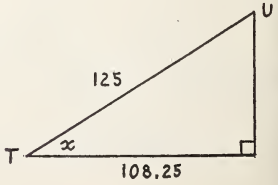
343–7 See the sketch below.
x, measure of angle of elevation of sun.
$112.75/62.5 \sim (\tan x)/1$.
61



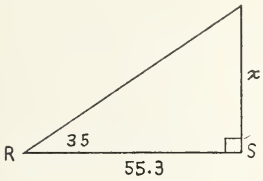
343–8 See the sketch below.
x, distance in feet from one corner of the pool to the opposite corner.
$18.5/x \sim (\cos 70)/1$.
54.1 ft.



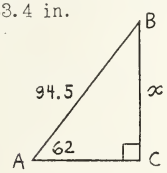
343–9 See the sketch below.
x, measure of angle of elevation from point T to point U.
$108.25/125 \sim (\cos x)/1$.
30



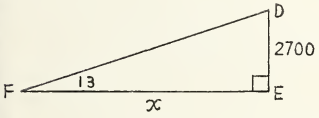
343–10 See the sketch below.
x, distance in feet from point S to the top of the antenna.
$(\tan 35)/1 \sim x/55.3$.
38.7 ft.



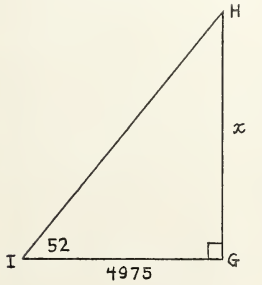
343–11 See the sketch below.
$x/94.5 \sim (\sin 62)/1$.
83.4 in.



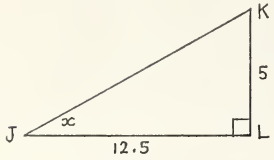
343–12 See the sketch below.
$2700/x \sim (\tan 13)/1$.
1.17×10^4 cm.



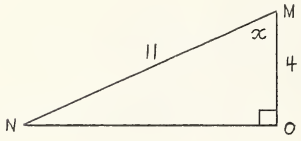
343–13 See the sketch below.
$x/4975 \sim (\tan 52)/1$.
6368 km.



343–14 See the sketch below.
$(\tan x)/1 \sim 5/12.5$.
22



343–15 See the sketch below.
$(\cos x)/1 \sim 4/11$.
69



Checking up

Test 150

344–1 less
344–2 6.2
344–3 GI
344–4 72
344–5 223 m.
344–6 1000
344–7 .0005 cm.
344–8 tenth
344–9 $\frac{1}{4}$ foot
344–10 $\frac{1}{6}$
344–11 77
344–12 100 in.

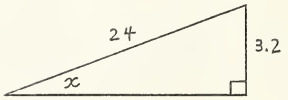
Test 151

344–13 T
344–14 F
344–15 F
344–16 T
344–17 T
344–18 F
344–19 T
344–20 F
344–21 T
344–22 F
344–23 F
344–24 T
344–25 T
344–26 T
344–27 F
344–28 F

345–29 x, average daily precipitation in inches.
$x/1 \sim 2.56/30$.
.0853 in.

345–30 x, number of feet higher McArthur Peak is than Mount Augusta.
$x = (1.44 \times 10^4) - (1.407 \times 10^4)$.
$\{.033 \times 10^4\}$. 3×10^2 ft.

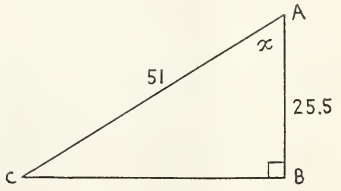
345–31 x, measure of angle of elevation.
$3.2/24 \sim (\sin x)/1$.
8



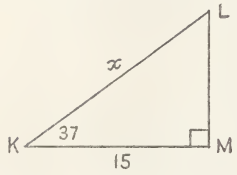
345–32 x, width of mainspring in inches.
$.394/1 \sim x/.520$.
.205 in.

345–33 x, altitude in kilometres.
$1000/1 \sim 25{,}644/x$.
25.644 km.

345–34 $(\cos x)/1 \sim 25.5/51$.
60

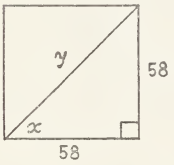


345–35 $(\cos 37)/1 \sim 15/x$.
19

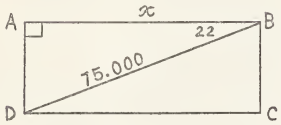


345–36 $35.2/29.54 \sim 43/x$.
36

345–37 $(\tan x)/1 \sim 58/58 \wedge (\cos x)/1 \sim 58/y$.
82 ft.



345–38 $(\cos 22)/1 \sim x/75.000$.
69.540 in.



Test 153

345–1 T
345–2 F
345–3 T
345–4 F
345–5 T
345–6 F
345–7 F
345–8 T
345–9 T. More correctly, the graph is a set that contains exactly one dot.
345–10 F

Test 154

345–11 $\{1\frac{1}{4}\}$
345–12 {33}
345–13 $\{m \mid m > -5\frac{2}{3}\}$
346–14 $\{m \mid 12.65 \leq m \leq 12.75\}$
346–15 {2.56}
346–16 $\{x \mid x \geq -3\frac{1}{3}\}$
346–17 $\{c \mid c \geq .2\}$
346–18 {(18,6)} for (a,b)
346–19 $\{(3\frac{1}{3}, \frac{5}{9}, 4\frac{4}{9})\}$ for (x,y,z)

Test 155

346–20 repeating
346–21 hypotenuse
346–22 $\frac{27}{64}$
346–23 {2,3}
346–24 0
346–25 3
346–26 well-defined
346–27 1.5948×10^2
346–28 difference
346–29 $1/(-1.4)$. Remember that this means $1 \div -1.4$.
346–30 17.7
346–31 irrational
346–32 density

Lesson 168

347–A 150 mi. * 120 mi.
347–B Yes * 13 gal. * 15 gal.
347–C 7 * Number of gallons of gasoline * Number of miles travelled
347–D Yes * $10/150 \sim 1/15$ because $10 \times 15 = 1 \times 150$.
347–E Yes * $12/180 \sim 1/15$ because $12 \times 15 = 1 \times 180$.
$15/225 \sim 1/15$ because $15 \times 15 = 1 \times 225$.
$8/120 \sim 1/15$ because $8 \times 15 = 1 \times 120$.
$13/195 \sim 1/15$ because $13 \times 15 = 1 \times 195$.
$11/165 \sim 1/15$ because $11 \times 15 = 1 \times 165$.
$9/135 \sim 1/15$ because $9 \times 15 = 1 \times 135$.
347–F $x/y \sim 1/15$. * D × D, the Cartesian set of real numbers
347–G Use the definition of equivalent rate pairs. * Use the commutative property of multiplication.
347–H Yes * Yes * Yes.
$-90 = 15 \cdot -6$.
347–I No * Yes * Yes
347–J Each member of $\{(x,y) \mid y = 15x\}$ is an ordered pair that is equivalent to each of the other members.
347–K Yes
348–L 15
348–M Yes * $(x,15x)$ indicates that the second component of each element of $\{(x,y) \mid y = 15x\}$ is equal to the product of 15 and the first component.
348–N Yes * x, y, and k * By using the definition of equivalent rate pairs and the commutative property of multiplication * Yes
348–O 3 * 3 * Each ordered pair whose second component is 3 times its first component satisfies $y = 3x$. Each ordered pair whose second component is 3 times its first component is indicated by $(x,3x)$.
348–P Yes * The second component of each solution is equal to the product of 3 and the first component.
348–Q The constant of variation in condition F is $-\frac{3}{4}$, in condition G is -2, and in condition H is $\frac{1}{2}$.
348–R For each condition, the second component of each solution is equal to the product of the constant of variation and the first

component.

348–S Because (x, kx) indicates that the second component of each solution is equal to the product of the constant of variation and the first component.

348–A $y = \frac{1}{3}x$.

348–B $y = \frac{1}{3}x$ is of the form $y = kx$. Therefore, $y = \frac{1}{3}x$ is a condition for direct variation and y varies directly as x varies. * $\frac{1}{3}$

348–C

x	-3	-2	0	1	3
y	-1	$-\frac{2}{3}$	0	$\frac{1}{3}$	1

348–D The replacements for y become greater. * $(-3,-1)$, $(-2,-\frac{2}{3})$, $(0,0)$, $(1,\frac{1}{3})$, and $(3,1)$

348–E $(0,0)$ * $(3,1)$ * $(-3,-1)$ * Yes.

349–F Yes (Point 0 is the origin.) * Yes

On your own

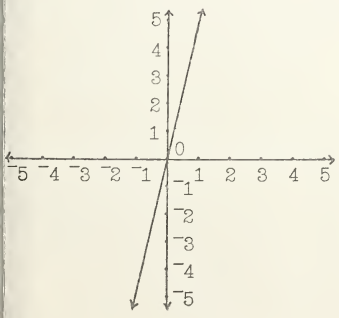
349–1 25¢ * 50¢ * 4 * 3

349–2 T = {(3,15), (2,10), (5,25), (10,50), (4,20)}.

349–3 Yes * 5

349–4 Yes * 5

349–5 See the graph below. * A line



350–6 Yes * Each point in ℓ_2 is associated with a member of $\{(x,y) \mid y = \frac{1}{2}x\}$ and each member of $\{(x,y) \mid y = \frac{1}{2}x\}$ is associated with a point in ℓ_2.

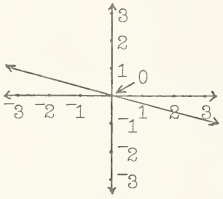
350–7 Yes * $\frac{1}{2}$

350–8 $y = 2\frac{1}{2}x$.

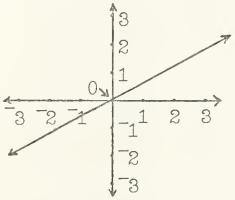
350–9 $y = -1\frac{1}{3}x$.

350–10 $y = -2x$.

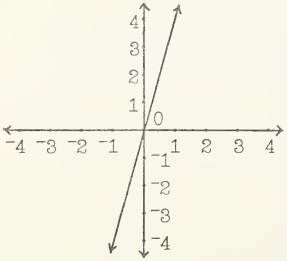
350–11



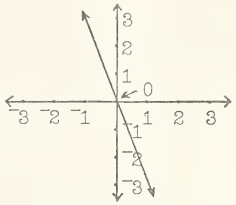
350–12



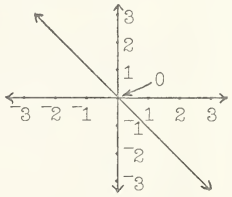
350–13



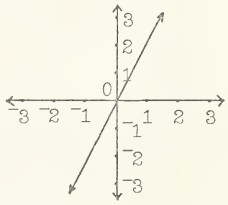
350–14



350–15



350–16



Lesson 169

350–A 16 ft. * 64 ft. * 144 ft. * 256 ft.

350–B Its speed increases * The first second the ball drops 16 ft. The second second the ball drops 64 − 16, or 48 ft. The third second the ball drops 144 − 64, or 80 ft. The fourth second the ball drops 256 − 144, or 112 ft.

350–C {(1,16), (2.64), (3,144), (4,256)}

350–D Yes

350–E No * You know that in a proportional relation, each member is equivalent to each of the other mem-

bers. The members of set M are not equivalent. For example, 1/16 is not equivalent to 2/64 because $1 \times 64 \neq 2 \times 16$.

351–F 16 * 32

351–G No * No

351–H A third row for x^2 has been added to the chart.

351–I 16 * 16 * 16 * 16

351–J Yes

351–K $y = 16x^2$. * Each member of set M is also a member of $\{(x,y) \mid y = 16x^2\}$.

351–L Yes

351–M D

351–N $16 * {}^{-}\frac{1}{2} * \sqrt{2}$

351–O Yes * In $y = {}^{-}\frac{1}{2}x^2$ the constant of variation is ${}^{-}\frac{1}{2}$. In $y = \sqrt{2}x^2$, the constant of variation is $\sqrt{2}$.

352–A $y = 2x^2$.

352–B

x	$^{-}2$	$^{-}1\frac{1}{2}$	$^{-}1$	0	1	$1\frac{1}{2}$	2
y	8	$4\frac{1}{2}$	2	0	2	$4\frac{1}{2}$	8

* $\{(^{-}2,8), (^{-}1\frac{1}{2},4\frac{1}{2}), (^{-}1,2), (0,0), (1,2), (1\frac{1}{2},4\frac{1}{2}), (2,8)\}$

352–C (2,8) * $(^{-}1\frac{1}{2},4\frac{1}{2})$ * $(1\frac{1}{2},4\frac{1}{2})$ * (0,0)

352–D Yes

On your own

352–1 $\{(^{-}2,4), (^{-}1,2), (0,0), (1,^{-}2), (2,^{-}4)\}$ * The ordered pairs satisfy a condition of the form $y = kx$. * $y = {}^{-}2x$.

352–2 $\{(^{-}3,9), (\frac{1}{2},\frac{1}{4}), (5,25), (0,0), (12,144)\}$ * The ordered pairs satisfy a condition of the form $y = kx^2$. * $y = x^2$.

353–3 $\{(3,4\frac{1}{2}), (4,8), (5,12\frac{1}{2}), (6,18), (7,24\frac{1}{2})\}$ * The ordered pairs satisfy a condition of the form $y = kx^2$. * $y = \frac{1}{2}x^2$.

353–4 $\{(5,^{-}3\frac{1}{3}), (10,^{-}6\frac{2}{3}), (15,^{-}10), (20,^{-}13\frac{1}{3}), (25,^{-}16\frac{2}{3})\}$ * The ordered pairs satisfy a condition of the form $y = kx$. * $y = {}^{-}\frac{2}{3}x$.

353–5 $\{(^{-}2,24), (0,0), (3,54), (5,150), (7,294)\}$ * The ordered pairs satisfy a condition of the form $y = kx^2$. * $y = 6x^2$.

353–6 Point D is associated with $(^{-}3,3)$, point A with $(^{-}2,1\frac{1}{3})$, point E with $(^{-}1,\frac{1}{3})$, point O with (0,0), point C with $(1,\frac{1}{3})$, point F with $(2,1\frac{1}{3})$, and point B with (3,3).

353–7 Yes * $y = \frac{1}{3}x^2$. * $\frac{1}{3}$

353–8 $\{(x,y) \mid y = \frac{1}{3}x^2\}$

353–9 Point K is associated with $(^{-}3,^{-}3)$, point G with $(^{-}2,^{-}1\frac{1}{3})$, point J with $(^{-}1,^{-}\frac{1}{3})$, point O with (0,0), point H with $(1,^{-}\frac{1}{3})$, point L with $(2,^{-}1\frac{1}{3})$, and point M with $(3,^{-}3)$.

353–10 $y = {}^{-}\frac{1}{3}x^2$. * $^{-}\frac{1}{3}$

353–11 $\{(x,y) \mid y = {}^{-}\frac{1}{3}x^2\}$

353–12 The locus of a solution set of a condition of the form $y = kx^2$ is a parabola.

Lesson 170

354–A 20 * $13\frac{1}{3}$ * 10

354–B 2 * 5

354–C $\{(1,40), (2,20), (3,13\frac{1}{3}), (4,10), (5,8)\}$

354–D Yes

354–E Yes * 40

354–F Yes * $4\frac{4}{9}$

354–G If set S were determined by a condition of the form $y = kx$, then there would be one number by which you could multiply the first component of each member of set S to obtain the second component.

354–H $1^2 = 1$. $2^2 = 4$. $3^2 = 9$. $4^2 = 16$. $5^2 = 25$.

354–I Yes * 5

354–J Yes * $\frac{10}{16}$

354–K If set S were determined by a condition of the form $y = kx^2$, then there would be one number by which you could multiply the square of the first component of each member of set S to obtain the second component.

355–L Yes * Yes * Yes

355–M $\frac{1}{3}$ * $\frac{1}{4}$ * $\frac{1}{5}$

355–N The chart in D3 includes a row for $\frac{1}{x}$.

355–O 40 * 40

355–P Yes * 40

355–Q Yes

355–R $y = 40(\frac{1}{x})$. * $y = \frac{40}{x}$. * D x D

355–S Yes * Yes * An infinite set

355–T Yes * Each member of set S satisfies condition B. Therefore, each member of set S is also a member of the solution set of condition B.

355–U If you replace x by 8 and y by 5 in $y = \frac{40}{x}$, you obtain the true statement $5 = \frac{40}{8}$. * Yes

355–V Yes

355–A $y = \frac{k}{x}$. * D

355–B 40 * 31.5 * $^{-}25$

355–C In condition E, the constant of variation is 31.5. In condition F, the constant of variation is $^{-}25$. * Yes

356–D $y = \frac{2}{x}$.

356–E 2

356–F

x	$^{-}3$	$^{-}2$	$^{-}1$	1	2	3
y	$\frac{2}{3}$	$^{-}1$	$^{-}2$	2	1	$\frac{2}{3}$

* $\{(^{-}3, \frac{2}{3}), (^{-}2,1), (^{-}1,^{-}2), (1,2), (2,1), (3,\frac{2}{3})\}$

356–G (1,2) * $(^{-}2,^{-}1)$ * $(^{-}1,^{-}2)$

356–H No * No * No

356–I There is no quotient of 2 and 0. The symbol $\frac{2}{0}$ is meaningless. Therefore, x cannot be replaced by 0 in $y = \frac{2}{x}$. If y is replaced by 0 in $y = \frac{2}{x}$, you obtain $0 = \frac{2}{x}$. There is no real number that satisifes $0 = \frac{2}{x}$.

356–J $xy = 2$ is equivalent to $y = \frac{2}{x}$. Therefore, the solution set of $xy = 2$ is the same set as the solution set of $y = \frac{2}{x}$.

356–K First quadrant * Third quadrant

356–L The second coördinate of point I is less than the second coördinate of point H. * The second coördinate of point J is less than the second coördinate of point I.

357–M b is less than d. * In the first quadrant, for the locus of the solution set of $y = \frac{2}{x}$, as the first coördinate increases, the second coördinate decreases.

On your own

357–1 $\{(^{-}3,^{-}\frac{1}{3}), (^{-}2,^{-}\frac{1}{2}), (1,1), (4,\frac{1}{4}), (5,\frac{1}{5})\}$ * $y = \frac{k}{x}$.

357–2 $\{(^{-}8,2), (^{-}4,1), (1,^{-}\frac{1}{4}), (2,^{-}\frac{1}{2}), (6,^{-}\frac{3}{2})\}$ * $y = kx$.

357–3 $\{(^{-}4,^{-}80), (^{-}3,^{-}45), (^{-}2,^{-}20), (1,^{-}5), (5,^{-}125)\}$ * $y = kx^2$.

357–4 $y = \frac{^{-}6}{x}$.

357–5 Point A: $(2,^{-}3)$
Point B: $(^{-}1,6)$
Point C: $(^{-}3,2)$
Point D: $(1,^{-}6)$
Point E: $(^{-}2,3)$
Point F: $(3,^{-}2)$
* $\{(x,y) \mid y = \frac{^{-}6}{x}\}$

357–6 No * No

357–7 If a point is in the first quadrant, both coördinates are positive. If a point is in the third quadrant, both coördinates are negative. To satisfy $y = \frac{^{-}6}{x}$, if x is positive, then y must be negative and if x is negative, then y must be positive.

357–8

x	$^{-}6$	$^{-}3$	$^{-}2$	$^{-}1$	1	2	3	6
y	$^{-}1$	$^{-}2$	$^{-}3$	$^{-}6$	6	3	2	1

357–9 $\{(^{-}6,^{-}1), (^{-}3,^{-}2), (^{-}2,^{-}3), (^{-}1,^{-}6), (1,6), (2,3), (3,2), (6,1)\}$

357–10 The first and third

357–11 y varies directly as x varies.

357–12 y varies inversely as x varies.

357–13 y varies directly as x varies.

357–14 y varies inversely as x varies.

357–15 y varies inversely as x varies.

357–16 y varies inversely as x varies.

Lesson 171

358–A D x D

358–B Yes * Yes

358–C Yes

358–D Replace k in $y = \frac{k}{x^2}$ by 16.

358–E k

358–F $B = \{(^{-}4,1), (^{-}2,4), (^{-}1,16), (1,16), (2,4), (4,1)\}$.

358–G Yes

359–H Yes * Each member of set B is also a member of $\{(x,y) \mid y = \frac{16}{x^2}\}$.

359–I Point C: (1,16)
Point D: (2,4)
Point E: (4,1)
Point F: $(^{-}1,16)$
Point G: $(^{-}2,4)$
Point H: $(^{-}4,1)$ * The first and second

359–J The square of 0 is 0. There is no quotient of 16 and 0. The symbol $\frac{16}{0^2}$ is meaningless. Therefore, x cannot be replaced by 0 in $y = \frac{16}{x^2}$. If y is replaced by 0 in $y = \frac{16}{x^2}$, you obtain $0 = \frac{16}{x^2}$. There is no real number that satisfies $0 = \frac{16}{x^2}$.

On your own

359–1 $y = \frac{1}{x^2}$. $y = ^{-}14(\frac{1}{x^2})$.

359–2

x	$^{-}4$	$^{-}2$	$^{-}1$	1	2	4
y	$^{-}\frac{1}{2}$	$^{-}2$	$^{-}8$	$^{-}8$	$^{-}2$	$^{-}\frac{1}{2}$

359–3 Yes

359–4 8

359–5

x	-4	-2	-1	1	2	4
y	$\frac{1}{2}$	2	8	8	2	$\frac{1}{2}$

359–6 The first and second quadrants

Lesson 172

360–A $y = -5x$. * Once * Yes

360–B Yes * -5 * -5

360–C The product of two negative numbers is positive. * If x is replaced by a positive number, the y that satisfies the condition must be negative.

360–D -2, -1, $-\frac{1}{2}$, 0, $\frac{2}{5}$, 1, and $\frac{3}{2}$ * $2\frac{1}{2}$ * -5

360–E Yes * Yes

360–F Point M: $(-\frac{1}{2}, 2\frac{1}{2})$
Point N: $(\frac{2}{5}, -2)$ * Yes

360–G $y = 3x^2$. * The second power of x * Yes

360–H Yes * -3

360–I Yes * $x^2 \geq 0$ is a property of real numbers. U = D. * y is negative.

360–J -3 * -3

361–K $-\frac{3}{4}$ * $-\frac{3}{4}$ * $-\frac{1}{3}$ * $-\frac{1}{3}$

361–L The same y satisfies the condition in both cases. The square of a given number is the same as the square of the additive inverse of the given number.

361–M Point A: $(1, -3)$
Point B: $(\frac{1}{2}, -\frac{3}{4})$
Point C: $(\frac{1}{3}, -\frac{1}{3})$
Point D: $(-\frac{1}{3}, -\frac{1}{3})$
Point E: $(-\frac{1}{2}, -\frac{3}{4})$
Point F: $(-1, -3)$

361–N Yes * Yes

361–O Points in the first and second quadrants have a positive second coördinate. The second component of each member of $\{(x,y) \mid y = -3x^2\}$ is not positive.

361–A Yes

362–B Yes * In a condition of the form $y = \frac{k}{x}$, y varies inversely as x varies. * 3

362–C The quotient of two positive numbers is positive. * It is negative.

362–D There is no quotient of 3 and 0. $0 = \frac{3}{x}$ cannot be satisfied by a real number.

362–E -3, -2, and -1 * 1, 2, and 3

362–F They become less. * As the replacements for x increase from 1 to 3, the corresponding replacements for y decrease from 3 to 1.

362–G Point H: (3,1)
Point I: $(2, 1\frac{1}{2})$
Point J: (1,3)
Point K: $(-3, -1)$
Point L: $(-2, -1\frac{1}{2})$
Point M: $(-1, -3)$

362–H The locus of the solution set of a condition of the form $y = \frac{k}{x}$ is a hyperbola. * The first and third quadrants * For each solution of $y = \frac{3}{x}$, both x and y are positive, or both x and y are negative.

362–I Yes

362–J y varies inversely as x^2 varies.

362–K Each x that satisfies $y = \frac{4}{x^2}$ is a non-zero real number. $x^2 > 0$ when x is a non-zero real number. The quotient of 4 and a positive number is positive. * No

362–L The same y satisfies each condition. * The same y satisfies each condition. * The same y satisfies each condition.

362–M They become less. * As the replacements for x increase from 1 to 3, the corresponding replacements for y decrease from 4 to $\frac{4}{9}$

362–N Point A: $(3, \frac{4}{9})$
Point B: (2,1)
Point C: (1,4)
Point D: $(-1, 4)$
Point E: $(-2, 1)$
Point F: $(-3, \frac{4}{9})$

363–0 First and second

On your own

363–1 y varies inversely as x^2.

363–2 y varies directly as x.

363–3 y varies directly as x^2.

363–4 y varies inversely as x.

363–5 y varies directly as x.

363–6 y varies directly as x^2.

363–7 y varies inversely as x.

363–8 y varies inversely as x^2.

363–9 y varies directly as x^2.

363–10

$y = -10x$.							
x	-4	-2	-1	0	1	2	4
y	40	20	10	0	-10	-20	-40

363–11

$y = \frac{3}{x^2}$.						
x	-3	-2	-1	1	2	3
y	$\frac{1}{3}$	$\frac{3}{4}$	3	3	$\frac{3}{4}$	$\frac{1}{3}$

363–12

$y = -4x^2$.							
x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y	-16	-4	-1	0	-1	-4	-16

363–13

$y = \frac{5}{x}$.						
x	$^-15$	$^-10$	$^-5$	5	10	15
y	$^-\frac{1}{3}$	$^-\frac{1}{2}$	$^-1$	1	$\frac{1}{2}$	$\frac{1}{3}$

363–14

$y = \frac{2}{3}x$.							
x	$^-9$	$^-6$	$^-3$	0	3	6	9
y	$^-6$	$^-4$	$^-2$	0	2	4	6

363–15

$y = \frac{1}{2}x^2$.							
x	$^-8$	$^-6$	$^-4$	0	4	6	8
y	32	18	8	0	8	18	32

363–16 Hyperbola

363–17 Parabola

Applying mathematics

365–1 x, number of miles per hour that is the difference of the average speed from London to Toronto and the average speed of the return flight.
$550 - 485 = x$.
{65}. 65 miles per hour

365–2 x, number of feet that top of television tower is above ground.
$475 + 352.1 = x$.
{827.1}. 827 ft.
Notice that rules for approximations are not used to obtain the solution set but are used to obtain the answer.

365–3 x, number of times as much iron ore produced in Quebec as produced in Ontario.
$x/1 \sim (1.2 \times 10^3)/(6 \times 10^6)$.
{2}. 2 times as much

365–4 x, number of pounds of raisins that the mixture contained.
$x + 10\frac{3}{4} \leq 25 \wedge x > 0$.
$\{x \mid 0 < x \leq 14\frac{1}{4}\}$.
The mixture contained some raisins but not more than 14 lb.

365–5 x, number of pounds of grass seed Mr. Page used.
$12.5 - x > 5.25 \wedge x > 0$.
$\{x \mid 0 < x < 7.25\}$.
Between 0 and 7.2 pounds.

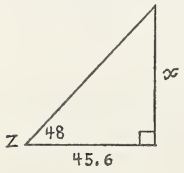
365–6 x, elevation of Lake Superior in feet.
$6.09/1 \sim x/95$.
{578.55}. 580 ft. ±5

365–7 x, total area of the three provinces in square miles.
$x = 251{,}000 + 251{,}700 + 255{,}285$.
{757985}. 757985 sq. mi.

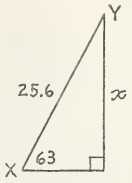
365–8 x, number of square rods of land Mr. Ryan bought.
$80.5/1 \sim x/35$.
{2817.5}. 2800 sq. rd. ±50

365–9 x, area of window in square inches.
$30.25/1 \sim x/26.5$.
{801.625}. 802 sq. in.

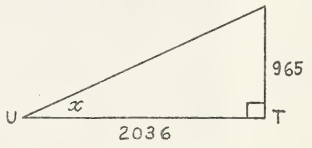
365–10 x, distance in feet from top of chimney to ground.
$(\tan 48)/1 \sim x/45.6$.
{45.6(tan 48)}. 50.6 ft.



365–11 See the sketch at the top of the next column.
x, height of pole in feet.
$(\sin 63)/1 \sim x/25.6$.
{25.6(sin 63)}. 22.8 ft.



365–12 x, measure of angle of elevation of airplane from point U.
$(\tan x)/1 \sim 965/2036$.
$\{x \mid \tan x = \frac{965}{2036}\}$. 25



365–13 x, length of post in feet
$(\cos 75)/1 \sim 3.75/x$.
{3.75/cos 75}. 14.5 ft.

Checking up

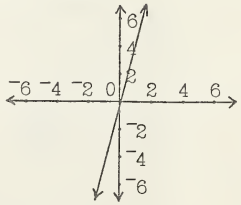
Test 156

366–1 {($^-2, ^-8$), ($^-1, ^-4$), (1,4), (2,8), (3,12)} * Yes

366–2 4

366–3 $y = 4x$.

366–4 Line *



Test 157

366–5 See the chart below. * {(5,⁻50), (3,⁻18), (1,⁻2), (⁻1,⁻2), (⁻3,⁻18)}

x	5	3	1	⁻1	⁻3
y	⁻50	⁻18	⁻2	⁻2	⁻18

366–6 See the chart below. * {(5,$2\frac{4}{5}$), (3,$4\frac{2}{3}$), (1,14), (⁻1,⁻14), (⁻3,$-4\frac{2}{3}$)}

x	5	3	1	⁻1	⁻3
y	$2\frac{4}{5}$	$4\frac{2}{3}$	14	⁻14	$-4\frac{2}{3}$

366–7 See the chart below. * {(5,⁻35), (3,⁻21), (1,⁻7), (⁻1,7), (⁻3,21)}

x	5	3	1	⁻1	⁻3
y	⁻35	⁻21	⁻7	7	21

366–8 See the chart below. * {(5,$2\frac{1}{2}$), (3,$1\frac{1}{2}$), (1,$\frac{1}{2}$), (⁻1,$-\frac{1}{2}$), (⁻3,$-1\frac{1}{2}$)}

x	5	3	1	⁻1	⁻3
y	$2\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-1\frac{1}{2}$

366–9 See the chart below. * {(5,30), (3,$10\frac{4}{5}$), (1,$1\frac{1}{5}$), (⁻1,$1\frac{1}{5}$), (⁻3,$10\frac{4}{5}$)}

x	5	3	1	⁻1	⁻3
y	30	$10\frac{4}{5}$	$1\frac{1}{5}$	$1\frac{1}{5}$	$10\frac{4}{5}$

366–10 See the chart below. * {(5,$\frac{1}{25}$), (3,$\frac{1}{9}$), (1,1), (⁻1,1), (⁻3,$\frac{1}{9}$)}

x	5	3	1	⁻1	⁻3
y	$\frac{1}{25}$	$\frac{1}{9}$	1	1	$\frac{1}{9}$

366–11 y varies directly as x varies. * ⁻14

366–12 y varies inversely as x^2 varies. * 5

366–13 y varies inversely as x varies. * ⁻19

Lesson 173

366–A The constant of variation

366–B x is a variable for the number of miles of road. * 4 * 4 = k(10).

366–C Yes

366–D Condition B can be obtained from condition A by using the well-defined property of multiplication. Condition C can be obtained by using the associative property of multiplication. Condition D can be obtained by using the reciprocal property and the definition of multiplication. Condition E can be obtained by using the identity-element property of multiplication.

366–E $\frac{2}{5}$ * Yes

367–F Yes

367–G Yes

367–H 30

367–I x is a variable for the number of months. * 10 * 10 = k(4).

367–J $2\frac{1}{2}$

367–K $y = 2\frac{1}{2}x$.

367–L x is a variable for the number of months.

367–M {30}. The crew can build 30 miles of road in 12 months.

367–N Yes

368–O Yes * 7 * Yes

368–P $\frac{4}{9}$ * $\frac{2}{3}$ * Yes

368–Q $(\frac{8}{5})^2 = \frac{64}{25}$ and $\sqrt{\frac{64}{25}} = \frac{8}{5}$. Therefore, $\sqrt{(\frac{8}{5})^2} = \frac{8}{5}$.

368–R For example: $(⁻7)^2 = 49$, but $\sqrt{49} = 7$. Therefore, $\sqrt{(⁻7)^2} = 7$, and $\sqrt{(⁻7)^2} \neq ⁻7$.

368–S Replace x by 16 and y by $\frac{64}{4}$. * Replace x by 16 and y by $\frac{64}{4}$. * Yes

368–T Yes * Yes

368–U x was replaced by 9 and y by $\frac{225}{25}$. * x was replaced by 9 and y by $\frac{225}{25}$. * Yes

368–A x is a variable for the speed of the car in miles per hour and y is a variable for the distance in feet that the car traveled after the brakes were applied. Since the distance that a car travels after brakes are applied varies directly as the square of the speed of the car, the condition for the problem is of the form $y = kx^2$.

368–B 30 * 63 * 63 = $k(30)^2$.

368–C 900 * 63 and 900 * .07

369–D $y = .07x^2$.

369–E y is a variable for the distance in feet the car traveled after brakes were applied. * $175 = .07x^2$.

369–F Use the reciprocal of .07 and the well-defined property of multiplication. Multiply both 175 and $.07x^2$ by $\frac{1}{07}$.

369–G x is a variable for the speed of the car in miles per hour and must be a positive number. * The universe for x is D. For each x, if $x \geq 0$, then $\sqrt{x^2} = x$. The universe

for x and y is D. For each x and y, if $x \geq 0$ and $y \geq 0$, and if $x = y$, then $\sqrt{x} = \sqrt{y}$. In other words, since x is positive you know that $\sqrt{x^2} = x$. Since 2500 is positive, x^2 is positive, and $2500 = x^2$, you know that $\sqrt{2500} = \sqrt{x^2}$, or $50 = x$.

369–H 50 * 50 miles per hour

369–I With twice as many pipes, it would take one half as long to fill the reservoir. * With one third as many pipes, it would take three times as long to fill the reservoir.

369–J No * Yes

369–K Yes

369–L 240 * 240

369–M 24

369–N It will take 10 of the pipes 24 hours to fill the reservoir.

370–O $.04 * .2 = \frac{k}{(.04)^2}$.

370–P Yes

370–Q Yes * .00032

370–R $y = \frac{.00032}{x^2}$.

370–S $y = \frac{.00032}{(.05)^2}$.

370–T Yes

370–U .00032 and .0025

370–V .128

370–W The electrical resistance of the second wire is .128 ohms.

On your own

370–I x, number of hours clock runs.
y, number of minutes clock gains.
$y = kx$. * $2\frac{1}{2} = k(12)$.
$k = \frac{5}{24}$. * $y = \frac{5}{24}(48)$. * 10 min.

370–2 x, number of boys.
y, number of hours it takes to wax car.
$y = \frac{k}{x}$. * $3 = \frac{k}{2\frac{3}{4}}$. $k = 8\frac{1}{4}$.
* $y = \frac{8\frac{1}{4}}{2}$. * $4\frac{1}{8}$ hr.

370–3 x, number of inches of precipitation Churchill had last year.
y, number of inches of precipitation Halifax had last year.
$y = kx$. * $3.6 = k(1)$.
$k = 3.6$. * $y = 3.6(15)$.
* 54.0 in.

370–4 x, number of presses.
y, number of hours.
$y = \frac{k}{x}$. * $2 = \frac{k}{3}$. $k = 6$.
* $y = \frac{6}{8}$. * $\frac{3}{4}$ hr.

370–5 x, altitude of triangle.
y, area of triangle.
$y = kx$. * $50 = k(6.4)$.
$k = 7.8125$. *
$y = 7.8125(4.8)$. *
37.5 sq. in.

370–6 x, time for one swing.
y, length of pendulum.
$y = kx^2$. * $60 = k(1.5)^2$.
$k = 26\frac{2}{3}$, or about 26.7.
* $y = 26\frac{2}{3}(2^2)$, or
$y = 26.7(2^2)$. * $106\frac{2}{3}$ cm.
or about 106.8 cm.

370–7 x, time in seconds.
y, distance rock travelled.
$y = kx^2$. * $400 = k(5^2)$.
$k = 16$. * $y = 16(10^2)$.
* 1600 ft.

370–8 x, distance from center of earth.
y, weight of object.
$y = \frac{k}{x^2}$. * $600 = \frac{k}{(4000)^2}$.
$k = 9.6 \times 10^9$. *
$y = (9.6 \times 10^9)/(5000)^2$.
* 384 lb.

Keeping skilful

371–1 10,000 lb.

371–2 $5\frac{2}{3}$ ft.

371–3 5280 yd.

371–4 400 sq. rd.

371–5 5 sq. yd.

371–6 1152 sq. in.

371–7 $2/4 \sim (\sin 30)/1$.

371–8 $3/4 \sim (\tan 37)/1$.

371–9 $2\sqrt{3}/4 \sim (\cos 30)/1$.

371–10 $4/5 \sim (\sin 53)/1$.

371–11 $2/2\sqrt{3} \sim (\tan 30)/1$.

371–12 $3/5 \sim (\sin 37)/1$.

371–13 $3/5 \sim (\cos 53)/1$.

371–14 $2\sqrt{3}/2 \sim (\tan 60)/1$.

371–15 $4/3 \sim (\tan 53)/1$.

371–16 $2\sqrt{3}/4 \sim (\sin 60)/1$.

371–17 $\{7\frac{1}{3}\}$

371–18 $\{3\}$

371–19 $\{h \mid h > 30\}$

371–20 $\{t \mid t > \bar{}1\frac{1}{2}\}$

371–21 $\{m \mid \bar{}12 < m < 2\}$

371–22 $\{n \mid n > \bar{}6\}$

Lesson 174

371–A No * $y/5$ * $20/1$

371–B k is a variable for the number of miles per hour and $k/1$ represents the distance travelled per one hour. $156/3$ represents distance travelled per 3 hr. The two rate pairs are equivalent.

372–C 1500 miles * Number of hours it takes to fly 1500 miles * The first component of 600/1 refers to 600 mi. and the second component refers to 1 hour.

372–D Yes

372–E t is the first letter of the word *time*.

372–F Yes * Use the definition of equivalent rate pairs.

372–G Yes

372–H 5 * 20

372–I Yes

372–J Replace d by 156 and t by 3. * $156 = r(3)$.

372–K $1500 = 600t$.

372–A No * Yes

372–B 1.25/1 * Yes

373–C Total cost of the 4 rolls of film * 4, which is the number of items * 1.25, which is the price

373–D Yes

373–E Use the definition of equivalent rate pairs and the commutative property of multiplication.

373–F For problem G, you can replace n by 4 in $c = np$ and p by 1.25 to obtain $c = 4(1.25)$ as a condition for the problem.

373–G Yes

373–H Because $80(1) = .8(100)$. * Yes * Yes

373–I A percentage of the base

373–J Yes.* Use the definition of equivalent rate pairs and the commutative property of multiplication.

373–K Replace p by 12 in $p = br$ and r by .8 to obtain $12 = b(.8)$ as a condition for the problem.

373–L The distance from home plate to the centre-field fence in yards * The comparison of the number of feet with the number of yards from home plate to the centre-field fence

374–M The comparison of the number of feet in one yard with one yard * Yes

374–N The number of lesser units * The number of greater units * The constant that represents the number of lesser units in 1 of the greater units

374–O Yes

374–P Use the definition of equivalent rate pairs.

374–Q Replace l by 426 in $l = kg$ and k by 3 to obtain $426 = 3g$ as a condition for the problem.

374–R Yes * Yes

374–S For the number of games * The average number of points per game

374–T Yes

374–U Use the definition of equivalent rate pairs and the well-defined property of multiplication. Multiply both t and an in $t = an$ by $\frac{1}{n}$. * a refers to the average number of points per game. t refers to the total number of points. n refers to the number of games.

374–V Replace t by 90 in $a = \frac{t}{n}$ and n by 8 to obtain $a = \frac{90}{8}$ as a condition for the problem.

375–A y/x represents the comparison of the measure of the side opposite an acute angle of a right triangle with the measure of the side adjacent to this angle. t is the measure of the acute angle. By definition, tan t is the first component of a rate pair that has a second component of 1 and that is equivalent to the rate pair that represents the comparison of the measures of the side opposite the acute angle with the side adjacent to the acute angle.

375–B $y/x \sim (\tan t)/1$.
$(\tan t)x = y$.
Definition of equivalent rate pairs.
$(\tan t)x \cdot \frac{1}{x} = y \cdot \frac{1}{x}$.
Well-defined property of multiplication.
$(\tan t) \cdot 1 = \frac{y}{x}$.
Reciprocal property.
$\tan t = \frac{y}{x}$.
Identity-element property of multiplication

375–C ΔABC has a right angle. * $\angle C$ * $\angle A$ and $\angle B$

375–D Yes * The tangent of $\angle A$ * 6 is the measure of the side opposite $\angle A$. * 8

375–E Replace y by 8 in $\tan t = \frac{y}{x}$ and x by 6 to obtain $\tan t = \frac{8}{6}$. In this case, t is the variable for the measure of $\angle B$.

375–F y/r represents the comparison of the measure of the side opposite an acute angle of a right triangle with the measure of the hypotenuse. t is the measure of the acute angle. By definition, sin t is the first component of a rate pair that has a second component of 1 and that is equivalent to the rate pair that represents the comparison of the measure

of the side opposite the acute angle with the measure of the hypotenuse.

375–G $y/r \sim (\sin t)/1$.
$(\sin t)r = y$.
Definition of equivalent rate pairs.
$(\sin t)r \cdot \frac{1}{r} = y \cdot \frac{1}{r}$.
Well-defined property of multiplication.
$(\sin t) = \frac{y}{r}$.
Reciprocal property and identity-element property of multiplication

375–H Replace y by 8 in $\sin t = \frac{y}{r}$ and r by 10 to obtain $\sin t = \frac{8}{10}$. In this case, t is a variable for the measure of $\angle B$.

375–I Replace y by 6 and r by 10.

375–J Yes

375–K Yes * $x/r \sim (\cos t)/1$.
$(\cos t)r = x$.
Definition of equivalent rate pairs.
$(\cos t)r \cdot \frac{1}{r} = x \cdot \frac{1}{x}$.
Well-defined property of multiplication.
$\cos t = \frac{x}{r}$.
Reciprocal property and identity-element property of multiplication.

375–L Replace x by 8 in $\cos t = \frac{x}{r}$ and r by 10 to obtain $\cos t = \frac{8}{10}$. * Replace x by 6 in $\cos t = \frac{x}{r}$ and r by 10 to obtain $\cos t = \frac{6}{10}$.

n your own

375–1 $l = kg$. * $l = 100(.75)$.

375–2 $c = np$. * $35 = n(5)$.

376–3 $\tan t = \frac{y}{x}$. * $\tan 40 = \frac{y}{3.5}$.

376–4 $p = br$. * $26 = b(.48)$.

376–5 $a = \frac{t}{n}$. * $2\frac{1}{3} = \frac{t}{5}$.

376–6 $\cos t = \frac{x}{r}$. * $\cos t = \frac{14}{22}$.

376–7 $c = np$. * $60 = 4p$.

376–8 $d = rt$. * $d = 7.5(15)$.

376–9 $l = kg$. * $l = 12(5\frac{1}{2})$.

376–10 $\sin t = \frac{y}{r}$. * $\sin 30 = \frac{8.2}{r}$.

376–11 $p = br$. * $40 = 8500r$.

376–12 $d = rt$. * $450 = 95t$.

376–13 $\tan t = \frac{y}{x}$. * $\tan t = \frac{9.45}{2.6}$.

376–14 $\cos t = \frac{x}{r}$. * $\cos t = \frac{5}{12}$.

376–15 $l = kg$. * $l = 1000(.125)$.

376–16 $c = np$. * $c = 4\frac{1}{2}(1.65)$.

376–17 $l = kg$. * $4 = 16g$

376–18 $p = br$. * $p = 8.2(1.75)$.

Lesson 175

376–A The distance the plane flew

376–B The rate at which the plane flew

376–C 3 hr. * The number of hours the plane flew

376–D The first component of (r,t,d) is r. The only r that satisfies $r = 400$ is 400. * 3

377–E (Column 1) $d = 400 \times 3$. * 1200

377–F (Column 1) {(400,3,1200)}

377–G (Column 1) r and t

377–H 400 * 3

377–I Yes

377–J 1200 * Yes

377–A The problem in D2 involves cost and $c = np$ is the cost formula.

377–B p

377–C 16,995 * 110 * Yes

377–D Use the reciprocal of 110 and the well-defined property of multiplication. Multiply both 16995 and $110p$ in $16995 = 110p$ by $\frac{1}{110}$. * 154.50

377–E (Column, 2) $154.50

377–F (Column 2) Yes

377–G (Column 2) p * b * 2.39 * 11.95

378–H The rate of discount

378–I Multiply both 2.39 and $11.95r$ in $2.39 = 11.95r$ by $\frac{1}{11.95}$. * .2

378–J 20 * 20%

378–K The problem in D4 involves average and $a = \frac{t}{n}$ is the average formula.

378–L t

378–M a * n

378–N Replace a by 950 and n by 6 in $a = \frac{t}{n}$. * {5700}

378–O 5700

378–P The problem in D5 involves converting from one standard unit to another and $l = kg$ is the conversion formula.

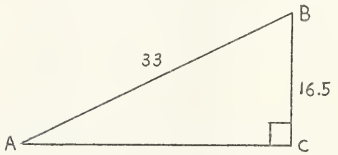
378–Q l * g

378–R 36 * 36

378–S Replace k by 36 and g by $1\frac{3}{8}$ in $l = kg$. * {$49\frac{1}{2}$}

378–T $49\frac{1}{2}$ inches

378–A See the sketch below. * 16.5 * 33



379–B For the problem in D6, you want to find the measure of $\angle A$. Since you know the measures of the side opposite $\angle A$ and the hypotenuse, you can find the sine of $\angle A^{\circ}$. This will enable you to find $\angle A^{\circ}$.

379–C 16.5 * 33 * $\sin t = \frac{16.5}{33}$. * .5000

379–D The measure of an angle

whose sine is .5000 is 30. * The measure of the angle of elevation of the top of the ladder from the bottom is 30.

379–E You know $\angle A^\circ$ and the measure of the hypotenuse. You want to find the measure of the side adjacent to $\angle A$.

379–F Replace t by 30 in $\cos t = \frac{x}{r}$ and r by 33.

379–G From the table, you can obtain .8660 as cos 30. Replace cos 30 by .8660 in $\cos 30 = \frac{x}{33}$.

379–H .8660 and 33

379–I 28.58

379–J The bottom of the ladder is 28.58 ft. from the base of the building.

379–K You want to find the measure of the side adjacent to $\angle A$. From the information given in problem A, you know the measure of the side opposite $\angle A$ is 16.5. From your answer to problem A you know $\angle A^\circ = 30$. Replace tan t by tan 30, or .5774 in $\tan t = \frac{y}{x}$ and y by 16.5. Then find the solution of the condition obtained. * {28.58}

On your own

379–1 $c = np$. * $4064 = 32p$. * $127

379–2 $l = kg$. * $l = 6076.10 \times 10$. * 60,761 ft.

379–3 $d = rt$. * $10 = r(\frac{1}{3})$. * 30 nautical miles per hour

379–4 $p = br$. * $3 = 15.95r$. * 19%

380–5 $a = \frac{t}{n}$. * $2\frac{1}{2} = \frac{t}{24}$. * 60 hr.

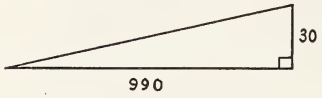
380–6 $p = br$. * $80000 = 35000r$. * 229%

380–7 $a = \frac{t}{n}$. * $a = \frac{3550}{7}$. * $507

380–8 $l = kg$. * $96 = 3600g$ * $\frac{2}{75}$ of 1 hr., or .02$\dot{6}$ hr., which is about .027 hr.

380–9 $d = rt$. * $1.2 = r(\frac{2}{75})$, or $1.2 = r(.027)$. * Using $1.2 = r(\frac{2}{75})$, you find that the monorail travelled 45 miles per hour. Using $1.2 = r(.027)$, you find that the monorail travelled 44 miles per hour.

380–10 See the sketch below. $\tan t = \frac{y}{x}$. * $\tan t = \frac{30}{990}$. * 2



Keeping skilful

380–1 $14\frac{5}{8}$

380–2 $-\frac{2}{5}$

380–3 $\frac{11}{36}$

380–4 $-14\frac{1}{2}$

380–5 $-\frac{170}{1}$

380–6 $-6\frac{13}{16}$

380–7 $\frac{120}{1}$

380–8 $1\frac{5}{8}$

380–9 $-15\frac{13}{20}$

380–10 $1\frac{1}{4}$

380–11 $-11\frac{1}{6}$

380–12 $236\frac{1}{4}$

380–13 $-\frac{15}{16}$

380–14 $-80\frac{15}{16}$

Lesson 176

380–A The connective "and" * Yes

381–B Condition D can be obtained from condition C by using the definition of equivalent rate pairs.

381–C From the well-defined property of multiplication, you know that, because $rp = x$, the product of x and t is the same as the product of rp and t. * Yes

381–D Replace p by 1500.

381–E Yes

381–F 247.5

381–G The principal * The interest * The principal and the interest

381–H Yes

381–I 2913.12 * Mr. Holland had $2913.12 in his savings account at the end of the year.

381–J c, o, and p * Yes

381–K 2.75 * The selling price should be $2.75 per box.

382–L The replacement for p is 0. * No

382–M The replacement for p is negative. * No

382–A The force and the distance

382–B 15 * Yes

382–C 375 * 375 foot-pounds of work are necessary to raise the 25 lb. object a distance of 15 ft.

382–D w is a variable for the work and t is a variable for the time. * The work and the time

382–E Replace w by 66,000 and t by 30.

382–F 2200 * The power of the machine is 2200 foot-pounds per second.

382–G $\frac{1}{2}$ * 132,000 foot-pounds per minute

383–H The number of volts and the number of ohms

383–I From problem N you know that there are 120 volts pressure and 15 ohms resistance. Therefore, you can replace E in $I = \frac{E}{R}$ by 120 and R by 15 to obtain $I = \frac{120}{15}$.

383–J 8 * 8 amperes of current will flow through the resistance.

383–K The number of volts of pressure * The number of amperes of current * The number of volts and the number of amperes

383–L W in $W = EI$ was replaced by 5500 and I was replaced by 50. * 110

383–M 110 volts of pressure will produce the 5500 watts of power.

383–N 212° on the Fahrenheit scale and 100° on the centigrade scale

383–O 100 * 180 * Yes

384–P F * Yes

384–Q 25

384–R $C = \frac{5}{9}(F - 32)$.
$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$.
Well-defined property of multiplication.
$\frac{9}{5}C = F - 32$.
Reciprocal property and identity-element property of multiplication.
$\frac{9}{5}C + 32 = F - 32 + 32$.
Well-defined property of addition.
$\frac{9}{5}C + 32 = F$.
Additive-inverse property.
$F = \frac{9}{5}C + 32$.
Symmetric property of equality

384–S 50 * Yes

384–T 122 * 122° F, is the same temperature as 50° C.

On your own

384–1 $s = c + o + p$. * $69.95 = 47.50 + 9.70 + p$. * $12.75

384–2 $C = \frac{5}{9}(F - 32)$. * $C = \frac{5}{9}(^{-}18 - 32)$. * $^{-}27\frac{7}{9}$° C.

384–3 $w = fd$. * $880 = f \times 55$. * 16 lb.

384–4 $a = p + i$. * $811.83 = 750 + i$. * $61.83

385–5 $I = \frac{E}{R}$. * $275 = \frac{850}{R}$. * $3\frac{1}{11}$ ohms

385–6 $i = prt$. * $1260 = 6000r(4)$. * $5\frac{1}{4}\%$

385–7 $p = \frac{w}{t}$. * $165000 = \frac{500000}{t}$. * $3\frac{1}{33}$ min.

385–8 $s = c + o + p$. * $14.79 = 18.00 + 4.50 + p$. * $7.71 loss

385–9 $W = EI$. * $W = 120 \times \frac{1}{2}$. * 60 watts

385–10 $i = prt$. * $i = 500 \times .065 \times 1$. * $32.50

385–11 $I = \frac{E}{R}$. * $20 = \frac{E}{8.5}$. * 170 volts

385–12 $F = \frac{9}{5}C + 32$. * $F = \frac{9}{5}(1535) + 32$. * 2795° F.

385–13 $s = c + o + p$. * $s = 14 + 1.95 + 5.15$. * $21.10

Keeping skilful

385–1 27.857

385–2 $^{-}1.39$

385–3 1.9

385–4 $^{-}16.325$

385–5 $^{-}15.28$

385–6 $^{-}8.39$

385–7 $^{-}24.\overline{3}$

385–8 4.6

385–9 25

385–10 $^{-}4.508$

385–11 $^{-}18.9$

385–12 $^{-}16.4$

385–13 57.97

385–14 $^{-}182.10$

385–15 $^{-}1092$

385–16 194.56

Lesson 177

385–A Yes

385–B Use the commutative and associative properties of addition. * Use the identity-element property of multiplication, the well-defined property of addition, and the distributive property. * Use the well-defined property of multiplication.

386–C 18 is given as the length of the rectangle and 15 is given as the width. * {66} * 66 ft.

386–D You can replace p and l in $p = 2l + 2w$. Then find the replacement for w that satisfies the condition you obtain.

386–E The measures of the sides are equal.

386–F The length and width of a square are both equal to the measure of a side of the square.

386–G Use the distributive property. * Use the well-defined property of multiplication.

386–H Replace p in $p = 4s$ by the given measure of the perimeter. Then find the replacement for s that satisfies the condition you obtain.

386–I The perimeter of the triangle * The measure of the sides of the triangle

386–J The measures of the sides are equal.

386–K Yes

386–A Yes * Use the definition of equivalent rate pairs.

387–B $1\frac{1}{2}$ * 3 * $\{4\frac{1}{2}\}$ * The area of the parallelogram is $4\frac{1}{2}$ sq. ft.

387–C Yes. The segment that represents the width is perpendicular to the lines that include a pair of opposite sides of the rectangle. * Yes * A rectangle is a parallelogram. You can use the width of the rectangle as a replacement for a in $A = ab$ because the width is the measure of the altitude. You can use the length of the rectangle as a replacement for b in $A = ab$ because the length is the measure of the base.

387–D Replace A in $A = lw$ by the area of the rectangle and w by the width of the rectangle. Then find the l that satisfies the condition obtained.

387–E They are equal. * Yes

387–F The measure of the base * The measure of the altitude * Think of a parallelogram that has a base and an altitude in common with the triangle. The condition $b/1 \sim 2A/a$ can be used to find the area of the parallelogram if you use $2A$ to refer to the area of the parallelogram. If a parallelogram and a triangle have a common base and a common altitude, then the area of the parallelogram is 2 times the area of the triangle. Thus, if $2A$ in $b/1 \sim 2A/a$ refers to the area of the parallelogram, then A refers to the area of the triangle.

387–G Use the definition of equivalent rate pairs.

387–H Use the reciprocal of 2 and the well-defined property of multiplication. Multiply both $2A$ and ab in $2A = ab$ by $\frac{1}{2}$.

387–I Replace a by 2 and b by 3.5.

387–J {3.5}

387–K 3.5 sq. in.

388–A (Column 1) Sides AB and CD and sides BC and AD

388–B (Column 1) Yes. If the intersection of two coplanar lines is the empty set, then the two lines are parallel. * Yes. Two segments included in parallel lines are parallel.

388–C No

388–D ABCD is a quadrilateral that has two, and only two, parallel sides.

388–E {G} * {F}

388–F Yes * Yes

388–G Yes

388–H $\overline{BE}$ is perpendicular to both of the lines that include the parallel sides of the trapezoid. The endpoints of $\overline{BE}$ are contained in the lines that include the parallel sides. * $\overline{HD}$ is perpendicular to both of the lines that include the parallel sides of the trapezoid. The endpoints of $\overline{HD}$ are contained in the lines that include the parallel sides.

388–I $\overline{AB}$ is not perpendicular to the lines that include the parallel sides of the trapezoid. * $\overline{CD}$ is not perpendicular to the lines that include the parallel sides of the trapezoid.

388–J Any parallelogram that has a right angle is a rectangle. $\angle E$ is a right angle.

388–K Yes * $\overline{BE}$ and $\overline{GF}$ are opposite sides of rectangle EBGF and opposite sides of a rectangle have the same measure.

388–L Yes

388–A (Column 2) $\angle J$ and $\angle L$ are one pair of opposite angles and $\angle K$ and $\angle M$ are the other pair. * Points J and L are one pair of opposite vertices and points K and M are the other pair.

388–B (Column 2) Segment KM

389–C $\overline{JK}$, $\overline{KM}$, and $\overline{JM}$ * $\overline{KL}$, $\overline{LM}$, and $\overline{KM}$

389–D The closed region determined by trapezoid JKLM * Yes

389–E Yes * Yes

389–F JM

389–G Yes * Yes

389–H $\overline{KL}$

389-I Yes * Yes

389-J The measure of the altitude of ΔJKM is the same as the measure of the altitude of ΔKLM.

389-K a represents the measure of the altitude of ΔJKM. b_1 represents the measure of the base of ΔJKM. Thus, the area of ΔJKM is represented by $\frac{1}{2}ab_1$. * a represents the measure of the altitude of ΔKLM. b_2 represents the measure of the base of ΔKLM. Thus, the area of ΔKLM is represented by $\frac{1}{2}ab_2$.

389-L Yes

389-M Distributive property * Yes * $\overline{JM}$ * $\overline{KL}$

389-N Replace a by 3, b_1 by 5, and b_2 by 7.

390-O Yes * 18 sq. in.

On your own

390-1 $p = a + b + c$.
$p = 3 + 7 + 9$.
{19}. 19 mm.

390-2 $A = lw$. $24 = l(5)$.
{4.8}. 4.8 ft.

390-3 $p = 4s$. $p = 4(9\frac{1}{2})$.
{38}. 38 ft.

390-4 $p = 3s$. $p = 3(5.2)$.
{15.6}. 15.6 cm.

390-5 $A = ab$. $A = 4.7 \times 6.2$.
{29.14}. 29.14 sq. yd.

390-6 $p = 2l + 2w$.
$p = 2(32) + 2(20.5)$.
{105}. 105 cm.

390-7 $A = s^2$. $A = (3\frac{1}{4})^2$.
$\{10\frac{9}{16}\}$. $10\frac{9}{16}$ sq. in.

390-8 $A = \frac{1}{2}ab$. $A = \frac{1}{2}(5.3)(6.2)$.
{16.43}. 16.43 sq. yd.

390-9 $A = ab$. $6.3 = (.7)b$.
{9}. 9 m.

390-10 $p = 2l + 2w$.
$39 = 2(14) + 2w$.
$\{5\frac{1}{2}\}$. $5\frac{1}{2}$ cm.

390-11 $A = \frac{1}{2}a(b_1 + b_2)$.
$A = \frac{1}{2}(1\frac{1}{2})(3 + 5\frac{1}{2})$.
$\{6\frac{3}{8}\}$. $6\frac{3}{8}$ sq. ft.

390-12 $A = \frac{1}{2}ab$. $A = \frac{1}{2}(1\frac{1}{2})(\frac{7}{10})$.
$\{\frac{21}{40}\}$. $\frac{21}{40}$ sq. m.

390-13 $p = 4s$. $13.6 = 4s$.
{3.4}. 3.4 mm.

390-14 $A = \frac{1}{2}a(b_1 + b_2)$.
$6.5 = \frac{1}{2}(a)(2.2 + 4.3)$.
{2}. 2 ft.

390-15 $p = 4s \wedge A = s^2$.
$35 = 4s \wedge A = s^2$.
$\{(8\frac{3}{4}, 76\frac{9}{16})\}$ for (s, A).
$76\frac{9}{16}$ sq. ft.

390-16 $A = \frac{1}{2}ab \wedge p = a + b + c$.
$A = \frac{1}{2}(4)b \wedge 16 = 5 + b + 5$. {(12,6)} for (A, b).
12 sq. ft.

Applying mathematics

391-1 $s = c + o + p$.
$14703 = 10247 + 1824.35 + p$. $2631.65

391-2 $p = \frac{w}{t}$. $75000 = \frac{w}{30}$.
2,250,000 foot-pounds

391-3 $i = prt$.
$i = (2400)(.055)(2)$. $264

391-4 $F = \frac{9}{5}C + 32$.
$F = \frac{9}{5}(810) + 32$. 1490° F.

391-5 $d = rt$. $250.2 = r(4.5)$.
55.6 mi. per hr.

391-6 $a = \frac{t}{n}$. $\frac{3}{4} = \frac{t}{30}$. $22\frac{1}{2}$ gal.

391-7 $I = \frac{E}{R}$. $10.5 = \frac{E}{9.6}$.
100.8 volts

391-8 $a = p + i$.
$a = 4500 + 146.25$.
$4646.25

391-9 $p = br$. $p = 6{,}049{,}000 \times .111$. 671,439 persons

391-10 $p = 2l + 2w$.
$34\frac{1}{2} = 2(10\frac{1}{2}) + 2w$. $6\frac{3}{4}$ ft.

391-11 $\cos t = \frac{x}{r}$. $\cos 77 = \frac{5.85}{r}$.
26 ft.

391-12 $A = lw$. $A = 295.5(1572)$.
464,526 sq. ft.

391-13 $W = EI$. $1.86 \times 10^5 = E(2.0 \times 10^1)$.
9.3×10^3 volts

391-14 $w = fd$. $w = 15(4)$.
60 foot-pounds

391-15 $s = c + o + p$.
$12.95 = 8.75 + .98 + p$.
$3.22

391-16 $l = kg$. $69 = 12g$.
$5\frac{3}{4}$ ft.

391-17 $p = 3s$. $p = 3(4\frac{1}{4})$.
$12\frac{3}{4}$ in.

391-18 $d = rt$. $1000 = 625t$.
1.6 hr.

391-19 $i = prt$.
$270 = 1500(r)3$. 6%

392-20 $C = \frac{5}{9}(F - 32)$.
$C = \frac{5}{9}(68 - 32)$. 20° C.

392-21 $p = br$. $395.30 = b(.059)$.
$6700

392-22 $A = \frac{1}{2}a(b_1 + b_2)$.
$A = \frac{1}{2}(12\frac{1}{2})(24 + 36\frac{1}{2})$.
$378\frac{1}{8}$ sq. ft.

392-23 $\tan t = \frac{y}{x}$. $\tan t = \frac{140}{155}$.
42

392-24 $p = 4s$. $p = 4(39\frac{1}{2})$.
158 in.

392-25 $c = np$. $91.80 = 12p$.
$7.65

392-26 $A = s^2$. $A = 150^2$.
22,500 sq. ft.

392-27 $\sin t = \frac{y}{r}$. $\sin t = \frac{24.5}{28}$.
61

392-28 $A = \frac{1}{2}ab$.
$A = \frac{1}{2}(20.4)(30.2)$.
308.04 sq. cm.

Checking up

Test 158

392-1 T

392-2 F

392-3 F

392-4 T

392-5 T

392-6 F

392-7 F

392-8 T

392-9 T
392-10 F

Test 159

392-11 (d) $(\bar{}3,2)$
392-12 (b) $(x,\bar{}4x)$
392-13 (c) parabola
392-14 (c) $y = \frac{5}{x}$.
392-15 (b) a two-part curve
392-16 (a) $x^2 \geq 0$.
392-17 (d) $12.435 < 12.\overline{4}$.
392-18 (b) $c = np$.
392-19 (d) 100
392-20 (c) $A = \frac{1}{2}a(b_1 + b_2)$.

Test 160

393-21 $\frac{3}{2}$
393-22 x^2
393-23 variation
393-24 hyperbola
393-25 $\frac{1}{3}$
393-26 distance
393-27 percentage
393-28 3
393-29 average
393-30 two

Test 161

393-31 $p = 2(12\frac{1}{2}) + 2(8\frac{1}{2})$. 42 ft.
393-32 $6 = 4s$. $1\frac{1}{2}$ in.
393-33 $p = 3(17)$. 51 m.
393-34 $12 = 8b$. $1\frac{1}{2}$ yd.
393-35 $32 = \frac{1}{2}(4)b$. 16 in.
393-36 $A = 5(8.6)$. 43 sq. ft.
393-37 $A = \frac{1}{2}(4)(8.9 + 6.7)$.
31.2 sq. mi.

Test 162

394-1 F
394-2 T
394-3 T
394-4 T
394-5 T
394-6 T
394-7 T
394-8 T
394-9 T
394-10 T

Test 163

394-11 Identity-element property of addition

394-12 Commutative property of addition

394-13 Additive-inverse property

394-14 Associative property of addition

394-15 Well-defined property of addition

Test 164

394-16 (c) Two points
394-17 (b) 180
394-18 (b) an equiangular
394-19 (a) $(\frac{\bar{}2}{11})^6$
394-20 (d) real numbers
394-21 (d) $< \sqrt{5}(\sqrt{8})$
394-22 (b) 100 cm.
394-23 (d) one
394-24 (b) 4
395-25 (c) sin ∠E/1

Test 165

395-26 x, increase in Mr. Meyer's salary.
$4500 + x < 8200 \wedge x > 0$.
$\{x \mid 0 < x < 3700\}$.
Less than \$3700

395-27 x, amount Ann spent.
y, amount Fred spent.
$x + y = 24.60 \wedge x = 2y$.
$\{(16.40, 8.20)\}$.
Ann spent \$16.40 and Fred spent \$8.20.

395-28 x, number of blocks Mr. Wagner had before he bought a load of 32.
$x < 48 \wedge x + 32 > 52 \wedge x \in I$.
$\{21, 22, 23, \ldots, 47\}$.
21 through 47 blocks

395-29 $A = (1\frac{1}{3})^2$. $\{1\frac{7}{9}\}$ $1\frac{7}{9}$ sq. in.

395-30 $x - y = \bar{}4 \wedge x = \frac{2}{3} + \frac{1}{2}$.
$\{(1\frac{1}{6}, 5\frac{1}{6})\}$

395-31 $x/y \sim 27/9 \wedge x + y = 16$.
$\{(12,4)\}$

395-32 $xy = 16 \wedge x = 2\frac{3}{4} + 2\frac{7}{12}$.
$\{(5\frac{1}{3}, 3)\}$

395-33 $xyz = 40\frac{1}{3} \wedge x = 3y \wedge y = 22 \div 6$.
$\{(11, 3\frac{2}{3}, 1)\}$

Test 165b

395-34 180
395-35 6
395-36 b
395-37 c
395-38 d
395-39 The starting point is S and the terminal point is K
395-40 4.75
395-41 b
395-42 2.5
395-43 .01 ft.
395-44 similar
395-45 directly
395-46 2300

Test 165c

396-47 coplanar
396-48 {L, M, N}
396-49 L
396-50 MS
396-51 page 5, D2

396 **-52** union
396 **-53** $\frac{7}{17}$
396 **-54** right
396 **-55** $\frac{4}{49}$
396 **-56** d
396 **-57** b
396 **-58** locus

Test 165d

396 **-59** 180
396 **-60** Side JK, side KL, side JL
396 **-61** Ra
396 **-62** a
396 **-63** b
396 **-64** d
396 **-65** b
396 **-66** 8584
396 **-67** {x/x<6 }
396 **-68** {(5,-3,1)}
396 **-69** 4
396 **-70** adjacent
396 **-71** a
396 **-72** Side RM
396 **-73** c
396 **-74** 8×10^{-5}

Lesson 178

397-A No. Because the two copies are new, they look alike; and because they are on the same shelf, one is not easier to reach than the other. Therefore, it does not seem likely that you would choose one copy in preference to the other.

397-B Because the pieces of pie look alike, the chance of someone's choosing one of the pieces is the same for each piece.

397-C Yes. Because there are 2 pieces of lemon candy for each piece of mint candy, it seems more likely that you would take a piece of lemon candy rather than a piece of mint candy.

398-D Taking the white marble * Yes

398-E If the marbles are the same size, each outcome is equally likely to occur.

398-F 1

398-G 1, 2, 3, 4, 5, and 6 * Yes

398-H 1

398-I 1 chance in 6

398-A Condition A requires that 1/2 and $P/1$ be equivalent to each other. * The number of chances in 1 the blue marble has of being taken

398-B Use the definition of equivalent rate pairs. * Use the well-defined property of multiplication. Multiply both $2P$ and 1 in $2P = 1$ by $\frac{1}{2}$. * $\frac{1}{2}$ * $\frac{1}{2}$

399-C No

399-D The rate that the numeral 4 will occur per 6 chances * 1 chance, or 1 experiment

399-E $\{\frac{1}{6}\}$ * $P(4) = \frac{1}{6}$. * The symbol $P(4)$ represents the outcome that you are considering.

399-F Yes * 3/10

399-G The probability that Mrs. Andrews will pick a hard-boiled egg * 1 experiment, or 1 chance

399-H $\{\frac{3}{10}\}$ * Yes * $P(\text{hard-boiled}) = \frac{3}{10}$.

400-I For the experiment described in D1, the favorable outcome is Jim's taking the blue marble. For the experiment described in D2, the favorable outcome is the pointer stopping on the numeral 4. For the experiment described in D5, the favorable outcome is Mrs. Andrews' picking a hard-boiled egg.

400-J The number of favorable outcomes and the number of possible outcomes

400–K f/t. This rate pair represents the favorable outcomes per total outcomes.

400–L Use the definition of equivalent rate pairs. * Use the well-defined property of multiplication. Multiply both Pt and f in $Pt = f$ by $\frac{1}{t}$.

400–M $\frac{3}{5}$ * $\frac{1}{4}$

400–N Yes. The quotient of any positive real number and itself is 1.

400–O The probability is 0. The quotient of 0 and any positive real number is 0.

400–P No. The greatest number of favorable outcomes is the total number of possible outcomes. * No. For the probability to be greater than 1, there would have to be more favorable outcomes than possible outcomes.

400–Q No. The least possible number of favorable outcomes is 0. * The quotient of any non-negative real number and a positive real number is a non-negative real number.

On your own

400–1 6 * 2 * The probability that Mrs. Emerson will open a can of peas is $\frac{1}{3}$.

401–2 6 * 4 * The probability that Mrs. Emerson will open a can of green beans is $\frac{2}{3}$.

401–3 6 * 0 * The probability that Mrs. Emerson will open a can of corn is 0.

401–4 12 * 4 * The probability that each player has of getting a ticket is $\frac{1}{3}$.

401–5 There are 495 combinations of the 12 members of the team arranged 4 at a time. Therefore, there are 495 possible outcomes when you consider which 4 boys will receive the tickets. * There is only 1 favorable outcome of Cal, Ernie, Ted, and Russ receiving the tickets. * The probability that Cal, Ernie, Ted, and Russ will get the 4 tickets is $\frac{1}{495}$.

401–6 6 * 6 * The probability that you will get a piece of chocolate is 1.

401–7 7 * 3 * The probability that Mr. Levy will take a carton of black coffee is $\frac{3}{7}$.

401–8 7 * 4 * The probability that Mr. Levy will take a carton of coffee with cream is $\frac{4}{7}$.

401–9 An example is: The chess club in Hancock High School consists of 4 girls and 5 boys. The president is chosen by drawing a name from a hat. What is the probability of having a girl as president of the club?

401–10 An example is: There are 100 marbles in a bag. All but 1 are white. What is the probability of drawing a white marble from the bag without looking?

401–11 An example is: In a game of musical chairs, 15 children remain in the game. What is each child's probability that he will be eliminated from the game the next time the music stops? Only one chair is removed at a time.

401–12 An example is: Miss Drake is the only eighth-grade English teacher in Strathdale Junior High School. Jim signed up for eighth-grade English in Strathdale Junior High School. What is the probability that Jim will have Miss Drake for an English teacher?

401–13 An example is: A manufacturer of photoflash bulbs tests 2 out of every 1000 bulbs he produces. What is the probability that a given bulb will be tested?

Lesson 179

401–A 1, 2, 3, and 4 * 1, 2, 3, 4, and 5

402–B Yes

402–C There are 4 ways of drawing a blue tag. There are 5 ways of drawing a red tag. Therefore, there are 4 × 5, or 20, ways of drawing a blue tag and a red tag.

402–D Yes. The numeral 2 appears on a blue tag and the numeral 3 on a red tag. * Yes. The numeral 1 appears on a blue tag and the numeral 5 on a red tag. * No. The numeral 5 does not appear on a blue tag. * No. The numeral 7 does not appear on a red tag.

402–E 20

402–F (2,1), (2,2), (2,3), (2,4), and (2,5) * 2 * Yes

402–G 4 * It is the numeral 4.

402–H (1,1), (2,2), (3,3), and (4,4) * (1,1), (1,2), (1,3), (1,4), and (1,5) * (1,5), (2,5), (3,5), and (4,5)

403–I B = {1, 2, 3, 4}.

403–J Yes * Yes * No

403–K R = {1, 2, 3, 4, 5}.

403–L Yes * Yes * No

403–M Yes. A Cartesian set is a set of all ordered pairs that can be formed by matching each member of one set in turn with each member of a second set.

403–N Yes

403–A Yes

403–B Yes * Yes * Yes

403–C See the chart below. * Yes * 24

a \ h	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)
5	(5,1)	(5,2)	(5,3)	(5,4)
6	(6,1)	(6,2)	(6,3)	(6,4)

403–D A = {1, 2, 3, 4, 5, 6}.

403–E H = {1, 2, 3, 4}.

403–F A × H * You know that the sample space represented in the chart was obtained by matching each element in set A with each element in set H. The Cartesian set A × H is obtained in the same way.

403–G The members of a Cartesian set are ordered pairs of objects rather than single objects.

404–A A = {(1,1), (1,2), (1,3), (1,4), (1,5)}.

404–B Yes. Each member of A is also a member of B × R. * Yes. Each member of A satisfies $b = 1$.

404–C Each member of B × R that has 1 as the first component is a favorable outcome of the experiment. Each element of A has a first component of 1.

404–D 20 * Yes. B × R contains all the possible outcomes of the experiment and each member of B × R is an outcome of the experiment.

404–E 5

404–F The number of favorable outcomes and the number of possible outcomes

405–G The event that we are considering is A. * 5 * 20 * P(A) = $\frac{5}{20}$.

On your own

405–I See the chart below.

405–2 42

405–3 A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}. P(A) = $\frac{6}{42}$. The probability of event A is $\frac{1}{7}$.

405–4 B = {(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5)}. P(B) = $\frac{15}{42}$. The probability of event B is $\frac{5}{14}$.

405–5 C = {(6,7)}. P(C) = $\frac{1}{42}$. The probability of event C is $\frac{1}{42}$.

405–6 D = {(1,1)}. P(D) = $\frac{1}{42}$. The probability of event D is $\frac{1}{42}$.

405–7 {s_1, s_2, s_3, u_1, u_2}

406–8 The set of favorable outcomes is {s_1, s_2, s_3}. P(sharpened) = $\frac{3}{5}$. The probability of taking a sharpened pencil from the box is $\frac{3}{5}$.

406–9 The set of favorable outcomes is {u_1, u_2}. P(unsharpened) = $\frac{2}{5}$. The probability of taking an unsharpened pencil from the box is $\frac{2}{5}$.

406–10 The set of favorable outcomes is { }. P(pen) = $\frac{0}{5}$. The probability of taking a pen from the box is 0.

Chart for 405-1

r \ c	1	2	3	4	5	6	7
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)

406–11 $P(Q) = \frac{8}{n}$. $\frac{2}{3} = \frac{8}{n}$. There are 12 elements in the sample space.

406–12 $P(R) = \frac{1}{25}$. The probability of event R is $\frac{1}{25}$.

406–13 $P(S) = \frac{m}{4}$. $1 = \frac{m}{4}$. There are 4 elements in event S.

406–14 $P(T) = \frac{m}{40}$. $\frac{3}{8} = \frac{m}{40}$. There are 15 elements in event T.

Lesson 180

406–A The number named on a blue tag * The number named on a red tag

406–B $S = \{(b,r) \mid 1 \leq b \leq 4 \wedge 1 \leq r \leq 5\}$. The universe for b and r is I_p.

406–C 20 * 20 * $\frac{1}{20}$

406–D {(2,1), (2,2), (2,3), (2,4), (2,5)} * Yes

407–E {(1,3), (2,3), (3,3), (4,3)} * Event B is the set of all outcomes in which you draw a red tag with the numeral 3.

407–F Yes * Yes

407–G There are 5 elements in event A. Therefore, $P(A) = \frac{5}{20}$, or $\frac{1}{4}$. * There are 4 elements in event B. Therefore, $P(B) = \frac{4}{20}$, or $\frac{1}{5}$.

407–H Yes

407–I Just the one member (2,3) * Yes

407–J Yes. Each member of {(2,3)} is also a member of S. * The event {(2,3)} contains 1 member and S has 20 members. Therefore, $P\{(2,3)\} = \frac{1}{20}$. * $A \cap B$ is the same set as {(2,3)} and, therefore, the probability of $A \cap B$ is the same as the probability of {(2,3)}.

407–K Yes

407–L (2,3) * (2,1), (2,2), (2,4), and (2,5) * (1,3), (3,3), (4,3) * Yes

407–M 8 * $P(A \cup B) = \frac{8}{20}$, or $\frac{2}{5}$, because $A \cup B$ has 8 members and the sample space has 20 members.

408–N Yes * $\frac{9}{20}$

408–O $P(A \cap B)$ is equal to $\frac{1}{20}$. * Yes * $\frac{2}{5}$

408–P $P(A \cup B)$ is equal to $\frac{2}{5}$. * $P(A \cup B)$ is the same number as $P(A) + P(B) - P(A \cap B)$.

408–A {(1,3), (2,2), (3,1)} * Event X is the set of all outcomes in which the numbers named on a blue tag and a red tag have a sum of 4.

408–B {(1,4), (2,4), (3,4), (4,4)} * Event Y is the set of all outcomes in which the red tag drawn has the numeral 4.

408–C No * If one of the tags has the numeral 4 on it, then the numerals on a blue tag and a red tag will express two numbers whose sum is greater than 4.

408–D Yes

408–E No

408–F Set S contains 20 members. There are 3 members in X. Therefore, $P(X) = \frac{3}{20}$. * There are 4 members in Y. Therefore, $P(Y) = \frac{4}{20}$, or $\frac{1}{5}$.

408–G 0. X and Y are mutually exclusive events. * The empty set * Since $X \cap Y$ does not contain any members, $P(X \cap Y) = \frac{0}{20}$, or 0.

409–H (Column 1) For each M and N, if M and N are events in a finite sample space, then $P(M \cup N) = P(M) + P(N) - P(M \cap N)$. Therefore, $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$. * Yes. Since $X \cap Y$ is the empty set, $P(X \cap Y) = 0$.

409–I (Column 1) $\frac{3}{20} + \frac{1}{5} - 0$, or $\frac{7}{20}$ * $\frac{7}{20}$

409–J Yes * Yes

409–K {(1,1)} * {(4,5)}

409–L Yes. Sets J and K do not contain any common members. * Yes. Events that have no elements in common are mutually exclusive events.

409–M $\frac{1}{20}$ * $\frac{1}{20}$ * 0 * $\frac{1}{20} + \frac{1}{20} - 0$, or $\frac{1}{10}$

409–N Yes

409–A {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5)}

409–B {(4,1), (4,2), (4,3), (4,4), (4,5)} * No * Yes

409–C Yes * Since $\{(b,r) \mid b \geq 4\}$ contains all the members of set S that are not members of set D, and since no member of $\{(b,r) \mid b \geq 4\}$ is contained in D, $\{(b,r) \mid b \geq 4\}$ is the complement of D. The universe is set S.

409–D Yes

409–E Yes

409–F Yes * D and $\bar{D}$ are events of a finite sample space that have no elements in common.

409–G Each member of S is a member of either D or $\overline{D}$. Each member of D and each member of $\overline{D}$ is a member of S.

409–H (Column 2) Because each possible outcome is a member of S, each possible outcome is a favorable outcome.

409–I (Column 2) The event you are considering is the same as the sample space. $P(S) = \frac{20}{20}$. Therefore, $P(S) = 1$.

410–J Yes * Yes

410–K $D \cup \overline{D}$ is the same as the sample space S, and the probability of S is 1.

410–L For each M and N, if M and N are mutually exclusive events of a finite sample space, then $P(M \cup N) = P(M) + P(N)$.

410–M Use the well-defined property of addition. Add the additive inverse of $P(D)$ to both $P(D) + P(\overline{D})$ and 1 in $P(D) + P(\overline{D}) = 1$. * Yes

410–N Yes

410–O Yes. The union of two complementary sets is the universe. * The probability of an event that is the same as the sample space is 1.

410–P Yes. Since complementary events are mutually exclusive, the probability of their union is the same as the sum of their separate probabilities. Since the probability of their union is 1, the sum of their separate probabilities must be 1. * Subtract the probability of the given event from 1.

410–Q 1 * For each M and $\overline{M}$, if M and $\overline{M}$ are two complementary events in a finite sample space, then $P(\overline{M}) = 1 - P(M)$. * Yes

On your own

410–1 Yes

410–2 No * The sample points 6 and 12 are in both A and B.

411–3 A = {3, 6, 9, 12, 15}. $P(A) = \frac{5}{15}$, or $\frac{1}{3}$. B = {2, 4, 6, 8, 10, 12, 14}. $P(B) = \frac{7}{15}$. $A \cap B = \{6, 12\}$. $P(A \cap B) = \frac{2}{15}$. Therefore, $P(A \cup B) = \frac{1}{3} + \frac{7}{15} - \frac{2}{15}$, or $\frac{2}{3}$. The probability that A or B will occur is $\frac{2}{3}$. * The probability that A and B will occur is $\frac{2}{15}$.

411–4 Yes * B and C do not have any common members.

411–5 C = {3, 5, 7, 11, 13}. $P(C) = \frac{5}{15}$, or $\frac{1}{3}$. $B \cap C = \{\ \}$. Therefore, $P(B \cap C) = 0$. * Since B and C are mutually exclusive, $P(B \cup C) = P(B) + P(C)$. Therefore, $P(B \cup C) = \frac{7}{15} + \frac{5}{15}$, or $\frac{4}{5}$.

411–6 The event that B does not occur is the event in which the tag drawn has a numeral for an odd number. * Yes

411–7 $\frac{7}{15}$ * $P(\overline{B}) = 1 - P(B)$. $P(\overline{B}) = 1 - \frac{7}{15}$, or $\frac{8}{15}$. * 0 * 1

411–8 {(G,H), (G,I), (G,J), (H,I), (H,J), (I,J)}

411–9 {(G,H), (G,I), (G,J), (H,I), (I,J)}. $\frac{5}{6}$ * {G, I}. $\frac{1}{6}$

411–10 {(C,J,E), (C,J,B), (C,J,T), (C,E,B), (C,E,T), (C,B,T), (J,E,B), (J,E,T), (J,B,T), (E,B,T)}

411–11 {(C,J,E), (J,E,B), (J,E,T)}. $\frac{3}{10}$ * {(C,J,E), (C,J,B), (C,J,T), (C,E,B), (C,E,T), (J,E,B), (J,E,T), (J,B,T), (E,B,T)}. $\frac{9}{10}$. You can determine this more easily by reasoning that there is only one committee possible that does not contain either Ed or Jim. The probability of this one committee is $\frac{1}{10}$. Since $1 - \frac{1}{10} = \frac{9}{10}$, the probability that either Ed or Jim will be on the committee is $\frac{9}{10}$.

411–12 $\frac{6}{10}$, or $\frac{3}{5}$ * $\frac{4}{10}$, or $\frac{2}{5}$

411–13 $P(M \cup N) = P(M) + P(N) - P(M \cap N)$. P(red) = $\frac{10}{25} + \frac{10}{25} - \frac{4}{25}$, or $\frac{16}{25}$.

411–14 $P(\overline{M}) = 1 - P(M)$. P(both green) = $1 - \frac{16}{25}$, or $\frac{9}{25}$.

Lesson 181

411–A The first component is a number named on a blue tag and the second component is a number named on a red tag. * Yes

412–B The event that the numeral on the blue tag drawn is 3 and the numeral on the red tag drawn is 2 * The event that the numeral on the blue tag drawn is 1, 2, or 3

412–C $\{(b,r) \mid r > 5\} * \{\ \}$

412–D $\{(b,r) \mid b = 2\}$

412–E $\{(2,1), (2,2), (2,3), (2,4), (2,5)\}$ * There are 5 elements in K and 20 in S. Therefore, $P(K) = \frac{5}{20}$, or $\frac{1}{4}$.

412–F $\{(b,r) \mid r = 4\}$

412–G $\{(1,4), (2,4), (3,4), (4,4)\}$ * There are 4 members in L. Therefore, $P(L) = \frac{4}{20}$, or $\frac{1}{5}$.

412–H Yes

412–I $\{(2,4)\} * \frac{1}{20}$ * There is 1 member in K ∩ L and 20 members in S. Therefore, $P(K \cap L) = \frac{1}{20}$.

412–J Yes * Yes * Yes * Yes

412–K Yes

412–L Yes

412–M $\{(b,r) \mid b \leq 2\}$

412–N $\{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5)\}$ * There are 10 members in A. Therefore, $P(A) = \frac{10}{20}$, or $\frac{1}{2}$.

412–O $\{(b,r) \mid r \geq 4\}$

412–P $\{(1,4), (2,4), (3,4), (4,4), (1,5), (2,5), (3,5), (4,5)\}$ * There are 8 members in B. Therefore, $P(B) = \frac{8}{20}$, or $\frac{2}{5}$.

412–Q $\{(1,4), (1,5), (2,4), (2,5)\}$ * There are 4 members in A ∩ B. Therefore, $P(A \cap B) = \frac{4}{20}$, or $\frac{1}{5}$.

412–R Yes

412–S Yes

413–T $\{(2,1), (2,2), (2,3), (2,4), (2,5), (4,1), (4,2), (4,3), (4,4), (4,5)\}$ * $\{(1,1), (2,1), (3,1), (4,1), (1,3), (2,3), (3,3), (4,3), (1,5), (2,5), (3,5), (4,5)\}$

413–U $\frac{10}{20}$, or $\frac{1}{2} * \frac{12}{20}$, or $\frac{3}{5}$

413–V Find the product of the probability of R and the probability of T.

413–W $P(R \cap T) = P(R) \cdot P(T)$. $P(R \cap T) = \frac{1}{2} \cdot \frac{3}{5}$, or $\frac{3}{10}$.

413–A $\{(1,3), (2,3), (3,3), (4,3), (1,4), (2,4), (3,4), (4,4), (1,5), (2,5), (3,5), (4,5)\}$ * $\{(1,4), (2,3), (3,2), (4,1)\}$

413–B $\frac{12}{20}$, or $\frac{3}{5} * \frac{4}{20}$, or $\frac{1}{5}$

413–C $\frac{3}{5} \cdot \frac{1}{5}$, or $\frac{3}{25}$

413–D $\{(2,3), (1,4)\} * 2 * \frac{2}{20}$, or $\frac{1}{10}$

413–E Yes * No * P(G ∩ H) does not equal P(G) · P(H). Hence, they are not independent events.

413–F By definition, mutually exclusive events are disjoint sets. The intersection of any two disjoint sets is the empty set.

413–G The probability of the event that has no members is 0.

413–H No * No * Neither X nor Y is the empty set.

413–I If the product of two numbers is 0, then at least one of the numbers must be 0. Neither the probability of X nor the probability of Y is 0. * The occurrence of X affects the occurrence of Y. If X occurs, Y cannot occur, and if Y occurs, X cannot occur. $P(X \cap Y) = 0$, but $P(X) \cdot P(Y) \neq 0$.

413–J No. It is not true that $P(X \cap Y) = P(X) \cdot P(Y)$.

On your own

414–1 See the chart below.

414–2 $\frac{10}{90}$, or $\frac{1}{9} * \frac{9}{90}$, or $\frac{1}{10}$ * Yes * The probability that the first number is 3 and the second number is 5 is $\frac{1}{90}$. $\frac{1}{90}$ is the product of $\frac{1}{9}$ and $\frac{1}{10}$.

414–3 $\frac{6}{90}$, or $\frac{1}{15} * \frac{18}{90}$, or $\frac{1}{5}$ * No * The probability of the intersection of the two events is $\frac{2}{90}$, or $\frac{1}{45}$. $\frac{1}{45}$ is not the product of $\frac{1}{15}$ and $\frac{1}{5}$.

414–4 $\frac{9}{90}$, or $\frac{1}{10} * \frac{9}{90}$, or $\frac{1}{10}$ * No * The probability of the intersection of the two events is $\frac{1}{90}$. $\frac{1}{90}$ is

Chart for 414–1

f \ s	0	1	2	3	4	5	6	7	8	9
1	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(1,9)
2	(2,0)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)	(2,9)
3	(3,0)	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)	(3,9)
4	(4,0)	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)	(4,9)
5	(5,0)	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,8)	(5,9)
6	(6,0)	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)	(6,7)	(6,8)	(6,9)
7	(7,0)	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,8)	(7,9)
8	(8,0)	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)	(8,8)	(8,9)
9	(9,0)	(9,1)	(9,2)	(9,3)	(9,4)	(9,5)	(9,6)	(9,7)	(9,8)	(9,9)

not the product of $\frac{1}{10}$ and $\frac{1}{10}$.

414–5 $\frac{9}{90}$, or $\frac{1}{10} * \frac{9}{90}$, or $\frac{1}{10} *$ No * The probability of the intersection of the two events is $\frac{1}{90}$. $\frac{1}{90}$ is not the product of $\frac{1}{10}$ and $\frac{1}{10}$.

414–6 $\frac{7}{90} * \frac{9}{90}$, or $\frac{1}{10} *$ The two events are mutually exclusive.

414–7 Yes * The event {3} has one member and {1, 2, 3, 4, 5} has five members. Therefore, P(third floor) = $\frac{1}{5}$. * {1, 2, 3, 4, 5} * $\frac{1}{5}$

414–8 $\frac{1}{5} \times \frac{1}{5}$, or $\frac{1}{25}$

414–9 $\frac{1}{5} \times \frac{1}{5}$, or $\frac{1}{25}$

414–10 $\frac{2}{5} \times \frac{2}{5}$, or $\frac{4}{25}$

414–11 $\frac{1}{7} * \frac{2}{7}$

414–12 $\frac{1}{7} \times \frac{2}{7}$, or $\frac{2}{49}$

414–13 $\frac{4}{7} \times \frac{3}{7}$, or $\frac{12}{49}$

Keeping skilful

415–1 $27\frac{3}{16}$

415–2 130.95

415–3 $-1\frac{5}{22}$

415–4 30.24

415–5 -211.6

415–6 $689\frac{2}{3}$

415–7 -490

415–8 107.5

415–9 $16\frac{11}{20}$

415–10 87.52

415–11 $178\frac{1}{4}$

415–12 24.7

415–13 -1072.45

415–14 288.38

415–15 $996\frac{13}{15}$

415–16 $2\frac{7}{19}$

415–17 -19.562

415–18 814

415–19 1.23

415–20 $88\frac{5}{12}$

415–21 146.15

415–22 1528

415–23 $\{y \mid 6.5 < y < 20.9\}$

415–24 $\{g \mid g \geq 20\frac{1}{5}\}$

415–25 {2}

415–26 $\{t \mid t \leq .25\}$

415–27 $\{m \mid m > 16\}$

415–28 $\{-4\frac{1}{2}, 4\frac{1}{3}\}$

415–29 { }

415–30 5040

415–31 120

415–32 362,880

415–33 39,916,800

415–34 720

415–35 30,240

415–36 15

415–37 56

415–38 $2^3 \times 3^2$

415–39 $2^1 \times 3^1 \times 5^2$

415–40 $2^6 \times 5^1$

415–41 $2^5 \times 3^3$

415–42 $3^2 \times 7^2$

415–43 $2^2 \times 3^1 \times 7^2$

415–44 $2^1 \times 5^1 \times 13^2$

415–45 $2^3 \times 5^3 \times 7^1$

415–46 4 * 40

415–47 6 * 248

415–48 9 * 217

415–49 8 * 270

415–50 15 * 403

415–51 12 * 1368

415–52 16 * 1080

415–53 18 * 728

Lesson 182

415–A 2

415–B No * Because a tack is not well balanced, it does not seem that the two outcomes of the experiment are equally likely.

416–C Yes * Point down * Point down * Point up * Point down

416–D Once * 3 times

416–E 4 times * Yes

416–F $\frac{27}{100}$

416–G 251 times

416–H $\frac{25,002}{100,000}$, or .25002 * Yes

416–I About .25 x .25, or about .0625

416–J Use x as a variable for the number of times U will occur in the next 100 tosses. You can use the condition $.25/1 \sim x/100$ to determine the number of times U will occur in the next 100 tosses. 25 satisfies $.25/1 \sim x/100$; therefore, it is likely that U will occur 25 times in the next 100 tosses.

417–A 971,804

417–B 951,483 * 677,771 * 181,765

417–C Out of the original 1,000,000 persons, 181,765 were still living at the age of 80 years. * .18

417–D 1,000,000 − 677,771, or 322,229 * .32

417–E 883,342 * 810,900

417–F Yes

417–G For every 883,342 persons of age 40, 883342 − 677771 will die by age 60. Therefore, the probability that Mr. Colfax will die by age 60 is $\frac{883342 - 677771}{883342}$, or $\frac{205571}{883342}$.

On your own

417–1 The event has occurred 75 times in 135 trials. The probability is $\frac{5}{9}$.

417–2 The event occurred twice in 1000 trials. The probability that a stapler picked from this lot is defective is .002. * $.002/1 \sim x/1500$. Three staplers can be expected to be defective.

417–3 The quotient of the number of hits he has made and the number of official times at bat is the relative frequency of the player's hits, which is an approximation of the probability that he will obtain a hit in one time at bat.

417–4 .320

417–5 .320 × .320, or .1024

418–6 The event occurred 975 times in 12,500 trials. The probability is .078.

418–7 .01 × .008, or .00008

418–8 .01 + .008 − .00008, or .01792

Applying mathematics

418–1 $P(M) = \frac{m}{n}$. $P(M) = \frac{2}{9}$. $\frac{2}{9}$

418–2 $P(M) = \frac{m}{n}$. $P(M) = \frac{3}{10}$. $\frac{3}{10}$

418–3 $P(M) = \frac{m}{n}$. $P(M) = \frac{2}{10}$. $\frac{1}{5}$

418–4 $P(M) = \frac{m}{n}$. $P(M) = \frac{0}{10}$. 0

418–5 $P(M) = \frac{m}{n}$. $P(M) = \frac{1}{5}$. $\frac{1}{5}$

Assume that the probability of getting each of the 5 kinds of flower seeds is equally likely.

418–6 $P(M) = \frac{m}{n}$. $P(M) = \frac{1}{6}$. $\frac{1}{6}$

Assume that the probability of getting each of the 6 kinds of vegetable seeds is equally likely.

418–7 $P(M \cap N) = P(M) \cdot P(N)$.
$P(M \cap N) = \frac{1}{5} \times \frac{1}{6}$. $\frac{1}{30}$

418–8 $P(M \cup N) = P(M) + P(N) - P(M \cap N)$.
$P(M \cup N) = \frac{1}{5} + \frac{1}{6} - \frac{1}{30}$. $\frac{1}{3}$

418–9 $P(M) = \frac{m}{n}$. $P(M) = \frac{42}{56}$. $\frac{3}{4}$

418–10 $P(M) = \frac{m}{n}$. $P(M) = \frac{5}{25}$. $\frac{1}{5}$

418–11 The probability that Jack will draw a red pencil is $\frac{12}{25}$. The probability that Jack will draw a green pencil is $\frac{8}{25}$. $P(M \cup N) = P(M) + P(N)$. $P(M \cup N) = \frac{12}{25} + \frac{8}{25}$. $\frac{4}{5}$

418–12 The probability that Jack will draw a green pencil from the box the first time is $\frac{8}{25}$. The probability that Jack will draw a green pencil from the box the second time is $\frac{8}{25}$. $P(M \cap N) = P(M) \cdot P(N)$. $P(M \cap N) = \frac{8}{25} \times \frac{8}{25}$. $\frac{64}{625}$

418–13 $P(M) = \frac{m}{n}$. $P(M) = \frac{65}{400}$. $\frac{13}{80}$

418–14 $P(M \cap N) = P(M) \cdot P(N)$.
$P(M \cap N) = \frac{13}{80} \times \frac{13}{80}$. $\frac{169}{6400}$

Checking up

Test 166

420–1 $\frac{3}{4}$

420–2 $\frac{3}{16}$

420–3 6

420–4 W x Y = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)}.

420–5 $\frac{6}{18}$, or $\frac{1}{3}$

420–6 $\frac{9}{18}$, or $\frac{1}{2}$

420–7 $\frac{3}{18}$, or $\frac{1}{6}$ * Yes

420–8 $\frac{12}{18}$, or $\frac{2}{3}$ * No

Lesson 183

420–A 1900 * 1960 * 6

420–B Each of these numerals expresses the world's population to the nearer 500,000,000. * At the present time it is impossible to obtain an exact number for the world's population.

420–C 1,500,000,000 * In 1920, it was 1,750,000,000. In 1930, it was 2,000,000,000. In 1940, it was 2,400,000,000. In 1950, it was 2,500,000,000. In 1960, it was 3,000,000,000.

421–D No * Yes

421–E From 1950 to 1960 * From 1900 to 1910

421–F 500,000,000

421–G 500,000,000

421–H They are the same.

421–I The statement that "The world's population will probably be greater in 1970 than in 1960" seems more reasonable.

421–J 100%

421–K 4,010,000

421–L Yes

421–M Yes

421–N 17.2%

422–O Yes

422–P 27° F. * 59° F. * 67° F.

422–Q July * January

423–A Yes

423–B Record D * Record C

423–C Record B * Record E * Record D

423–D 64 * By finding the total number of red disks drawn by the 4 students

423–E The three other numbers were obtained by finding the sums of the numbers of disks of each color that were drawn by the four students.

423–F 200 * 200

423–G Yes

423–H Yes

423–I There were probably fewer

green disks than red, blue, or white disks in the bag.

423–J From the data given in D7, you know that the total number of disks drawn was 200. You can use a condition for equivalence to find the probability of drawing a disk of a certain color in one drawing. For example, the probability of drawing a red disk in one drawing can be found by using the condition $64/200 \sim x/1$. The solution of this condition is .32. If 400 more drawings were to be made, you can use the condition $.32/1 \sim x/400$ to predict the number of times a red disk would be drawn. Thus, you could predict that 128 red disks would be drawn in the next 400 drawings. Using a similar method for each of the other colors, you could predict that 76 blue disks, 148 or 152 white disks, and 44 or 48 green disks would be drawn in the next 400 drawings.

424–K 46

424–L Program B * Program D

424–M A histogram pictures rectangles rather than rectangular regions and the pictures of the rectangles touch each other.

424–N Yes * Yes * Yes

424–O He might use a histogram because it is very easy to tell at a glance that more viewers watch program B than any of the other programs. With a histogram, it would not be necessary to compare the number of viewers of each program as it is with a frequency array.

On your own

424–1 100%

424–2 Rent

424–3 Clothing

424–4 Some of the expenses might be for taxes, insurance, entertainment, telephone bills, doctors' bills, furniture, and recreation.

424–5 They spend $140 for rent, $115 for food, $80 for clothing, $50 for transportation, $80 for other expenses, and they save $35.

425–6 The attendance at the Star Theater each day during a certain week

425–7 Saturday * Monday

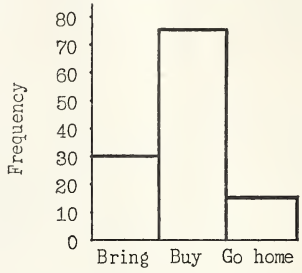
425–8 Wednesday * Tuesday

425–9 He will probably need more popcorn on the days when the attendance is high than on the days when the attendance is low.

425–10

How students obtain lunch	Number of students
Bring from home	30
Buy in cafeteria	75
Go home	15

425–11 HOW STUDENTS OBTAIN LUNCH

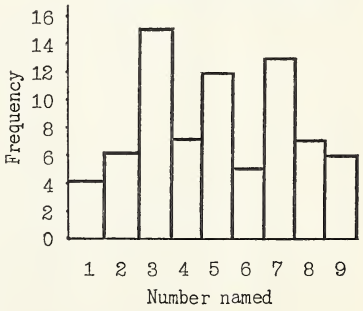


425–12

Number	Number of times named
1	4
2	6
3	15
4	7
5	12
6	5
7	13
8	7
9	6

425–13

PROBABILITY EXPERIMENT



Keeping skilful

425–1 $\{(\bar{}106\frac{1}{4}, 21\frac{1}{4})\}$

425–2 $\{(11.8, 14.3)\}$

425–3 $\{(20, 16)\}$

425–4 $\{(\bar{}8.3, \bar{}.06)\}$

425–5 $\{(2, 3)\}$

425–6 $\{(24, 3)\}$

425–7 $\{(12.3, 2.46)\}$

425–8 $\{(\bar{}6, \bar{}6)\}$

425–9 {(28.75, $^{-}$24.75)}
425–10 {(135, 137)}
425–11 {($^{-}11\frac{2}{3}$, $^{-}16\frac{2}{3}$)}

Lesson 184

426–A Wednesday * Monday * 59
426–B The number that is the average * The number that is the total of the measures * The number of measures whose sum is the total
426–C 5 * Find the sum of 485, 538, 544, 535, and 492.
426–D 2594
426–E 518.8
426–F The average daily attendance was 518.8.
427–G 94 * 78 * 16
427–H x two * x one hundred * x n
427–I Yes * $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$
427–J Yes
427–K The number that is the total of the measures * Yes
427–L a was replaced by $\bar{x}$ and t was replaced by $x_1 + x_2 + x_3 + \ldots + x_n$.
427–M Since 87 is the first measure given in the problem, x_1 was replaced by 87; since 92 is the second measure, x_2 was replaced by 92; since 78 is the third measure, x_3 was replaced by 78; since 90 is the fourth measure, x_4 was replaced by 90; since 86 is the fifth measure, x_5 was replaced by 86; since 94 is the last measure, x_n was replaced by 94. Since there are 6 measures, n was replaced by 6.
427–N Find the standard name of 87 + 92 + 78 + 90 + 86 + 94. * $87\frac{5}{6}$ * Yes
428–A $\bar{x} = \frac{x_1+x_2+x_3+\ldots+x_n}{n}$.

$\bar{x} = \frac{405 + 461 + 490}{3}$.

$\bar{x} = \frac{1356}{3}$. $\bar{x} = 452$. The mean number of persons attending the play per evening was 452.

428–B Yes * 461 − 452, or 9 * 490 − 452, or 38
428–C $^{-}47 + 9 + 38$, or 0
428–D $\bar{x} = \frac{5\frac{2}{3}+3\frac{3}{4}+8\frac{1}{2}+7\frac{2}{3}+6\frac{1}{2}}{5}$.

$6\frac{5}{12} * 5\frac{2}{3} - 6\frac{5}{12} = {}^{-}\frac{3}{4}$.
$3\frac{3}{4} - 6\frac{5}{12} = {}^{-}2\frac{2}{3}$.
$8\frac{1}{2} - 6\frac{5}{12} = 2\frac{1}{12}$.
$7\frac{2}{3} - 6\frac{5}{12} = 1\frac{1}{4}$.
$6\frac{1}{2} - 6\frac{5}{12} = \frac{1}{12}$.
$^{-}\frac{3}{4} + {}^{-}2\frac{2}{3} + 2\frac{1}{12} + 1\frac{1}{4} + \frac{1}{12} = 0$.

428–E $\bar{x} = \frac{610+552+484+729+692+477}{6}$

$590\frac{2}{3} * 610 - 590\frac{2}{3} = 19\frac{1}{3}$.
$552 - 590\frac{2}{3} = {}^{-}38\frac{2}{3}$.
$484 - 590\frac{2}{3} = {}^{-}106\frac{2}{3}$.
$729 - 590\frac{2}{3} = 138\frac{1}{3}$.
$692 - 590\frac{2}{3} = 101\frac{1}{3}$.
$477 - 590\frac{2}{3} = {}^{-}113\frac{2}{3}$.
$19\frac{1}{3} + {}^{-}38\frac{2}{3} + {}^{-}106\frac{2}{3} + 138\frac{1}{3} + 101\frac{1}{3} + {}^{-}113\frac{2}{3} = 0$.

428–F 23 students * The middle score would be the score of the twelfth student because 11 students have either the same score or a lower score and 11 students have either the same score or a higher score.
428–G If you consider the students whose scores were 75 as the first and second students, the student whose score was 78 as the third student, and the students whose scores were 83 as the fourth through ninth students, then you find that the 12th student is one of the students whose score was 85.
428–H Yes * The sum of the first two measures
428–I Yes * 1980
429–J $86\frac{2}{23}$
429–K 4.1
429–L 6.36 * 6.7

On your own

429–1 6
429–2 32
429–3 $2\frac{1}{32}$
429–4 2
429–5 3400 * 3880 * 3500
429–6 5.6 * 4.94 * 4.6
429–7 $2\frac{23}{24}$ * $7\frac{11}{40}$ * Between $7\frac{1}{2}$ and $7\frac{2}{3}$
429–8 16 * $24\frac{3}{8}$ * 25
429–9 .021 * .023 to the nearer thousandth * .024
429–10 26 * $177\frac{7}{11}$ * 176

Applying mathematics

429–1 x, amount of money Roger has in all.
$x = 130.95 + 16.42 + 1.90$.
{149.27}. $149.27
429–2 x, number of dollars that Mr. Martin spent for each dollar that Mr. Nelson spent.
$145.35/116.28 \sim x/1$.
{1.25}. 1.25 times
429–3 x, number of hours Ken worked on Friday.
y, number of hours Ken worked on Saturday.
$x + y = 10 \wedge y + \frac{3}{4} = 6$.

{$(4\frac{3}{4},5\frac{1}{4})$}. $4\frac{3}{4}$ hr. on Friday and $5\frac{1}{4}$ hr. on Saturday

429–4 Use the formula $p = br$.
$p = 875(.008)$.
{7}. 7 students

429–5 x, number of pounds of hickory nuts Mrs. Wilson used.
$2\frac{1}{2} - x > 2\frac{1}{8} \wedge x > 0$.
$\{x \mid 0 < x < \frac{3}{8}\}$.
Between 0 and $\frac{3}{8}$ lb.

429–6 x, number of jackets Mr. Craft sold during April.
$152 + x = 321$.
{169}. 169 jackets

429–7 x, number of degrees that was the temperature in Toronto at 6:00 P.M.
$x > \bar{}10 \wedge x - 4 < \bar{}6$.
$\{x \mid \bar{}10 < x < \bar{}2\}$.
Between 2 degrees below zero and 10 degrees below zero

430–8 x, number of ounces of mints Beth had before she bought the $10\frac{1}{2}$-oz. package.
$x > 8 \wedge x + 10\frac{1}{2} < 24$.
$\{x \mid 8 < x < 18\frac{1}{2}\}$.
Between 8 and $18\frac{1}{2}$ oz.

430–9 x, number of packages of envelopes the company should send.
$100/1 \sim 3300/x \vee 275/1 \sim 3300/x$.
{33, 12}. Either 33 packages of 100 or 12 packages of 275

430–10 x, amount Betty earned.
y, amount Alice earned.
$\frac{4}{5}/1 \sim 12.80/x \wedge x + y = 25.40$.
{(16,9.40)}. Betty earned \$16.00 and Alice earned \$9.40.

430–11 x, number of quarts of wax Mrs. Andrews used.
$3 - x \leq 1\frac{1}{8} \wedge x < 3$.
$\{x \mid 1\frac{7}{8} \leq x < 3\}$.
At least $1\frac{7}{8}$ qt. and less than 3 qt.

430–12 x, amount the school spent for the tennis rackets.
y, amount the school spent for the tennis balls.
z, amount the school spent in all for the tennis rackets and the tennis balls.
$9.95/1 \sim x/15 \wedge 2.25/3 \sim y/18 \wedge x + y = z$.
{(149.25,13.50,162.75)}.
\$162.75

430–13 x, number of pounds of hamburger.
y, number of pounds of bacon.
z, number of pounds of pork chops.
$x + y + z = 9\frac{3}{4} \wedge 2/1 \sim x/y \wedge y + z = 5\frac{3}{4}$.
{(4,2,$3\frac{3}{4}$)}. 4 lb. of hamburger, 2 lb. of bacon, and $3\frac{3}{4}$ lb. of pork chops.

430–14 $x - 16\frac{8}{9} \geq 20\frac{1}{3}$.
$\{x \mid x \geq 37\frac{2}{9}\}$.

430–15 x, first number.
y, second number.
z, third number.
$3/1 \sim x/y \wedge y - 4 = 2.5 \wedge z = x + 9.5$.
{(19.5,6.5,29)}

430–16 x, first number.
y, second number.
$\sim(x - y > \bar{}6) \wedge y + 2 = 10$.
$\{(x,y) \mid x \leq 2 \wedge y = 8\}$

Checking up

Test 167

430–1 sample point

430–2 and

430–3 mutually exclusive

430–4 empirical

430–5 arithmetic mean

430–6 probability

430–7 range

430–8 relative frequency

430–9 independent

430–10 one

Test 168

430–11 See the chart below.

f \ l	1	2	3	4
0	(0,1)	(0,2)	(0,3)	(0,4)
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

431–12 $\frac{4}{20}$, or $\frac{1}{5}$

431–13 $\frac{4}{20}$, or $\frac{1}{5}$ * $\frac{5}{20}$, or $\frac{1}{4}$ * Yes

431–14 $\frac{4}{20}$, or $\frac{1}{5}$ * $\frac{3}{20}$ * Yes

431–15 $\frac{8}{20}$, or $\frac{2}{5}$ * $\frac{6}{20}$, or $\frac{3}{10}$ * No

431–16 1.6

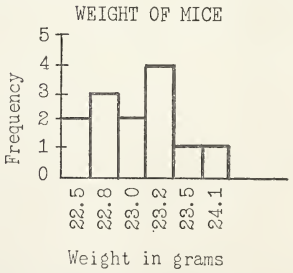
431–17

Weight	Frequency
22.5	2
22.8	3
23.0	2
23.2	4
23.5	1
24.1	1
	13

431–18 23.06 to the nearer hundredth

431–19 23.0

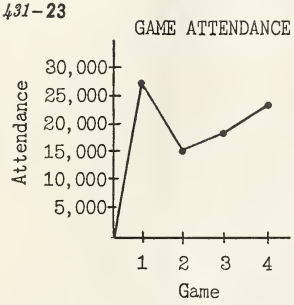
431–20



431–21 $\frac{4}{5}$

431–22 One

431–23



431–24 .935

Test 169

431–1 R, R_p, I, and I_p

431–2 R, R_n, I, and I_n

431–3 $\overline{R}$

431–4 R and R_p

431–5 $\overline{R}$

431–6 $\overline{R}$

Test 170

432–7 T

432–8 F

432–9 F

432–10 T

432–11 T

432–12 T

Test 171

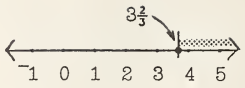
432–13 $13\frac{7}{8}$

432–14 secant

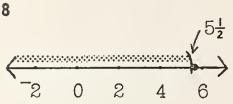
432–15 23^{11}

432–16 3

432–17



432–18



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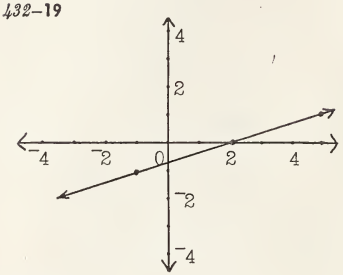
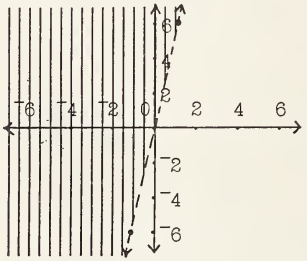
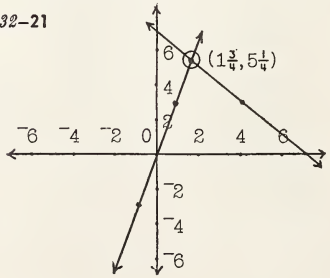


432–20



432–21



Test 172a

432–22 Use the formula $p = br$.
$1 = 17\frac{1}{3}r$. $\{\frac{3}{5\frac{2}{}}\}$. 5.77%

432–23 Use the formulas $i = prt$ and $a = p + i$.
$i = 473.50(.04)(1) \wedge$
$a = 473.50 + i$.
$\{(18.94, 492.44)\}$ for (i,a).
$492.44

432–24 x, number of quarts of antifreeze.
y, number of quarts of water.
$x/y \sim 2/5 \wedge x + y = 16$.
$\{(4\frac{4}{7}, 11\frac{3}{7})\}$. $4\frac{4}{7}$ qt. of antifreeze and $11\frac{3}{7}$ qt. of water

432–25 $x \div \bar{}3 = 7\frac{1}{4}$. $\{\bar{}21\frac{3}{4}\}$

432–26 x, first number.
y, second number.
$x = \bar{}.87y \wedge y = 4.1 - 6.9$.
$\{(2.436, \bar{}2.8)\}$

432–27 x, first number.
y, second number.
z, third number.
$x + y + z = \bar{}13\frac{1}{2} \wedge$
$y = 3x \wedge z = \frac{1}{2}x$.
$\{(\bar{}3, \bar{}9, \bar{}1\frac{1}{2})\}$

Test 172b

432–28 complementary

432–29 third and fourth

432–30 $\frac{\bar{}8}{8}$

432–31 10.5 sq. in.

432–32 event

432–33 complementary

432–34 $\frac{1}{6}$

432–35 36

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